ENGINEERING fundamentals

ADVANCED RESERVOIR ENGINEERING

(Up-to-date Treatment of Fundamentals)

PROBLEMI CONNESSI CON LA MESSA IN PRODUZIONE DI GIACIMENTI A OLIO E GAS.

PART 1

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FOREWORD

The treatment of reservoir engineering contained herein covers the most advanced developments and findings made by engineers and scientists working on the many subsurface problems concerned with oil and gas. This material, as a whole, is available from no other source.

All subjects, or divisions thereof, for convenience of the petroleum-engineer reader, are presented in single-page units. In many of the discussions a "problem and answer" treatment is used to emphasize the practical handling—and to facilitate solution of specific problems by the user.

The articles start with the first installment (No. 300) of The Oil and Gas Journal's regular weekly feature, "Engineering Fundamentals," as prepared by Dr. John C. Calhoun, Jr., Chairman, Petroleum Engineering School, University of Oklahoma.

INDEX OF ARTICLES

No. 300

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Reservoir Fluids: Behavior of Gases

On the basis of the kinetic theory of gases, one can arrive at a law giving the relationship existing between pressure, volume, and temperature for a given mass of a gas. This relationship, called the Perfect Gas Law, mathematically is:

$$PV = nRT \qquad (1)$$

where

P = absolute pressure on the gas
V = volume which it occupies
T = absolute temperature of gas
n = the number of mols of gas, equal to its mass divided by its gaseous molecular weight, and
R = the gas constant

From this relationship a volume can be calculated if number of mols, pressure, and temperature are known. Units in the above relationship must, of course, be consistent. Usually the consistency in units is arranged with the gas constant, R, which is numerically the same for all gases. Values of R for the pressure, temperature, volume, and mols in certain units are:

P	V	T	n	R
Atm.	cc.	°Absolute Centigrade Scale	gram-mols	82.05
Atm.	cu.ft.	°Rankine	lb.-mols	0.729
Psi.	cu.ft.	°Rankine	lb.-mols	10.71
(*)	cu.ft.	°Rankine	lb.-mols	1,544

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*Pounds per square foot.

Perhaps a word of clarification is necessary concerning the definition of mols. As stated above, the number of mols of gas is equal to its weight divided by its gaseous molecular weight. Since weight is given in either pounds or grams, depending upon the system of units used, two different molar quantities are possible, either pound-mols or gram-mols. For example, methane, CH_4, has a molecular weight of 16. Therefore, 32 lb. of methane is equal to 2 pound-mols, or 32 grams of methane is equal to 2 gram-mols.

The perfect gas law reduces to Boyle's Law,

$$P_1V_1 = P_2V \qquad (2)$$

when the temperature is constant, and to Charles' Law,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad (3)$$

when the pressure is constant.

The perfect gas law states that one mol of any perfect gas will occupy the same volume at a given pressure and temperature. This is a convenient relationship and we thus utilize 359 cu. ft. as the volume occupied by 1 lb.-mol of any perfect gas at one atmosphere and 32° F., and 379 cu. ft. as the volume occupied by 1 lb.-mol of any perfect gas at one atmosphere and 60° F. This relationship allows the engineer to convert rapidly from standard cubic feet to pound-mols or vice versa.

Density of any substance is defined as mass per unit volume of the substance. Thus, density of a perfect gas at a given temperature and pressure can be arrived at by rearranging the perfect gas law as follows:

$$\text{density} = \frac{m}{V} = \frac{P \times M}{RT} \qquad (4)$$

where

m = the mass of the gas under consideration and
M = molecular weight of the gas

This states that density varies directly with pressure at a given temperature and inversely with temperature at a given pressure.

Since density does vary with temperature and pressure, it is not a very convenient physical quantity to use to characterize a gas. A more convenient term is specific gravity, which is defined generally as the ratio of the density of a substance to the density of a standard substance. For gases, air is usually used as the standard. The density of air is also variant with pressure and temperature, so both must be specified. Then it is most convenient to define the specific gravity of a gas as the ratio of its density at a given temperature and pressure to the density of air at the same temperature and pressure. Thus, specific gravity will be independent of pressure and temperature.

From the perfect gas law, the density of air is

$$d_{air} = \frac{P \times 28.96}{RT} \qquad (5)$$

where 28.96 is the apparent molecular weight of air. Dividing this into Equation 4 for gas density, gives for the specific gravity,

$$S.G. = \frac{d_{gas}}{d_{air}} = \frac{M}{28.96} \qquad (6)$$

This equation states that specific gravity as defined above can be calculated from the molecular weight of the gas alone, and is not dependent upon pressure and temperature.

Example 1: What is the density of propane gas, formula C_3H_8, in pounds per cubic foot at 100° F. and 30 psig.?

Solution: Equation 4 is used. Since density is required in pounds per cubic foot and the pressure is in pounds per square inch, the value of R to use will be 10.71. The molecular weight is found from the formula C_3H_8. Carbon has an atomic weight of 12 and hydrogen of 1. Therefore, $(3 \times 12) + (8 \times 1) = 44$, the molecular weight. Therefore,

$$\text{Density} = \frac{44.7 \times 44}{10.71 \times 560}$$

$$= .328 \text{ lb. per cu. ft. at 30 psig. and } 100° \text{ F.}$$

Example 2: What is the specific gravity of ethane gas, formula C_2H_6?

Solution: The molecular weight is found as above from the atomic weight of carbon = 12 and hydrogen = 1. Thus, $M = (2 \times 12) + (6 \times 1) = 30$. The specific gravity is then found from Equation 6.

$$S.G. = \frac{30}{28.96} = 1.035$$

which is the value for specific gravity of ethane at any temperatures and pressures at which it can be considered a perfect gas.

Example 3: What will be the volume in cubic feet occupied by 1,000 lb. of air at 3 atmospheres and 150° F.?

Solution: Using Equation 1, and taken $R = 0.729$ since pressure is given in atmospheres,

$$3 \times V = \frac{1{,}000}{28.96} \times 0.729 \times 6.0$$

$$V = 5{,}110 \text{ cu. ft.}$$

Nomenclature: P = Pressure; V = Volume; n = Mols; R = Gas Constant; T = Temperature; m = Mass; M = Mol Weight; d = Density; S.G. = Specific Gravity.

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ENGINEERING fundamentals

No. 301

Standard Calculations for Gases

GAS compositions are expressed in terms of mol per cent or fraction, volume per cent, and weight per cent. The mol fraction of any component in a mixture of gases is equivalent to the number of mols of that particular component divided by the total number of mols of all the components. The mol per cent of a component is equivalent to 100 times the mol fraction. Similarly, the weight per cent of a component in a mixture of gases is 100 times the ratio of the weight of that particular component to the total weight of all the components.

For gases which obey the perfect gas law, the volume per cent of a given component in a mixture is equivalent to the mol per cent of that component. This arises from the fact that molar quantities of perfect gases under a given pressure and temperature occupy identical volumes. It should be stressed that mol per cent equals volume per cent only for gases.

Conversion from molar composition or volume composition to weight composition is readily made by recalling the definitions of mol and density. If the mol fraction of a given component in a gaseous mixture is y, then the weight of that component is y × M, where M is the molecular weight of the component. By definition of weight per cent, therefore,

$$\%m = \frac{100 \times y \times M}{\Sigma\, y \times M} = \frac{100 \times \%n \times M}{\Sigma\, \%n \times M} \qquad (1)$$

where, $\Sigma\, y \times M$, or $\Sigma\, \%n \times M$, means the sum of the weights of the individual components.

Similarly, since density, $d = m/V$ one can convert from volume per cent to weight per cent by the following:

$$\%m = \frac{100 \times \%V \times d}{\Sigma\, \%V \times d} = \frac{100 \times \%V \times SG}{\Sigma\, \%V \times SG} \qquad (2)$$

where again $\Sigma\, V \times d$ or $\Sigma\, V \times SG$ means the sum of the weights of all components.

To convert from weight per cent to volume per cent or mol per cent, one can use the following relationship:

$$\%V = \%n = \frac{100 \times \%m/d}{\Sigma\, \%m/d} = \frac{100 \times \%m/M}{\Sigma\, \%m/M} \qquad (3)$$

where $\Sigma\, \%m/d$ is equivalent to the sum of the volume of all the components and $\Sigma\, \%m/M$ is equivalent to the sum of the mols of all the components.

An apparent molecular weight of a gaseous mixture can be calculated from its mol composition and utilized in the same fashion as is the true molecular weight of any gas. Molecular weight signifies the weight of one mol. In a mixture of gases the sum of the number of mols of the individual gases is taken as the number of mols of mixture. The weight of the mixture divided by this total number of mols gives the weight per mol, or apparent molecular weight. By formula, this is

$$M = \frac{1}{100}\, \Sigma\, \%n \times M \qquad (4)$$

The specific gravity of a gaseous mixture is the ratio of its apparent molecular weight to that of air. Therefore, for the mixture,

$$SG = \frac{M}{28.96} = \frac{\Sigma\, \%n \times M}{100 \times 28.96} \qquad (5)$$

Since the value of M for each component is equivalent to its SG times 28.96, the formula for computing the specific gravity of a mixture can be given in terms of the specific gravity of each component, instead of in terms of molecular weight, thus,

$$SG = \frac{\Sigma\, \%n \times SG}{100} \qquad (6)$$

Example: Gas from the Hill zone, Rodessa field, Caddo Parish, Louisiana, is reported in U.S.B.M., R.I. 3715, to have the components and per cent volume composition as given in Columns 1 and 2 of the table below. What is (a) the mol per cent, (b) the weight per cent of each component in the gas, (c) the apparent molecular weight of the gas, and (d) its specific gravity?

Solution: (a) For gases mol per cent is equal to volume per cent; therefore, the figures given in Column 2 for volume per cent are used in Column 3 for mol per cent.

(b) The weight per cent composition is computed according to Equation 1, the molecular weight of the individual components being found from the molecular formulas or from tables of data such as that given on Page 106 in the Technical Manual (Fifth Edition) of the Natural Gasoline Supply Men's Association. These molecular weights are listed in Column 4. Since no molecular weight is given for heptanes +, that for heptane alone is used. Entries in Column 5 give $\%n \times M$ for each component, and the total gives $\Sigma\, \%n \times M = 1{,}870.3$. Per cent by weight is then calculated in Column 6 by dividing each entry in Column 5 by the total of Column 5. A check on the validity of the weight per cent values is obtained by noting that their sum which should be 100 per cent is equal to 100.04.

(c) The apparent molecular weight of this gas is calculated from Equation 4.

$$M = \frac{1}{100}\, \Sigma\, \%n \times M = \frac{1{,}870.3}{100} = 18.70$$

(d) The specific gravity of the gas is found from Equation 5

$$SG = \frac{18.70}{28.96} = .645$$

New nomenclature.—%m = weight per cent composition; y = Mol fraction, in gas; %V = Volume per cent composition; %n = Mol per cent composition.

Given data		Solution			
Column 1	Column 2	Column 3	Column 4	Column 5	Column 6
Component	% by vol.	% by mols	Mol wt.	3 × 4	% by weight (100 × V ÷ 1870.3)
Methane	87.09	87.09	16.04	1,398	74.8
Ethane	4.42	4.42	30.07	133	7.12
Propane	1.60	1.60	44.09	70.5	3.76
Iso-Butane	0.40	0.40	58.12	23.2	1.24
Normal-butane	0.52	0.52	58.12	30.3	1.62
Pentanes	0.46	0.46	72.15	33.2	1.77
Hexanes	0.29	0.29	86.17	25.0	1.33
Heptanes +	0.06	0.06	100.20	6.0	0.32
Nitrogen	4.76	4.76	28.02	133.5	7.14
Carbon dioxide	0.40	0.40	44.00	17.6	0.94
Total	100.00	100.00		1,870.3	100.04

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No. 302

Pseudo-Critical Temperature Of Gases

THE Perfect Gas Law is the basic relationship for calculating the behavior of gases. Although a few actual gases conform to this law over a considerable range of pressure and temperature, most gases follow the perfect gas law only over limited ranges. The reasons for this are that the perfect gas law is derived by considering that the molecules of a gas do not occupy space and do not exert attraction between themselves, which is not the case for actual gases. For single-component gases a formula for calculating departures from ideality has been worked out by Van der Waals. His equation introduces two corrections, one for the size of gas molecules, and the other for the attraction between molecules.

A useful method to account for departure of gases from ideality consists merely of inserting a correction factor, Z, into the perfect gas law such that

$$PV = ZnRT \qquad (1)$$

This factor Z is a dimensionless quantity. It is nothing more than the number necessary to introduce in order to make observed pressures, volumes, and temperatures on a given mass of gas fit into an equation. This factor is called the compressibility or supercompressibility of a gas. It is a function of pressure, temperature, and the gas under consideration.

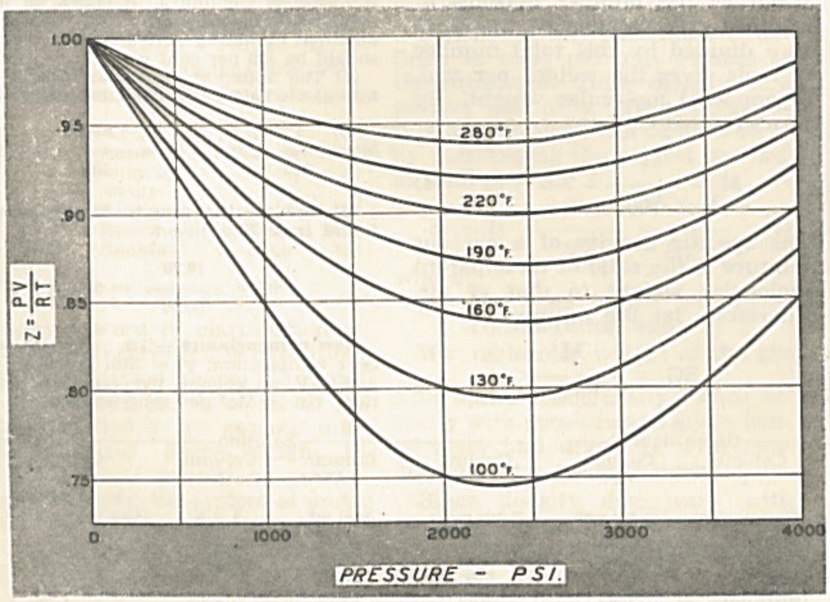

Fig. 1-

The pressure plotted above on the X or pressure axis is the "pseudo reduced pressure." (From Sage & Reamer, A.I.M.E.)

The value of Z to use for a given gas at a given temperature and pressure may be determined by experimentation, that is, by actually measuring all quantities in Equation 1 except Z. This is frequently done on reservoir gases and the results follow a very definite pattern. Fig. 1 illustrates the results of such measurements as reported for a trap-gas sample taken from Rio Bravo field, in California.* For this reservoir gas, for example, at 100° F. and 2,000 psi., Z = .75, and volumes would be in error by 33 per cent if calculated by the perfect gas law at this temperature and pressure.

Individual Experiments Not Necessary

Fortunately, it is not necessary to run individual experiments to construct a set of curves similar to Fig. 1 for every reservoir gas. Sufficient is known about the variation of compressibility with pressure and temperature to permit a correlation. This correlation is not made directly with temperature and pressure, but rather with a "reduced temperature" and a "reduced pressure."

To "reduce" an actual temperature or pressure means to divide that temperature or pressure by the critical temperature or critical pressure of the gas, whichever the case may be. In equation form this is usually written

$$P_R = \frac{P}{P_c} \text{ and } T_R = \frac{T}{T_c} \qquad (2)$$

where the pressure and temperature are in absolute units and the subscripts "c" and "R" refer to critical value and reduced value respectively. For example, ethane has a critical temperature of 90.1° F. or 550.1° R., and a critical pressure of 708 psia. Therefore, ethane at 1,000 psia. and 200° F. would have a value of reduced pressure,

$$P_R = \frac{1{,}000}{708} = 1.41$$

and a value of reduced temperature,

$$T_R = \frac{660}{550.1} = 1.20$$

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*Sage and Reamer, Volumetric Behavior of Oil and Gas From the Rio Bravo field, A.I.M.E., Transactions Vol. 142, Page 181.

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Engineering Fundamentals—(continued)

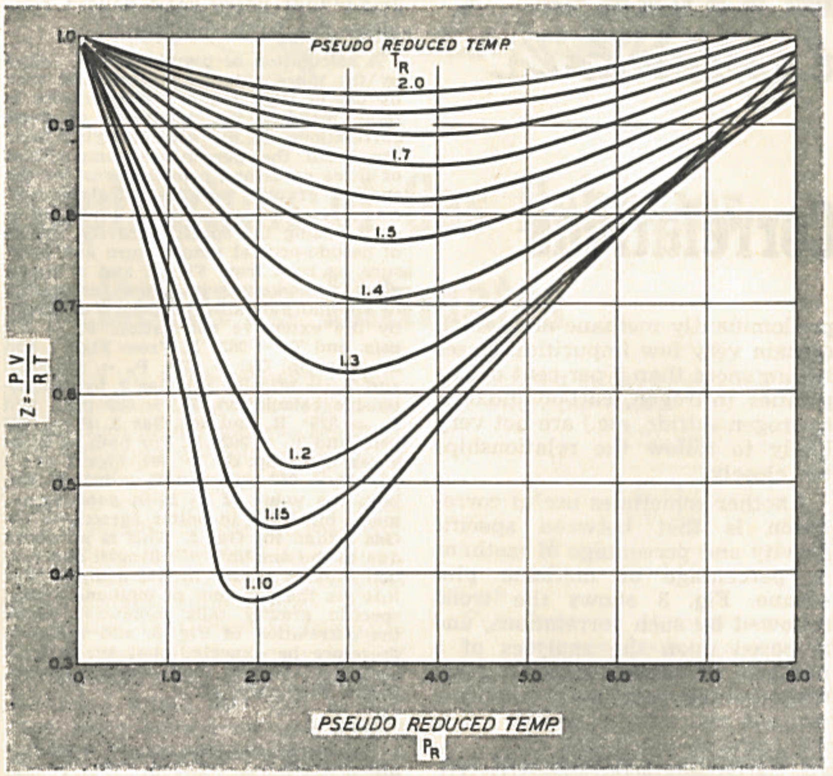

Fig. 2—Compressibility factors for gaseous mixtures containing methane and natural gas substantially free from nitrogen (Natural Gasoline Supply Men's Association)

Only single-component gases possess distinct values of critical temperature and pressure. For mixtures of gases, such as reservoir gases, there is a range of pressure over which a gas will liquefy at a given temperature, and a range of temperatures at which liquid may exist at a given pressure. There is, therefore, no single value for critical temperature and pressure for a mixture of gases. It then becomes necessary to define some quantities that can be used in place of critical temperature and pressure in order to reduce the pressure. These quantities are called "pseudo-critical temperature" and "pseudo-critical pressure."

Pseudo-critical values for mixtures of gases are calculated on the assumption that each component in the mixture contributes to the pseudo-critical value in proportion to the number of mols of the component. Thus, we write

$$\text{pseudo } T_c = \frac{\%m}{100} \times T_c$$

$$\text{pseudo } P_c = \frac{\%m}{100} \times P_c$$

This calculation is illustrated in the following table. Column 1 gives the components in a gas, Column 2 is the volume per cent or mol per cent of each component, Columns 3 and 4 list the critical temperatures and pressures of the components (taken from Technical Manual, Natural Gasoline Supply Men's Association), Columns 5 and 6 are the products of Columns 2 × 3 and Columns 2 × 4 respectively. The totals of the last two columns give the pseudo-critical temperature (382.8° R.) and pseudo-critical pressure (663.2 psia.).

Column 1	Column 2	Column 3	Column 4	Column 5	Column 6
Component—	Volume % or Mol %	Critical temp., °R.	Critical pressure, psia.	$\frac{2 \times 3}{100}$	$\frac{2 \times 4}{100}$
Methane	86.02	343.5	673	296	572
Ethane	7.70	550.1	708	42.4	54.5
Propane	4.26	666.2	617	28.4	26.3
Isobutane	0.57	733.2	530	4.2	3.0
Normal butane	0.87	765.6	551	6.7	4.8
Isopentane	0.11	830.0	482	0.9	0.5
Normal pentane	0.14	847.0	485	1.2	0.7
Hexanes	0.33	914.6	434	3.0	1.4
Totals	100.00			382.2	663.2

A typical correlation between reduced temperatures and pressures and Z factors for gaseous mixtures is shown in Fig. 2 (From Technical Manual, Natural Gasoline Supply Men's Association). This particular chart, it should be noted, is a correlation for gases that are principally methane and substantially free from nitrogen. This chart can be used to a fair degree of accuracy for most natural gases, although strictly speaking different correlations will result for gaseous mixtures having different types of molecules. For example, paraffinic hydrocarbons and aromatic hydrocarbons would require two different correlations.

Example: For the gas whose composition is given in the accompanying table, what will be the compressibility factor, Z, at 750 psia. and 150° F.?

Solution: Pseudo-critical temperature = 382.8° R. Pseudo-critical pressure = 663.2 psia.

Therefore,·

$$P_R = \frac{750}{663.2} = 1.13$$

$$T_R = \frac{610}{382.8} = 1.59$$

and Z from Fig. 2 is read as 0.91.

New nomenclature.—Z = Compressibility factor. T_c = Critical temperature, °R. ·P_c = Critical pressure, psia. T_R = Reduced temperature, dimensionless. P_R = Reduced pressure, dimensionless.

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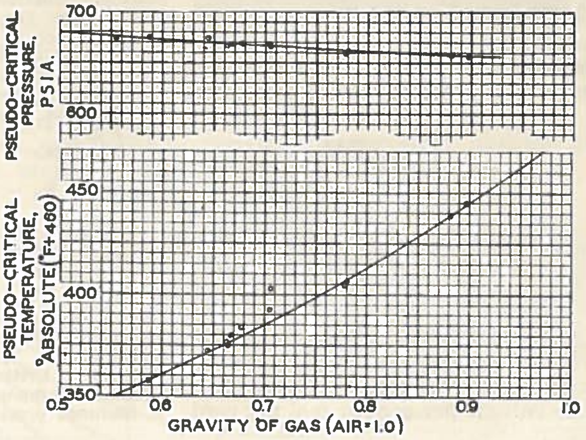
Specific-Gravity Correlations

GASEOUS mixtures, such as natural gases, are characterized, strictly speaking, only by a detailed analysis of components. Such data are often not known or are obtained only at great expense and it is convenient to be able to characterize natural gases by some simpler means. One correlation is that between specific gravity and pseudo-critical temperature. Another is that between specific gravity and pseudo-critical pressure. A third is that between specific gravity and percentage of methane or methane plus ethane.

Figs. 1 and 2 are correlations as given in the Fifth Technical Manual of the Natural Gasoline Supply Men's Association. The use of these correlations rules out the necessity of a detailed calculation of pseudo-critical values from the percentage composition analysis of a gas. It must be noted that these correlations are good only for natural gases which are predominantly methane and which contain very few impurities. Gases having more than 5 per cent of impurities (nitrogen, carbon dioxide, hydrogen sulfide, etc.) are not very likely to follow the relationships too closely.

Another sometimes useful correlation is that between specific gravity and percentage of methane or percentage of methane plus ethane. Fig. 3 shows the trend followed by such correlations, and is based upon the analyses of a number of actual natural gases which were principally methane and substantially free of impurities. This empirical relationship cannot be regarded as being so exact as those in Figs. 1 and 2. Individual gases may depart considerably from the trend.

A calculation of pseudo-critical values by the more detailed method and then by use of Figs. 1 and 2 will indicate in some measure the reliability of these correlations. In the accompanying table are listed the percentage compositions of three gases and computations for the specific gravity, pseudo-critical temperature, and pseudo-critical pressure of each. Using the specific gravity, values of pseudo-critical temperature and pressure, as read from Figs. 1 and 2, are in good agreement with those calculated by the more detailed process. For Gas 1, by the extensive calculation, $P_c = 672$ psia. and $T_c = 362°$ R. From Figs. 1 and 2, using Sp. Gr. = .610, $P_c = 672$ psia. and $T_c = 363°$ R. For Gas 2, by the extensive calculation, $P_c = 639$ psia. and $T_c = 379°$ R, and for Gas 3, $P_c = 673$ psia. and $T_c = 392°$ R. For both of these gases, using Sp. Gr. = .701, Figs. 1 and 2 give $P_c = 667$ psia. and $T_c = 385°$ R. For both the value of T_c is in good agreement, but P_c is in better agreement for Gas 3 than for Gas 2. This is no doubt due to the amounts of nitrogen and carbon dioxide shown in the analysis. For this gas the per cent of methane versus specific gravity falls somewhat below the correlation of Fig. 3, and it would therefore be expected that the estimation of P_c and T_c from specific gravity would be slightly off.



Figs. 1 and 2

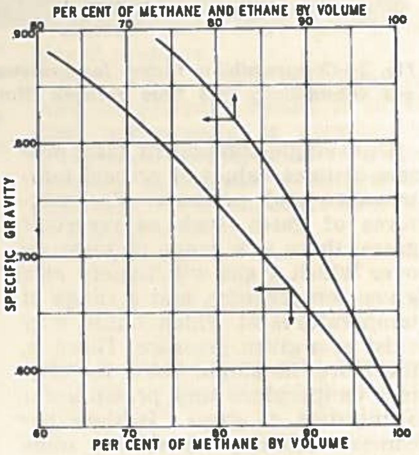


Fig. 3

				Gas No. 1,* sp. gr. = 0.610				Gas No. 2,† sp. gr. = 0.701				Gas No. 3,‡ sp. gr. = 0.701			
Component—	Mol. wt.	T_c	P_c	% by vol.	% × mol. wt.	% × P_c	% × T_c	% by vol.	% × mol. wt.	% × P_c	% × T_c	% by vol.	% × mol. wt.	% × P_c	% × T_c
Methane	16.04	343.5	673	93.40	14.98	629	321	79.08	12.70	5.32	272	82.55	13.25	556	284
Ethane	30.07	550.1	708	3.85	1.16	27.3	21.2	8.71	2.62	61.7	48	9.01	2.71	64	49.6
Propane	44.09	666.2	617	1.33	0.59	8.2	8.9	3.29	1.45	20.2	21.9	4.62	2.04	28.5	30.8
i-Butane	58.12	733.2	530	0.46	0.26	2.5	3.4	0.43	0.25	2.3	3.2	0.74	0.43	3.9	5.4
n-Butane	58.12	765.6	551	0.38	0.22	2.1	2.9	0.98	0.57	5.4	7.5	1.26	0.73	7.0	9.7
i-Pentane	72.15	830.0	842	....	....	....	....	0.43	0.31	2.1	3.6	0.32	0.23	1.5	2.6
n-Pentane	72.15	847.0	485	....	....	....	....	....	....	....	....	0.22	0.16	1.1	1.9
Pentanes+	72.15	840	484	0.58	0.42	2.8	4.9	....	....			....	....	....	....
Hexanes	86.17	914	434	....	....	....	....	0.17	0.15	0.7	1.6				
Hexanes+	86.17	914	434	....	....	....	....	....	....	....	....	0.38	0.33	1.6	3.5
Heptanes+	100.20	952	400	....	....	....	....	0.10	0.10	0.4	1.0	....	....	....	....
CO_2	44.0	548.0	1,073	....	....	....	....	1.46	0.64	6.9	8.0	0.90	0.40	9.6	4.9
N_2	28.02	227.2	492	....	....	....	....	5.33	1.50	7.4	12.1	....	....	....	....
				100.00	17.63	671.9	362.3	99.98	20.29	639.1	378.9	100.00	20.28	673.2	392.4

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*From Gloyd zone, Rodessa field, page 79, "Secondary Recovery in Arkansas," Arkansas Oil and Gas Commission. †From Jones sand, page 224, Ibid. ‡From Paloma field, A.I.M.E., Vol. 160 (1945) Olds, Sage and Lacey.

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Application of Gas Behavior To Reserve Estimates

ONE of the most important applications of the gas laws is to the calculation of natural-gas reserves. The following will outline some of the details, assuming (1) that the reservoir contains no gas in solution in liquid, (2) that the reservoir pore space is constant in volume, and (3) that the reservoir remains at a constant temperature.

Consider that there is originally in the reservoir the equivalent of V_t standard cubic feet of gas (60° F. and 1 atm.), and that at any time there have been produced a total of V_g standard cubic feet of gas. Let V_r be the equivalent number of standard cubic feet remaining in the reservoir. The following must then be true:

$$V_r = V_t - V_g \qquad (1)$$

The standard volumes V_t and V_r can be represented in terms of the temperature and pressure under which they exist by means of the perfect gas law. This is, in general:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \qquad (2)$$

Specifically then, since V_r and V_t are defined at 60° F. and 14.7 psia., the following hold:

$$V_t = \frac{P_oV_p\,520}{T_f\,14.7} \qquad (3)$$

$$V_r = \frac{PV_p\,520}{T_f\,14.7} \qquad (4)$$

where

T_f = reservoir temperature in degrees absolute, a constant

P_o = original reservoir pressure

V_p = volume of reservoir pore space in cubic feet occupied by gas, also a constant, and

P = reservoir pressure at the time when V_g standard cubic feet of gas has been produced

Substituting these into Equation 1 gives:

$$P = P_o - \frac{14.7\,T_f}{520\,V_p}\,V_g \qquad (5)$$

This equation states that the total standard cubic feet of gas produced will plot linearly against the pressure on the reservoir and that the slope of the line will be $\frac{14.7\,T_f}{520\,V_p}$.

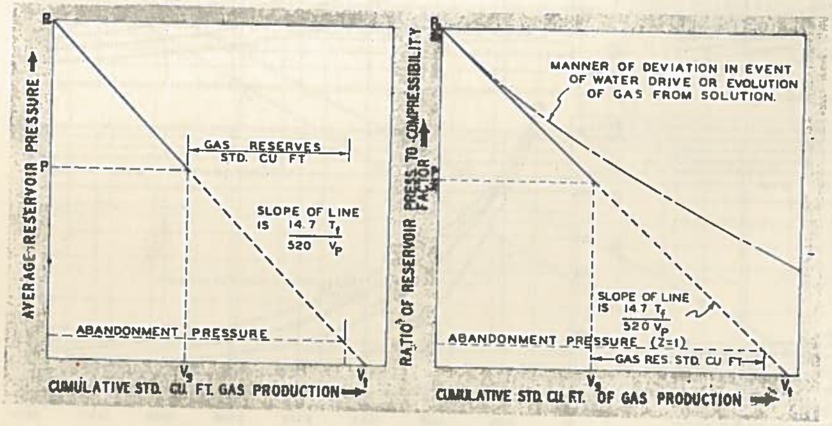

Fig. 1—(Left) Pressure decline of a gas reservoir as a function of withdrawal, on basis of perfect gas law. Fig. 2—(Right) Pressure decline of a gas reservoir as a function of withdrawal, on basis of PV = ZnRT

An ideal representation of this equation is shown in Fig. 1.

Of course, it has been shown that natural gases deviate from the perfect gas law, and the above representation does not hold when such deviations occur. By using the compressibility factor, a similar representation for actual gases can be developed. With the compressibility factor, Equation 2 becomes:

$$\frac{P_1V_1}{Z_1T_1} = \frac{P_2V_2}{Z_2T_2} \qquad (6)$$

and Equations 3 and 4 for V_t and V_r become:

$$V_t = \frac{P_o}{Z_o}\,\frac{V_p}{T_f}\,\frac{520}{14.7} \qquad (7)$$

$$V_r = \frac{P}{Z}\,\frac{V_p}{T_p}\,\frac{520}{14.7} \qquad (8)$$

where Z_o represents the compressibility factor at P_o and Z the compressibility factor at P. The final equation, reached by substituting Equations 7 and 8 into Equation 1 is:

$$\frac{P}{Z} = \frac{P_o}{Z_o} - \frac{14.7\,T_f}{520\,V_p}\,V_g \qquad (9)$$

which states that the total standard cubic feet of gas produced should plot linearly against P/Z at any time. Fig. 2 shows an ideal representation of this relationship. The slope of this line is again $\frac{14.7\,T_f}{520\,V_p}$, the same as that in Fig. 1.

In using either of these relationships, observed pressures and cumulative gas production can be used to estimate formation volume, V_p, if it cannot be accurately calculated otherwise. Reserves are, of course, the standard cubic feet represented on the horizontal axis between the present pressure and abandonment pressure.

In the event that the basic assumptions do not hold, deviation from these ideal relationships will occur. Either the intrusion of water into the gas reservoir or the evolution of gas from liquid solution within the reservoir would tend to maintain pressure and give the type curve illustrated by the dotted line of Fig. 2.

New nomenclature:

V_p = available reservoir pore space, cubic feet

V_t = total standard cubic feet of gas in reservoir originally

V_g = total standard cubic feet of gas produced

V_r = total standard cubic feet of gas remaining in reservoir

P_o = original reservoir pressure, psia.

Z_o = original value of compressibility

T_f = reservoir temperature, degrees Rankine

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Reservoir Volumes—Barrels of Free-Gas Space

THE gas law, $PV = ZnRT$, is used to convert volumes of gas at one pressure and temperature to other pressures and temperatures. In particular, gas removed from a reservoir, measured at some base temperature and pressure, can be converted to an equivalent volume representing the space that this produced gas would occupy in the reservoir, assuming that it remained in the gaseous state.

To convert 1 standard cubic foot of gas to the volume it would occupy as free gas in the reservoir, proceed as follows:

$$\frac{P_1V_1}{Z_1T_1} = \frac{P_2V_2}{Z_2T_2} \qquad (1)$$

or if standard conditions are 14.7 psia. and 520° R. (at which conditions Z is 1.0), then for 1 standard cubic foot,

$$\frac{14.7}{520} = \frac{PV}{ZT_r} \qquad (2)$$

where V is the equivalent cubic feet of free gas in the reservoir at P and T_r.

It is frequently desirable to know this gas volume in barrels rather than in cubic feet. If **v** designates the barrels of free gas space occupied in the reservoir by 1 standard cubic foot of gas compressed to reservoir conditions, then from Equation 2:

$$v = \frac{14.7}{520} \cdot \frac{T_r}{5.62} \cdot \frac{Z}{P} \qquad (3)$$

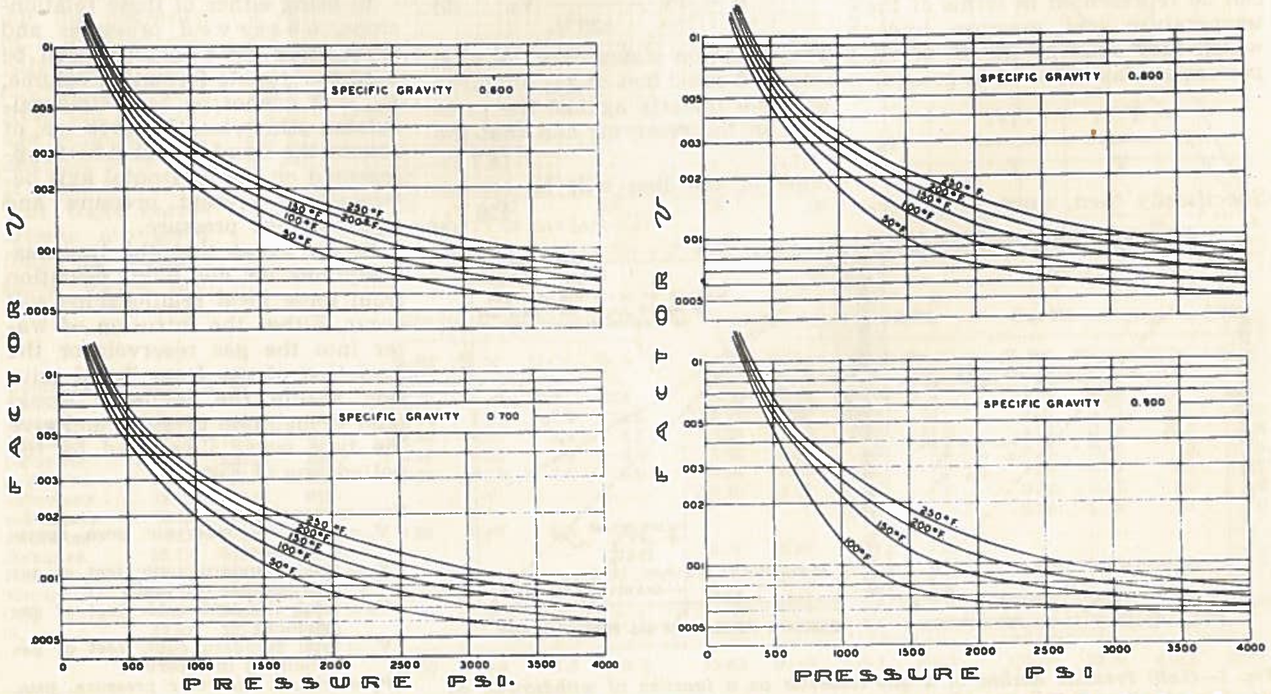
since there are 5.62 cu. ft. per barrel.

This relationship has been used to prepare the accompanying charts from which may be read the quantity **v** at various temperatures and pressures for gases of varying specific gravity. These were prepared by estimating pseudocritical temperature and pseudocritical pressure from the gas gravity by use of the charts in Engineering Fundamentals No. 303, and using these values to get Z factors from the charts in Engineering Fundamentals No. 302. The use of the accompanying graphs is self evident.

Example.—A gas has a gravity of .675. How much space would be occupied by this gas in a reservoir at 170° F. and 2,000 psia.?

Solution.—Use charts for Sp. Gr. = .600 and Sp. Gr. = .700 and interpolate. For Sp. Gr. = .600, v = .00145. For Sp. Gr. = .700, v = .00137. By a direct ratio, .075/.100 (.00145 — .00137) = .00006.

Therefore, at Sp. Gr. = .675 the value of v is .00139, which is the number of barrels of free gas space occupied at 170° F. and 2,000 psia. by 1 standard cubic foot of gas.



These four charts give the "v" factor—or barrels of free gas space—occupied in the reservoir by one standard cubic foot of gas, for various stipulated pressures, temperatures, and specific gravities of the gas

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ENGINEERING *fundamentals*

Example of Reserve Estimates By Gas Law

IN previous discussions, it was pointed out that one of the applications of the gas law was to the estimation of gas reserves. The following data are taken from the literature to illustrate this application.[1]

It was noted that cumulative production should follow, according to the perfect gas law, the equation

$$P = P_o - \frac{14.7\,T_f}{520\,V_p} V_g \quad (1)$$

where

P_o = original reservoir pressure
T_f = reservoir temperature
V_p = pore volume occupied by gas
V_g = cumulative gas production, S.C.F., and
P = pressure when V_g S.C.F. has been produced.

Fig. 1 is reproduced from the reference given to illustrate how observed data on Monroe gas field follow this theoretical relationship. From this, the reserves are estimated by the authors at 5,500 billion cubic feet, the average productive volume pressure being felt to be more accurate than the average acreage pressure.

The slope of the line giving pressure decline is —.172 psi. per billion cubic feet withdrawal. On an estimated reservoir temperature of 110° F., this would give a value of V_p as follows:

$$V_p = \frac{14.7 \times 570}{520 \times .172}$$

= 94.0 billion cubic feet of reservoir pore space

This reservoir existed under an initial pressure at which compressibility would be important. On the basis of PV = ZnRT the pressure decline equation is

$$\frac{P}{Z} = \frac{P_o}{Z_o} - \frac{14.7\,T_f}{520\,V_p} V_g \quad (2)$$

Using a gas specific gravity of 0.6 to calculate Z factors at the various pressures and a temperature of 110° F., the pressure decline of Fig. 2 is constructed. Drawing a straight line through the points gives again approximately 5,500 billion cubic feet of initial reserves. The slope of this line is —.192. The value of V_p is then calculated as 84.0 billion cubic feet.

Neither of these approaches is presented as being the most accurate representation of reserves in this instance, since there are other factors which must be weighed in making a final judgment on reserves. The data are here used primarily to indicate the adherence of field performance to theoretical considerations.

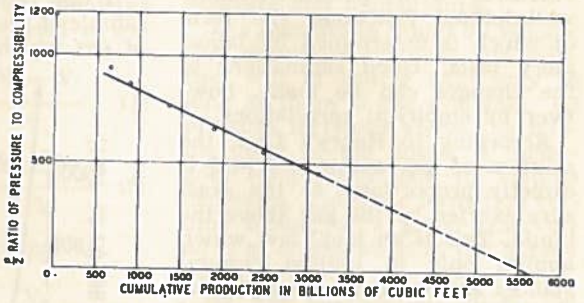
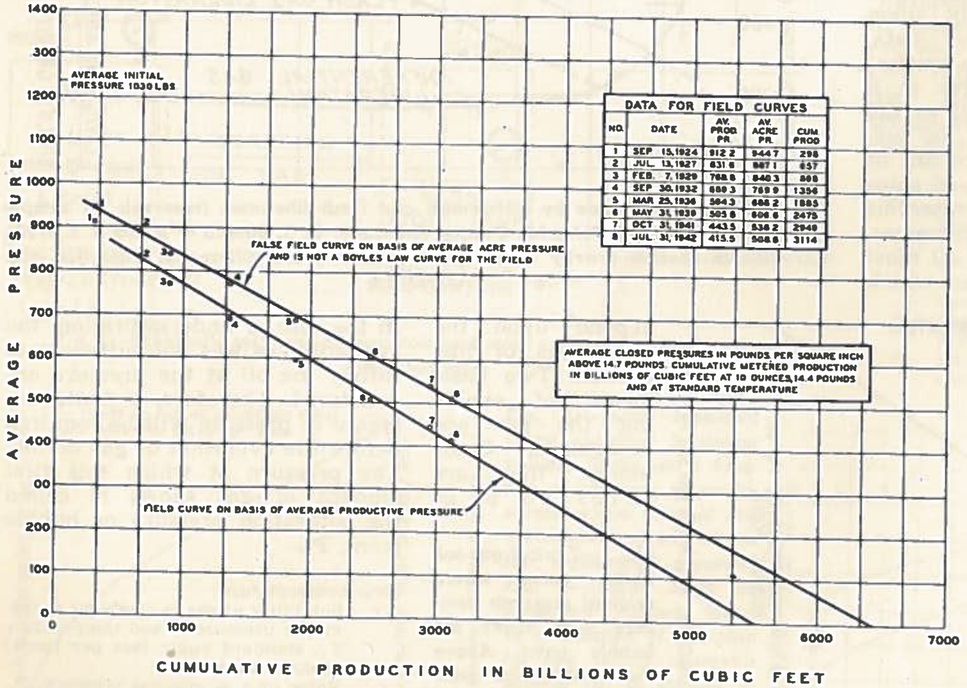


Fig. 2—Production-pressure decline on basis of corrected pressure



DATA FOR FIELD CURVES

NO.	DATE	AV. PROD. PR.	AV. ACRE PR.	CUM PROD
1	SEP 15, 1924	912.2	944.7	298
2	JUL. 13, 1927	831.6	887.1	657
3	FEB. 7, 1929	768.8	840.3	868
4	SEP 30, 1932	689.3	769.9	1356
5	MAR 25, 1936	584.3	688.2	1885
6	MAY 31, 1939	505.6	606.6	2475
7	OCT 31, 1941	443.5	538.2	2948
8	JUL. 31, 1942	415.5	508.6	3114

Fig. 1—Production-pressure decline of Monroe gas field[1]

Reference

1. The Monroe Gas field and Its Pressure Behavior, Preston Fergus and D. T. Mac Roberts, A.P.I. Mid-Continent Division meeting, Amarillo, Tex., May 22-23, 1947.

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ENGINEERING *fundamentals*

Solubility of Gas in Crude Oil

ANY physical characteristic of a liquid that is a function of pressure deserves consideration in reservoir engineering problems. Most important of such characteristics in present usage are compressibility. amount of gas solubility, change in liquid volume upon gas solution, and viscosity. Data which have been procured to the present time indicate that the changes in these variables with pressure follow well-defined functions, the form of which is determined by laboratory tests. Good estimations of the changes can be made, however by empirical correlations.

According to Henry's Law, the solution of gas within a liquid is directly proportional to the pressure exerted by the gas above the liquid. This is an ideal law which applies only in limited circumstances and it cannot be used correctly throughout the range of pressures commonly encountered in petroleum reservoirs. Therefore, one must rely primarily upon experimentally determined data. Rather than determine the amount of gas which will dissolve in a given stock-tank crude oil as the pressure is increased, it is customary to measure the amount of gas which will come out of solution from a sample of reservoir crude oil as the pressure decreases.

The solubility of gas in oil must be referred to some basis, which might be either stock-tank or reservoir oil. It is customary to use the former. The definition will be made, therefore, of solubility, **r**, as the number of cubic feet of gas measured at standard conditions which will dissolve per barrel of stock tank oil (defined at 1 atmosphere and 60° F.) when subjected to reservoir pressure P and reservoir temperature T_r.

If one releases pressure from a sample of reservoir oil the quantity of gas which will be evolved will depend upon the conditions of liberation. Two basic means of evolving the gas are recognized. Commonly, they are referred to as "flash" and "differential" gas liberation. In the former it is required that the gas evolved during a finite reduction in pressure be kept in contact with the liquid from which it was evolved until equilibrium is established between all the gas and liquid at the final conditions. In the latter, it is required that the gas being evolved from the liquid be removed continuously from contact with the liquid, and thus the liquid is in equilibrium only with the gas being evolved at a given pressure, not with that evolved over an entire pressure interval.

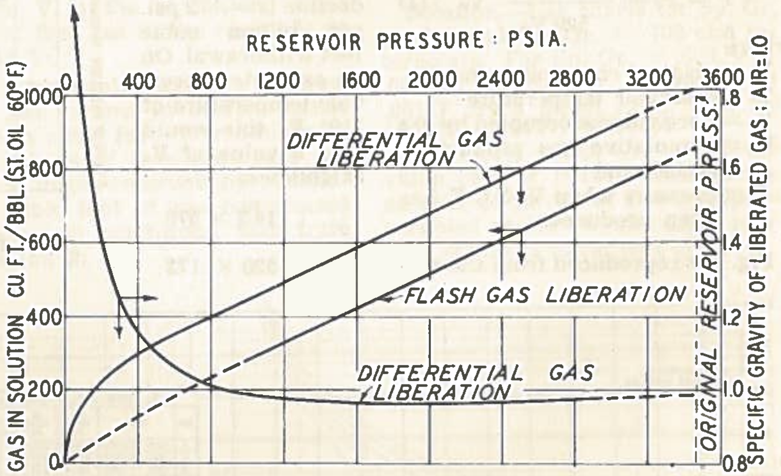

Fig. 2—Separation of gas by differential and flash liberation (reservoir oil sample is from Magnolia field, Columbia County, Arkansas; U. S. Bureau of Mines R. I. 3720). Variation in specific gravity of gas evolved during the differential liberation also is plotted

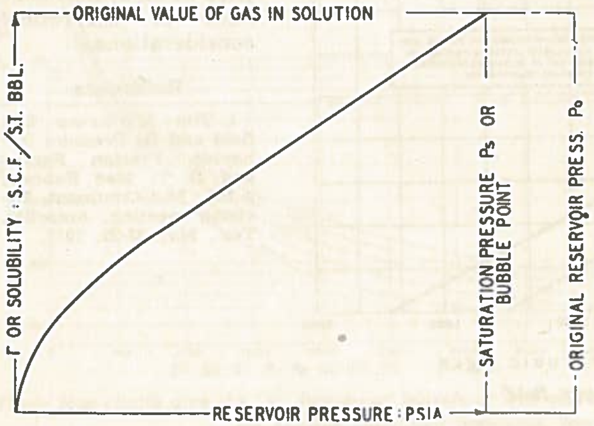

Fig. 1—Typical gas-solubility curve where original reservoir pressure was above the bubble point. Above the bubble-point pressure, amount of gas in solution is constant

Beginning with a given quantity of reservoir oil, therefore, and going to atmospheric pressure, the final quantity of stock tank oil remaining, its density, the total amount of gas evolved, and its density will vary depending upon the manner of gas separation.

Reservoir oils may be found completely saturated with gas, or undersaturated. In the case of saturation, there was sufficient or an excess of gas available. An infinitesimal drop in pressure will suffice to evolve gas from such an oil. In the case of undersaturation, the available gas was not sufficient to satisfy the oil at the pressure encountered. Therefore, a finite decrease in pressure will be required before the evolution of gas occurs. The pressure at which this first amount of gas shows is called the saturation pressure or bubble point, $\mathbf{P_s}$.

New nomenclature:

r = Solubility of gas in crude oil at reservoir pressure P and temperature T_r, standard cubic feet per barrel of stock tank oil.
r_o = Value of r at original pressure, P_o.
P_s = Saturation pressure or bubble point of a reservoir oil.

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ENGINEERING *fundamentals*

No. 308

Change in Oil Volume With Gas Solution

As gas dissolves in crude oil there is an increase in the total liquid volume. Best means of obtaining this volume increase is by an empirical measurement on a sample of reservoir oil, as the pressure is brought from its original reservoir value to atmospheric or some other desired base.

Change in oil volume, upon liberation of gas, must be referred to some basis. This basis may be the reservoir oil or the stock-tank oil. Usually, the latter is preferred since it is this quantity that is measured as a matter of field operation.

The change in oil volume upon reduction in pressure is commonly expressed in two fashions, (1) as a relative volume ratio, or (2) as a percentage volume change. This can best be explained by reference to Fig. 1.

Consider a sample of reservoir liquid of volume V_1 (Fig. 1a) which is brought to stock-tank conditions (Fig. 1b) at which point the liquid volume measures V_2, some quantity less than V_1. The following four relationships are used to express the change:

$$\text{Shrinkage} = Sh_1 \frac{V_1 - V_2}{V_2} \qquad (1)$$

$$\text{Shrinkage} = Sh_2 \frac{V_1 - V_2}{V_2} \qquad (2)$$

$$\text{Formation volume factor}$$

$$= \text{f.v.f.} = \beta = \frac{V_1}{V_2} \qquad (3)$$

$$\text{Shrinkage factor} = \frac{V_2}{V_1} \qquad (4)$$

These ratios all express the same change and the relationship between them is as follows:

$$\beta = (1 + Sh_1)$$

$$= \frac{1}{\text{Shrinkage factor}}$$

$$= \frac{1}{(1 - Sh_2)} \qquad (5)$$

$$\text{Shrinkage factor} = (1 - Sh_2) \qquad (6)$$

Perhaps the most commonly used of these quantities is the formation volume factor designated as β. It is defined as the volume in barrels at reservoir pressure P and temperature T_f occupied by 1 bbl. of stock-tank oil, plus the gas which will go into solution at that pressure and temperature.

The increase in liquid oil volume that occurs as gas goes into solution will of course hold only so long as there is gas available. As pressure increases above the bubble point of the liquid, it undergoes compression and its volume decreases. Thus, the formation volume factor for a given reservoir oil is at a maximum at the bubble point.

As was pointed out under the discussion of amount of gas in solution, two methods, "flash" and "differential" liberation, are used to measure the amounts of gas evolved. Since these two methods give different quantities of gas, they must also give different shrinkage values or formation volume factors.

New nomenclature:

Sh = Shrinkage, referred either to stock tank oil or reservoir oil.

β = Formation volume factor, or the volume occupied at reservoir conditions by one barrel of stock tank oil plus its gas in solution.

β_o = Value of β at original reservoir pressure.

β_s = Value of β at saturation pressure.

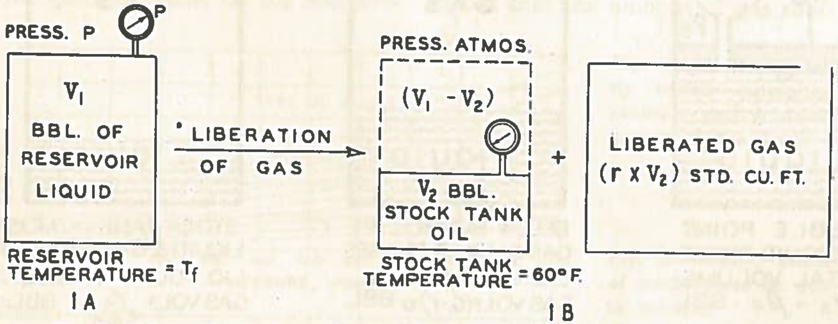


Fig. 1—Schematic relations of reservoir and stock-tank oil to dissolved and liberated gas

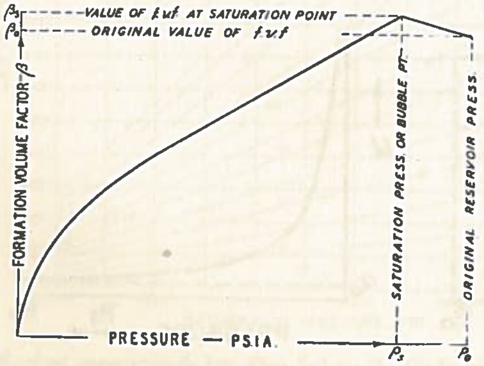
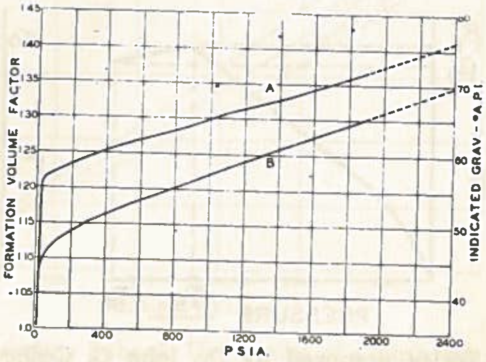


Fig. 2—(Left) Idealized curve showing formation volume factor as a function of pressure in an instance where original reservoir pressure was above the saturation pressure. Fig. 3—(Right) Data on formation volume factor as a function of pressure for an actual reservoir sample (U. S. Bureau of Mines R.I. 3715, Rodessa field).



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ENGINEERING *fundamentals*

Reservoir Volumes—Space Occupied by Stock-Tank Oil and Its Original Gas

IT is convenient for some reservoir calculations to know the space that would be occupied in the reservoir at any pressure below the original by the contents of a unit volume of original reservoir oil, or by a unit volume of stock-tank oil and all the gas which was originally in it. An explanation of this calculation will also aid in clarifying the concepts of gas solubility and formation volume factor which were defined in previous discussions.

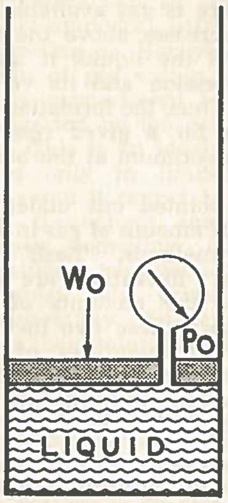
Space changes which occur when pressure is released from reservoir oil are shown in the accompanying set of diagrams. On decrease in pressure the total space occupied by liquid and gas must increase, provided no withdrawals are made. The total volume based on a barrel of stock tank oil is designated as **u** and is expressed mathematically as follows:

$$u = \beta + (r_o - r)\,v$$

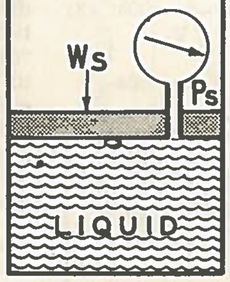
Where β represents the liquid volume at a given pressure, and $(r_o - r)\,v$ represents the free gas volume at that pressure. The latter follows from the fact that r standard cubic feet of gas go into solution at the pressure considered and, therefore, of the original r_o standard cubic feet there are $(r_o - r)$ standard cubic feet remaining as free gas. At pressure P this gas volume occupies less space than at standard conditions, the conversion factor being v, previously defined as

$$\frac{14.7 \times T_r \times Z}{520 \times 5.62 \times P}$$

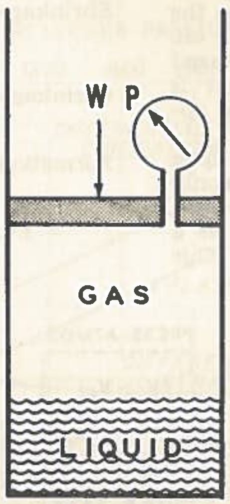
The units of **u** as given are barrels. The graphs accompanying the diagrams indicate the general manner in which β, r, and **u** vary with pressure. It is noted that **u** is an increasing quantity as pressure decreases regardless of whether there are two phases present or one.



INITIAL RESERVOIR
ALL LIQUID PHASE
TOTAL VOLUME=
$u_o = \beta_o$ BBL.

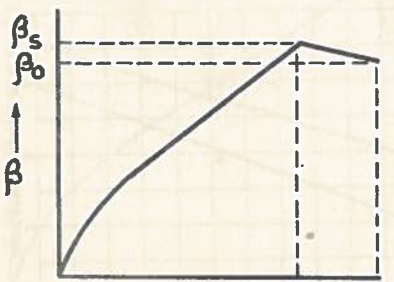


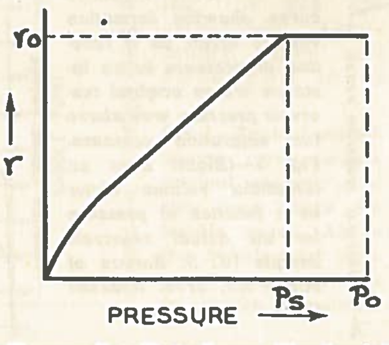
BUBBLE POINT
ALL LIQUID PHASE
TOTAL VOLUME=
$u_s = \beta_s$ BBL.

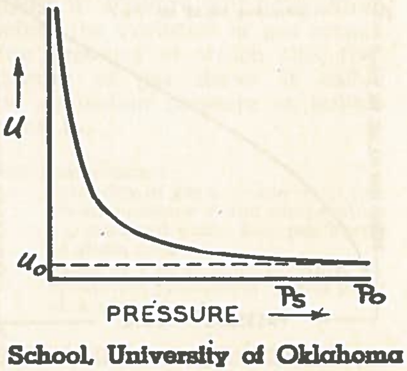


BELOW BUBBLE PT.
GAS & LIQUID PHASES
LIQ. VOL. = β BBL.
GAS VOL: $(r_o - r)v$ BBL.
TOT. VOL: $u = \beta + (r_o - r)v$ BBL.

STOCK TANK (ATMOS.)
LIQUID & GAS PHASES
LIQ. VOL. = 1 BBL.
GAS VOL = $\frac{r_o}{5.62}$ BBL.
TOT. VOL. = $\left(1 + \frac{r_o}{5.62}\right)$ BBL.







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ENGINEERING fundamentals

Estimation of Solution Gas

AMOUNT of gas in solution in reservoir oil is not always a known measured quantity and an estimate may be necessary to pursue engineering calculations. Accurate estimates of this quantity can be made because sufficient data have been taken on reservoir oils throughout the petroleum industry to enable correlations to be drawn. The amount of solution gas can be estimated within reasonable limits, over a wide range of pressures, temperatures, and fluid characteristics. The simplest estimate is made from a knowledge of only the A.P.I. gravity of the stock-tank oil. The most accurate estimation would require a knowledge of gas composition and liquid composition, temperature, and pressure.

An analysis of the phase behavior of liquids and gases leads to the consideration of the following variables in predicting solution gas: (1) the composition of the liquid phase, (2) the composition of the gaseous phase, (3) the temperature, and (4) the pressure. Since both the liquid and gaseous phases are made up of members of a homologous series, a regular change of gas solubility with molecular complexity would be expected. The simplest manner of characterizing the composition of the liquid and gas is by specific gravity for the gas and by A.P.I. gravity for the oil.

Solubility of Given Gas

The lower the specific gravity of a gas, the greater the percentage of light components indicated and the smaller would be the expected solution in a given oil at a given temperature and pressure. The higher the A.P.I. gravity of an oil, the greater would be the expected amount of solubility of a given gas at a specific temperature and pressure. For a given oil and a given gas at a given pressure, the solubility will decrease as temperature increases. It has previously been noted that the amount of gas solubility increases almost directly with pressure, other things being constant.

The above considerations on the effect of the four variables are qualitative. The quantitative magnitude of the changes can be found only by making specific correlations. A direct correlation of the amount of gas in solution as a function of pressure and A.P.I. gravity of the stock-tank oil involved, but neglecting variations with temperature and gas specific gravity was first made by Katz,[1] and later extended by Beal.[2]

Fig. 1 is a duplication of the chart presented by Beal for estimating gas in solution. The use of this chart is obvious. For using this chart to construct a specific curve for gas solubility versus pressure, A.P.I. gravity of stock-tank oil is the only information necessary. In its use, one cannot differentiate between two instances in which reservoir temperature differs.

A more extensive correlation of gas solubility against pressure, using gas gravity, A.P.I. gravity of oil, and reservoir temperature has been made by Standing.[3] From his correlation, the decrease of gas solubility with temperature rise, all other things being constant, is of the order of 2 per cent per 1° F.

For example, Fig. 2 is obtained by a replotting of data from Standing's general set of curves, and shows the effect of pressure upon gas in solution at three temperatures for a 0.70 specific gravity gas in a 40° A.P.I. oil.

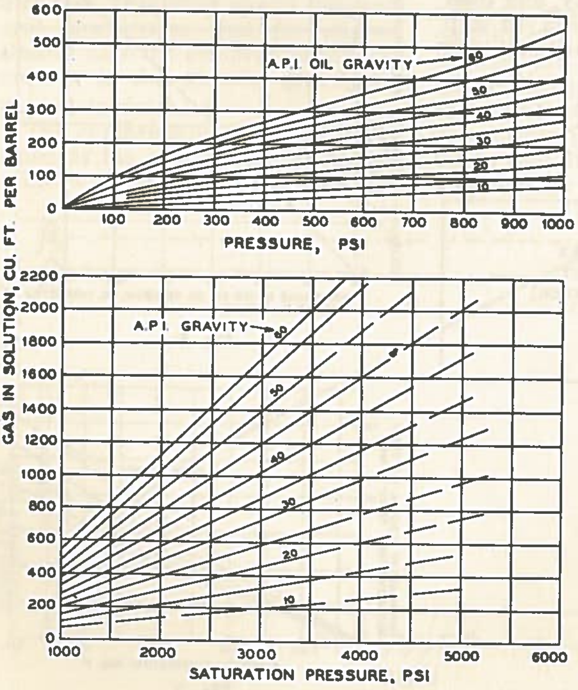

Fig. 1—(Left) Solubility versus saturation pressure and crude-oil gravity (From Beal[2])

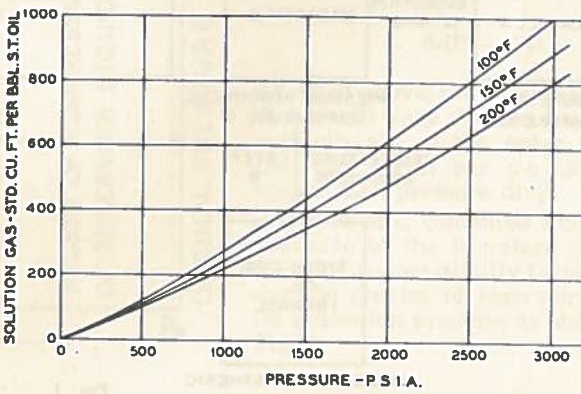

Fig. 2—(Below) Effect of temperature on gas in solution

References

1. Prediction of the Shrinkage of Crude Oils, D. L. Katz, A.P.I. Drilling and Production Practice (1942) 137-147.

2. The Viscosity of Air, Water, Natural Gas, Crude Oil, and Its Associated Gases at Oil-Field Temperature and Pressures, Beal, A.I.M.E. Trans, 1946, Vol. 165.

3. A General Pressure-Volume-Temperature Correlation for Mixtures of California Oils and Gases, M. B. Standing. Presented A.P.I. Pacific Coast district, Division of Production, May 15-16, 1947. Paper also published in The Oil and Gas Journal, May 17, 1947, p. 95.

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ENGINEERING fundamentals

No. 311

Estimation of Formation Volume Factor

As pointed out in a previous discussion, complete data describing the behavior of reservoir liquids and gases is not at hand in all cases. When it is not, estimates can be made by use of charts which correlate measurements that have been made on many reservoir oils. The same correlations that have been presented in the literature to estimate gas in solution have also been used to estimate formation volume factors. The simplest basis of estimating requires a knowledge of the stock tank A.P.I. gravity, the reservoir temperature, and the pressure at which the formation volume factor is desired.

Factors which contribute to the variation in amount of solution gas also contribute to the variation in formation volume factor. However, since an increase in volume of oil occurs chiefly due to gas going into solution, the use of this quantity alone will yield good correlations. In effect, it is the gas going into solution at a given pressure and not the pressure itself which gives rise to the increased volume. Standing[1] has presented a complete correlation giving the effects of (1) temperature, (2) amount of solution gas, (3) gas gravity, and (4) oil gravity. Qualitatively, an increase in the value of the formation volume factor occurs with higher gas specific gravities, higher A.P.I. oil gravities, higher temperatures, and increased amounts of gas in solution.

Katz[2] has presented a method for estimating formation volume factors based upon the analysis of both the gas and oil produced along with a knowledge of the amount of gas in solution. He also presented a simpler method based entirely upon an empirical correlation. His charts are reproduced here as one of the simpler means of estimating formation volume factor.

In illustrating the use of this correlation, consider the evolution of gas and the reduction of oil to stock tank conditions in a two-stage process. Refer to Fig. 1 in which step A shows the gas evolved at reservoir temperature with consequent shrinkage in oil volume. Step B shows the lowering of the temperature on the gas-free oil formation temperature to 60° (stock-tank temperature). The formation volume factor is arrived at by estimating the changes in steps A and B separately, and then combining the two effects. In step B, the shrinkage based on the stock tank oil is

$$\mathrm{Sh}_2 = \frac{V_2 - 1}{1}.$$

In step A the shrinkage based on volume V_2 is

$$\mathrm{Sh}_1 = \frac{\beta - V_2}{V_2}.$$

Eliminating V_2 from the two equations gives $\beta = (1 + \mathrm{Sh}_1)(1 + \mathrm{Sh}_2)$ which is the formation volume factor desired. Figs. 2 and 3 read directly the values of Sh_1 and Sh_2 respectively. The gas in solution used on these charts may be a known measured quantity or an estimated quantity.

Example.—It is desired to know the formation volume factor on an oil produced as 40° A.P.I. from a reservoir temperature of 200° F. and 2,000 psia. At this pressure and temperature the gas in solution is known to be 500 standard cubic feet per stock tank barrel.

Solution.—Reading Fig. 1 for 500 cu. ft. of gas in solution, $\mathrm{Sh}_1 = .25$. Reading Fig. 2 for drop from 200° to 60° gives $\mathrm{Sh}_2 = .068$. Therefore, $\beta = (1.068)(1.25) = 1.34$ the desired formation volume factor.

References

1. A General Pressure-Volume-Temperature Correlation for Mixtures of California Oils and Gases, M. B. Standing. Presented A.P.I. Pacific Coast District, Division of Production, May 15-16, 1947. Paper also published in The Oil and Gas Journal, May 17, 1947, p. 95.

2. Prediction of the Shrinkage of Crude Oils, B. L. Katz, A.P.I. Drilling and Production Practice (1942) 137-147.

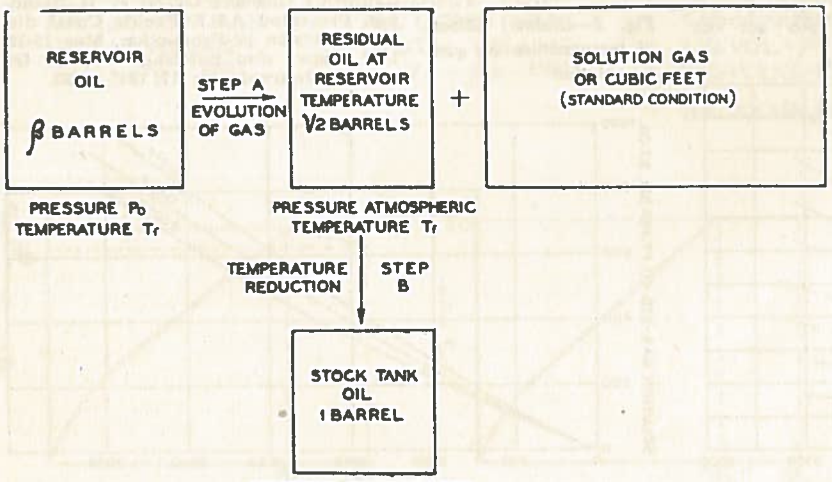

Fig. 1

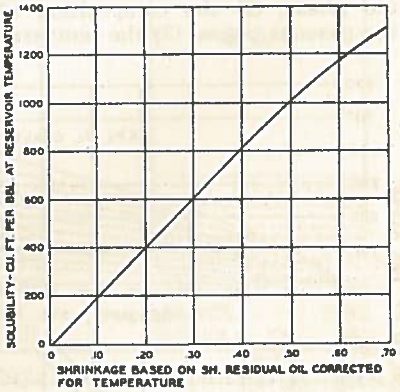

Fig. 2

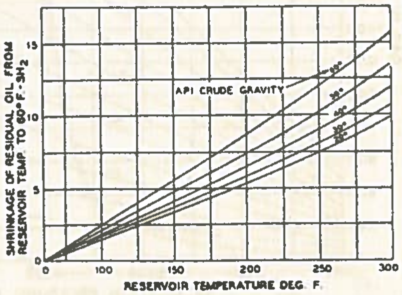

Fig. 3

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Compressibility of Reservoir Fluid Above the Bubble Point

RESERVOIR oil, above the pressure at which all available gas has gone into solution, is a liquid which has the properties of a single-phase system. In particular, further compression will decrease the total liquid volume. Hence, the formation volume factor decreases as has been shown previously by a typical curve. Therefore, a given volume of reservoir oil existing above its saturation pressure, when taken from original pressure to the pressure at which gas first appears, will undergo an expansion in volume. The magnitude of this expansion is useful in certain reservoir calculations.

The degree of compressibility of a liquid is dependent upon the type of liquid. There is no general relationship expressing the compressibility of all liquids. For a homologous series of compounds, compressibility varies more or less regularly with the variation in molecular size. This gives rise to the general observation that for oils of similar base the compressibility decreases as the specific gravity of the oil increases.

The compressibility factor is defined as the rate of change in volume with pressure increase per unit volume of liquid, all variables other than pressure being constant. In general, the compressibility factor will change with pressure. The factor is ordinarily determined empirically over a definite pressure interval by direct measurement. For the average compressibility factor over a pressure interval the following equation can be used:

$$V_2 = V_1 [1 - C (P_2 - P_1)] \quad (1)$$

where V_1 represents a volume liquid at some reference pressure P_1, V_2 represents the volume occupied by the same liquid under pressure P_2 and, therefore, a minus sign is arbitrarily used to yield C as a positive quantity.

One can consider the expansibility of a liquid under pressure release as the inverse of compressibility. According to a strict definition, the factor of expansibility is the same as that for compressibility but differing in sign. In defining an average expansibility, a different numerical value will result because of the differing volume used for a reference. Thus, for expansion the equation corresponding to Equation 1 would be:

$$V_1 = V_2 [1 + C_e (P_2 - P_1)] \quad (2)$$

where the smaller volume V_2 at the higher pressure P_2 is used as the reference. C_e differs from C by the ratio of V_1/V_2. As the limit at a definite pressure is taken, the values of C_e and C become identical.

Equation 1 can be applied to calculate the average compressibility of a reservoir oil above its saturation pressure (region of a single phase) from the plot of formation volume factor, β, versus pressure (Fig. 1). The reference pressure is chosen as P_s to P_o the relative volume change is from β_s to β_o. Substituting in Equation 1 gives:

$$\beta_o = \beta_s [1 - C (P_o - P_s)] \quad (3)$$

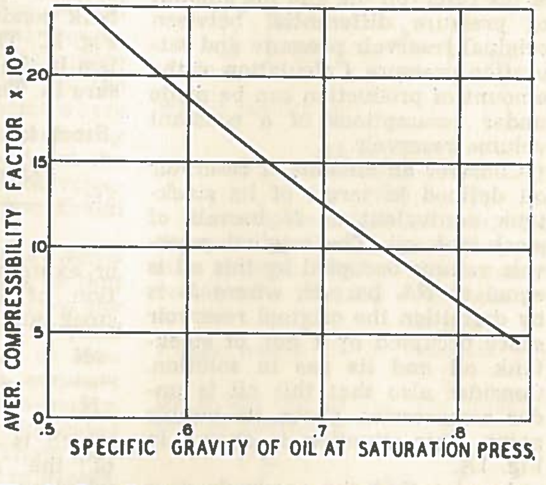

Fig. 2—Barrels compression per pound per square inch per barrel of liquid at bubble point, as a function of specific gravity at bubble point

Solving for C, yields:

$$C = \frac{\beta_s - \beta_o}{\beta_s (P_o - P_s)} \quad (4)$$

As noted above, the compressibility factor will vary. Values for reservoir oils are of the order of 10^{-6} bbl. per barrel per pound per square inch pressure drop.

Values of C computed from data available in the literature show a trend of compressibility factor with specific gravity of reservoir oil at its saturation pressure as shown in Fig. 2.

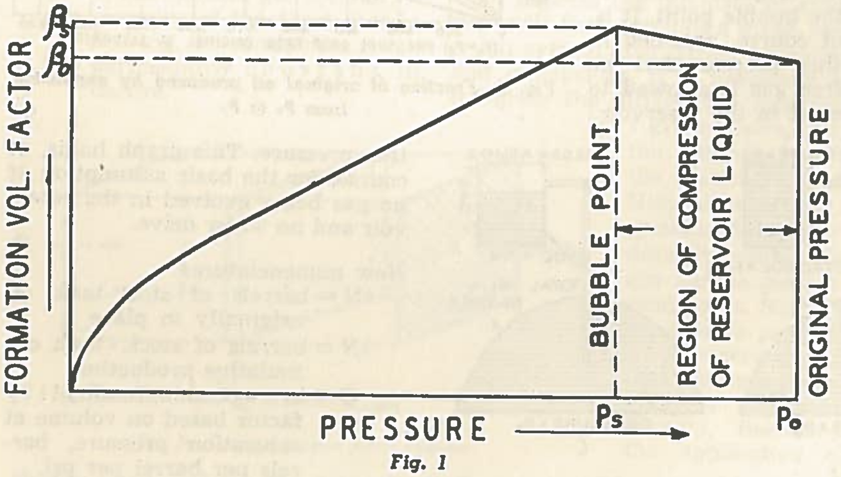

Fig. 1

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ENGINEERING *fundamentals*

No. 313

Production by Expansion of Reservoir Fluid Above Bubble Point

IN instances where reservoir liquid is originally under compression above its bubble point, expansion of that liquid alone is sufficient for production of oil. The amount of oil which can be withdrawn from a reservoir before the liquid undergoes complete expansion to its bubble point is dependent upon the total amount of oil in place, the compressibility factor of the reservoir oil, and the amount of pressure differential between original reservoir pressure and saturation pressure. Calculation of the amount of production can be made under assumptions of a constant volume reservoir.

Consider an amount of reservoir oil defined in terms of its stock-tank equivalent as N barrels of stock-tank oil. The original reservoir volume occupied by this oil is equal to $N\beta_o$ barrels, where β_o is by definition the original reservoir space occupied by 1 bbl. of stock-tank oil and its gas in solution. Consider also that this oil is under compression above its bubble point. This situation is shown in Fig. 1A.

Assume that the reservoir pore volume occupied by the oil remains constant and that the reservoir pressure falls to P_s. To accomplish this, the oil must expand from its original space. If all the liquid were kept at P_s, it would occupy a total space of $N\beta_s$ barrels, where β_s is by definition the space occupied at P_s by 1 bbl. of stock-tank oil plus its gas in solution. This condition is shown in Fig. 1B.

The increase in total space necessary to hold the oil is given by $N(\beta_s - \beta_o)$. In practice, however, the removed oil is not held at such a pressure but is reduced to atmospheric pressure. The amount of oil production will therefore be represented by that in the increased space, reduced to stock-tank conditions. This is shown in Fig. 1C. This amount of oil will be less in the stock tank than at pressure P_s. The conversion is given by:

Stock tank production =

$$\Delta N = \frac{N(\beta_s - \beta_o)}{\beta_s} \qquad (1)$$

or, expressed as a fraction of the original stock tank oil in place:

$$\frac{\Delta N}{N} = \frac{\beta_s - \beta_o}{\beta_s} \qquad (2)$$

which is the fraction of the original oil which can be produced by decline of the reservoir pressure from its original value to the bubble point. It is, of course, assumed in this process that no free gas is allowed to exist in the reservoir.

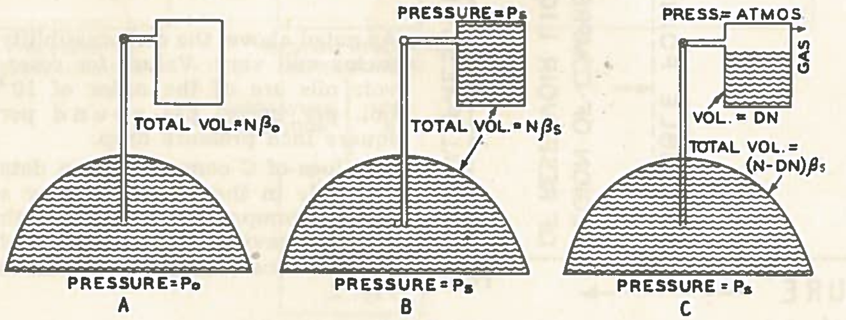

Fig. 1

The average compressibility factor, C, of the reservoir oil has already been defined (Engineering Fundamentals No. 312) as

$$C = \frac{(\beta_s - \beta_o)}{\beta_s (P_o - P_s)}$$

Therefore, Equation 2 could be written as:

$$\frac{\Delta N}{N} = C(P_o - P_s) \qquad (3)$$

The fractional production from original pressure to any pressure P between P_s and P_o would be given by:

$$\frac{\Delta N}{N} = C(P - P_o)$$

In the event that water encroachment ensues into the reservoir, the increase in total space between Figs. 1A and 1B would have been modified to $N(\beta_s - \beta_o) + W$ and the final equation would be:

$$\frac{\Delta N}{N} = \frac{\beta_s - \beta_o}{\beta_s} + \frac{W}{\beta_s} \qquad (4)$$

Using Equation 3, Fig. 2 has been prepared to show the fractional production $\Delta N/N$ as a function of the compressibility factor and the pressure differential between original reservoir pressure and saturation pressure. This graph holds, of course, for the basic assumption of no gas being evolved in the reservoir and no water drive.

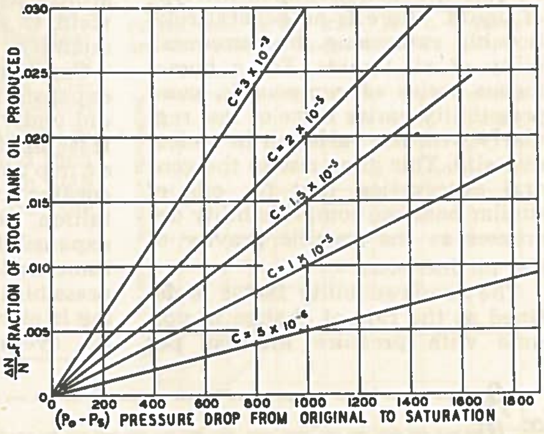

Fig. 2—Fraction of original oil produced by expansion from P_o to P_s

New nomenclature:

N = barrels of stock-tank oil originally in place
ΔN = barrels of stock-tank cumulative production
C = average compressibility factor based on volume at saturation pressure, barrels per barrel per psi.

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Permeability—Darcy's Law

ONE of the necessary characteristics of a petroleum reservoir is its ability to allow the flow of fluids through it. The quantitative expression of this characteristic is of basic importance in the production of oil. This property of a reservoir rock is given the name of permeability.

The quantitative expression of permeability is known as Darcy's law, so named after its discoverer, a Frenchman, Henry Darcy. Darcy's experiments were made with water flowing through horizontal sand-filter beds and resulted in his formulation that the rate of flow through a sand bed was proportional to the pressure head above the filter and the cross-sectional area of the filter, but inversely, proportional to the thickness of the filter. This law has since been extended to apply generally to any fluid flowing in any direction through a porous material so long as conditions of flow are viscous.

The formulation given by Darcy is an integrated equation applying to the specific system he studied. The general formulation of his law is usually made in a differential form, thus:

$$v = -\frac{k}{\mu}\,\frac{dp}{dl} \qquad (1)$$

where

v = macroscopic velocity of flow, in centimeters per second.

μ = viscosity of the fluid that is flowing, in centipoises.

k = permeability constant, in darcys.

$\frac{dp}{dl}$ = pressure gradient in the direction of flow, in atmospheres per centimeter.

In writing this equation, it must be noted that several conditions are necessary. First, it is assumed that the entire pore space of the reservoir material is filled with the fluid flowing and only with that fluid. Second, the velocity of flow, v, is not an actual but an apparent velocity. It is equivalent to Q/A where Q is the volume rate of flow in cubic centimeters per second through the cross-sectional area A. The area A is expressed in square centimeters perpendicular to the direction of flow at the point where v is desired. Since flow occurs only through the pore area and not the entire area A, the apparent velocity v is smaller than the actual fluid velocity.

A permeability of one darcy may, therefore, be defined as that permeability which will allow the flow of 1 cc. per second of fluid of 1 cp. viscosity through an area of 1 sq. cm. under a pressure gradient of 1 atm. per centimeter. Units other than those indicated may be used if desired. Those given are such to produce k in darcys, the commonly accepted unit. *A millidarcy is one-thousandth of a darcy.*

To express the quantity of fluid that will flow through a given reservoir system of specified geometry and dimensions, it is necessary to integrate the differential Equation 1 given above, over the boundaries of the system desired. Natural reservoir systems do not ordinarily conform to any simple geometrical form. Nor are we able to produce from reservoirs so that a simple geometry is inherent. Usually, therefore, the application of Darcy's law to a reservoir system must be in the nature of a simplification. Two ideal systems are used as being most widely adaptable to producing systems. These are the linear flow system and the radial flow system.

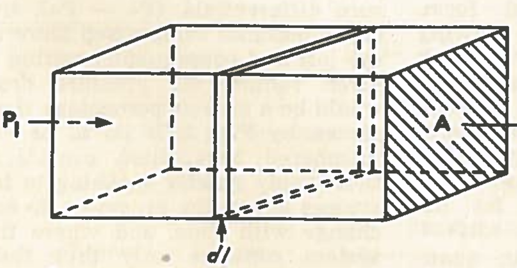

Fig. 1—Typical linear flow system

In the linear system, it is assumed that flow occurs through a constant cross-sectional area, that the ends of the system are parallel planes, and that the pressures at either end of the system are constant over the end surface. A typical representation of this is given in the accompanying figure.

It will aid in the understanding of permeability and fluid flow if the equations holding for this system are developed. Darcy's law for a dl segment of this system is:

$$\frac{Q}{A} = -\frac{k}{\mu}\,\frac{dp}{dl}$$

If a noncompressible fluid is flowing, all of the quantities are constant and the integration can be performed directly, thus:

$$\frac{Q}{A}\int_{0}^{L} dl = -\frac{k}{\mu}\int_{P_1}^{P_2} dp$$

which results in:

$$Q = \frac{kA\,(P_1 - P_2)}{\mu L} \qquad (2)$$

If a compressible fluid is flowing, the quantity Q varies with the pressure change. The usually assumed variation is that PQ = constant = $P_m Q_m$ where $P_m = \frac{P_1 + P_2}{2}$ and Q_m is the rate at P_m. The integration is, therefore:

$$\int_{0}^{L} dl = \frac{-kA}{P_m Q_m}\int_{P_1}^{P_2} P\,dp$$

$$Q_m = \frac{kA\,(P_1 - P_2)}{\mu L} \qquad (3)$$

Thus, it is evident for the linear system that gas flow and liquid flow can be calculated by the same equation provided the rate is measured at the pressure of the system. These equations are the ones most commonly used for the determination of permeability in the laboratory.

New nomenclature:

μ = viscosity of fluid flowing

k = permeability

P_m = arithmetic mean pressure of flow

Q_m = volume rate of flow measured at P_m

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ENGINEERING *fundamentals*

Darcy's Law—Radial Flow System

THE radial flow system has been used as an approximation of the quantitative expression of flow into a well from a surrounding reservoir. The present discussion will indicate the form of this equation and some of its characteristics.

Fig. 1 illustrates the properties of a radial system. Flow is assumed to occur between two concentric cylindrical surfaces, the outer of radius R_e and the inner of radius R_w. Over the cylindrical surface at R_e the pressure is a constant value P_e and over the cylindrical surface at R_w the pressure is a constant value P_w. Flow in this system converges from the outer cylinder to the central cylinder.

When Darcy's law in differential form is applied to a dr cylinder of the system, the resultant integrated equation of flow is:

$$Q_m = \frac{2\pi kh\,(P_e - P_w)}{\mu \ln R_e/R_w} \tag{1}$$

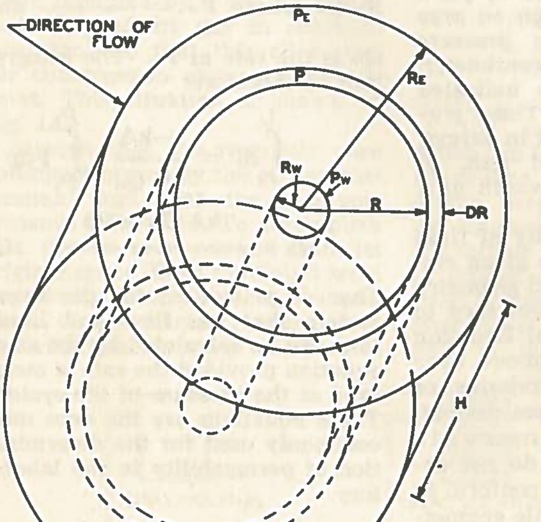

Fig. 1

where Q_m = volume rate of flow measured at the arithmetic mean pressure $\dfrac{P_e + P_w}{2}$

h = height of the radial flow system

and the remaining terms signify quantities as previously defined. This equation expresses either gas or liquid flow since the volume rate is given at a mean pressure.

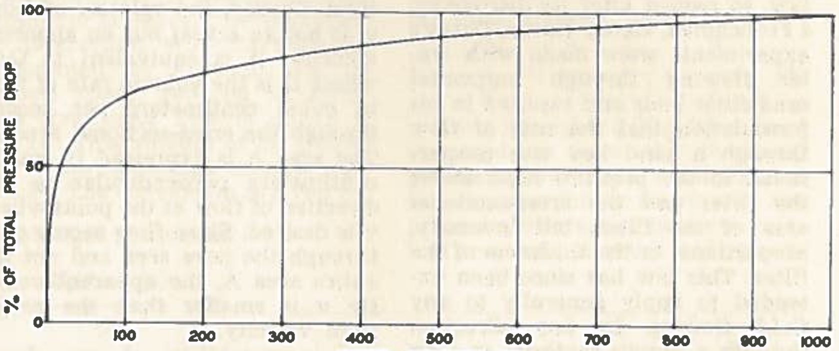

Fig. 2—Ideal pressure drop through a radial system

It has been demonstrated by authorities on fluid flow that Equation 1 will hold to a good degree of accuracy even when the outer boundary of the system deviates considerably from the ideal cylindrical form. Thus, a well drawing from a uniform block system is represented by the radial flow formula within reasonable limits, an average cylindrical surface at radius R_e being substituted for the actual irregular surface.

The change in quantity of flow from a given radial system with a change in well radius R_w is demonstrated by considering the term $\ln R_e/R_w$. To double the rate of flow for a given R_e means to halve $\ln R_e/R_w$. For example for R_e = 1,000 ft., R_w would have to be increased from ¼ ft. to almost 16 ft. to double the rate of flow.

From the equation of flow, the variation of pressure with radius is easily demonstrated. Darcy's law holds for any or all parts of the system and, therefore, from R_w out to any radius R, the equation for a liquid is:

$$Q = \frac{2\pi kh\,(P - P_w)}{\mu \ln R/R_w} \tag{2}$$

For a definite rate of flow, Q, in a system of given k, h, and μ, ln R must plot as a straight line against P. For example, Fig. 2 indicates the variation of pressure with distance from the well in a system in which the outer radius was chosen as 1,000 ft. and the well radius as ¼ ft. Over 50 per cent of the entire pressure drop occurs within the 16-ft. radius and over 75 per cent within the 125-ft. radius. This applies, no matter what the total pressure differential. When gas is flowing, the same curve applies for the pressure distribution if the pressure differential of flow approaches zero. For larger total pressure differentials, $(P_e - P_w)$, the curve becomes compressed more to the left and consequently within a given radius, the pressure drop would be a greater percentage than shown by Fig. 2. It is to be remembered that these considerations apply strictly speaking to instances where the pressures do not change with time, and where the system contains only that fluid which is flowing.

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No. 316

ENGINEERING *fundamentals*

Darcy's Law: Resume of Equations

DARCY'S law has been applied to many ideal systems, and has been presented with many sets of units. The following tabulation is presented to bring together some of these equations and units.

It should first be noted that the dimensions of permeability are L^2. Therefore the permeability unit in the C.G.S. system would be $cm.^2$ and in the foot-pound-second system would be $ft.^2$ Neither of these is a commonly accepted unit, but for certain considerations, fundamental units may at times be necessary. The common unit is the darcy, defined to give a permeability constant of reasonable magnitude and in usable units.

The nomenclature used below is as follows:

k = permeability constant.
A = area
P = pressure
L = length
Q = volume rate of flow
μ = viscosity
R = radius
h = thickness of pay zone
d = distance between unlike wells in a flooding network

Subscripts "w" and "e" refer to well conditions and reservoir conditions respectively.

ln = a logarithm to base e and log designates a logarithm to Base 10
T_f = flowing temperature
ΔP = a pressure difference between unlike wells in a flooding network.

No.	Type flow system	Fluid flowing	Form of Darcy's law
1.	Linear	Liquid	$Q = \dfrac{kA\,(P_1 - P_2)}{\mu L}$ k = darcies A = sq. cm. $P_1 - P_2$ = atm. μ = cp. L = cm. Q = cc. per second
2.	Linear	Gas	$Q_m = \dfrac{kA\,(P_1 - P_2)}{\mu L}$, or $Q_a = \dfrac{kA\,(P_1^2 - P_2^2)}{2\mu L}$ Q_m = cc. per second at ($P_m = \dfrac{P_1 + P_2}{2}$) and flowing temperature Q_a = cc. per second at 1 atm. and flowing temperature k = darcies A = sq. cm. $P_1 - P_2$ = atm. μ = cp. L = cm.
3.	Linear	Liquid	$Q = \dfrac{kA\,(P_1 - P_2)}{\mu L}$ Q = cc. per second k = sq. cm. A = sq. cm. $P_1 - P_2$ = dynes per sq. cm. μ = poises L = cm.
4.	Linear	Liquid	$Q = \dfrac{1.013 \times 10^6\, kA\,(P_1 - P_2)}{\mu L}$ Q = cc. per second A = sq. cm. $P_1 - P_2$ = atm. μ = cp. L = cm. k = sq. cm.
5.	Linear	Liquid	$Q = \dfrac{7.07}{2\pi} \dfrac{kA\,(P_1 - P_2)}{\mu L}$ Q = bbl. per day A = sq. ft. μ = cp. k = darcies $P_1 - P_2$ = psia. L = ft.
6.	Radial	Liquid	$Q = \dfrac{2\pi kh\,(P_e - P_w)}{\mu \ln R_e/R_w}$ Q = cc. per second k = darcies h = cm. R_e and R_w = cm. (P_e and P_w) = atm. μ = cp.
7.	Radial	Liquid	$Q = \dfrac{7.07\, kh\,(P_e - P_w)}{\mu \ln R_e/R_w}$, or $Q = \dfrac{3.07\, kh\,(P_e - P_w)}{\mu \log R_e/R_w}$ Q = bbl. per day k = darcies h = ft. R_e and R_w = ft. μ = cp. P_e and P_w = psia.
8.	Radial	Gas	$Q_m = \dfrac{7.07\, kh\,(P_e - P_w)}{\mu \ln R_e/R_w}$, or $Q_a = \dfrac{7.07\, kh\,(P_e^2 - P_w^2)}{2\mu \ln R_e/R_w}$, or $Q_s = \dfrac{.0382\, kh\,(P_e^2 - P_w^2)\, T_f}{\mu \ln R_e/R_w}$ Q_m = bbl. per day at ($P_m = \dfrac{P_e + P_w}{2}$) and flowing temperature Q_a = bbl. per day at 1 atm. pressure and flowing temperature Q_s = cu. ft. at 60° F., and 1 atm. k = darcies h = ft. μ = cp. P_e and P_w = psia. R_e and R_w = ft. Q_f = °R.
9.	Five-Spot	Liquid	$Q = \dfrac{7.07\, kh\, \Delta P}{2\mu\,(\ln d/R_w - 0.619)}$, or $Q = \dfrac{1.54\, kh\, \Delta P}{\mu\,(\log d/R_w - 0.269}$ Q = bbl. per day k = darcies h = ft. ΔP = psia. μ = cp. d = ft. R_w = ft.

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ENGINEERING fundamentals

Effective and Relative Permeability

It was noted in a previous discussion that Darcy's law for flow in porous material was predicated upon the condition that the material was entirely saturated with the flowing fluid. Such a circumstance does not often exist in nature, particularly in petroleum reservoirs. Gas or oil is usually found coexistent with water, and frequently gas, oil, and water may occupy together the pores in a portion or all of a reservoir. The present discussion will outline the use of Darcy's law for flow when the fluid under consideration occupies less than all of the pore space.

Ability of a porous material to conduct a fluid when the saturation of that fluid in the material is less than 100 per cent of the pore space is known as the **effective permeability** of the material to that fluid. The effective permeability is written by using a subscript to designate the fluid under consideration. For example, k_o, k_g, and k_w mean respectively the effective permeability to oil, the effective permeability to gas, and the effective permeability to water. The effective permeability is used mathematically in Darcy's law in place of permeability. For example, the expression for flow through the linear system under a partial saturation of oil is written:

$$Q_o = \frac{k_o A (P_1 - P_2)}{\mu_o L} \qquad (1)$$

where Q_o designates the volume rate of oil flow at the particular oil saturation in mind, μ_o is the viscosity of the oil flowing, and the other quantities are as previously defined.

Of course, it would be expected that for a given system (A, P_1, P_2, μ_o, and L being constant) the value of Q_o would increase as the pore spaces of the material in question contained more oil. It has been found experimentally that at a given value of fluid saturation the value of the effective permeability to that fluid is constant. Thus, a change in effective permeability depends only upon a change in saturation. This conclusion has been tested upon specific porous systems under varying pressure differentials, varying values of interfacial tension, and varying viscosity ratios.

Effective permeability will, of course, vary from zero to the value of the permeability at 100 per cent saturation. For a given reservoir material and given fluids there is no means of predicting the effective permeability at a given fluid saturation. One must rely upon experimental measurement. No two porous bodies will necessarily have the same variation of effective permeability with saturation.

A term more useful for correlating purposes is **relative permeability.** This is defined as the ratio of the effective permeability to a given fluid at a definite saturation to the permeability at 100 per cent saturation. The terminology most widely used is simply k_g/k, k_o/k, k_w/k, meaning the relative permeability to gas, oil, and water respectively. Since k is a constant for a given porous material, the relative permeability varies with the fluid saturation in the same fashion as does the effective permeability. The relative permeability to a fluid will vary from a value of zero at some low saturation of that fluid to a value of 1.0 at 100 per cent saturation of that fluid.

Although relative permeability is strictly defined as given above, another useful definition is sometimes given as the ratio of the amount of fluid flowing at a given saturation to the amount which would flow at 100 per cent saturation, conditions of pressure being the same. By formula:

$$k_o/k = Q_o/Q$$

Likewise, the ratio of any two effective permeabilities is equivalent to the ratio of the rates of flow at the two saturations considered, pressure differentials being constant. By formula:

$$k_o/k_o' = Q_o/Q_o'$$

Relative permeability for a given system cannot be calculated but must be measured experimentally. Nevertheless, there are certain characteristics of the relationship between relative permeability and saturation which can be noted. For example, Fig. 1 shows the relative permeability to gas and oil versus saturation in a given porous material. Observe that zero permeability to oil exists at a definite oil saturation, that the sum of the relative permeabilities at a given saturation value do not add up to 1.0, that a definite gas saturation is necessary before gas will flow, and that a small amount of oil does not decrease the relative permeability to gas as rapidly as a small amount of gas decreases the relative permeability to oil. These observations apply quite generally to most relative-permeability curves.

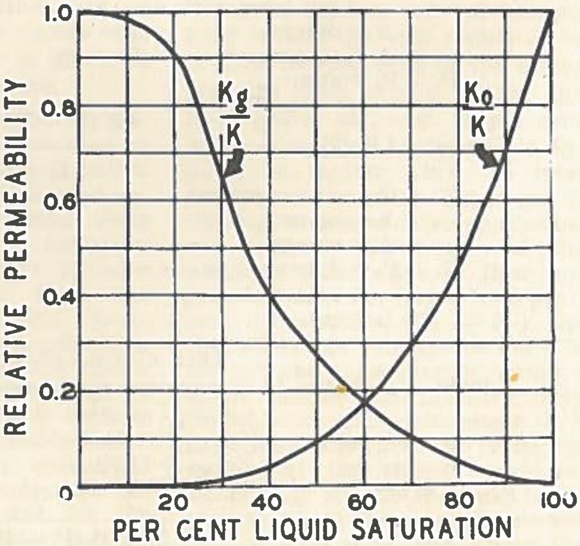

Fig. 1—A typical relative permeability saturation relationship (After Muskat et al. Trans. A.I.M.E., Vol. 123)

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Combination of Permeabilities—Parallel Beds

THE permeability of a reservoir is determined from measurements made upon small samples taken from the reservoir. In most reservoirs there will be both a lateral and vertical variation in permeability. It is desirable to know how to re-combine the individual permeability measurements, representing several portions of the reservoir, into an average value. The present discussion will outline the calculation of average permeability on beds in parallel, assuming that each individual bed is homogeneously permeable.

Consider the situation of several beds lying in parallel (Fig. 1), each of which is separated from the others so that no cross flow can occur. For sake of illustration, consider that the beds are in a radial system, although the application can be made as readily to other systems. It is also assumed that the physical dimensions and pressure boundaries are the same for all beds. By Darcy's law, for the first of the beds, the rate of fluid flow through it is

$$Q_1 = \frac{2\pi k_1 h_1 (P_e - P_w)}{\mu \ln R_e/R_w} \qquad (1)$$

and for each succeeding bed the flow Q_2, Q_3, etc. is expressed by a similar equation with the quantities k_2h_2, k_3h_3, etc. Now, the flow through the entire system of beds equals the sum of the flow through the individual beds. Thus

$$Q_t = Q_1 + Q_2 + Q_3 + Q_4 + \ldots \qquad (2)$$

And the total flow can also be expressed by the use of the average permeability desired as

$$Q_t = \frac{2\pi k_{avg} h_t (P_e - P_w)}{\mu \ln R_e/R_w} \qquad (3)$$

where h_t is the sum of the thicknesses of the individual beds. Therefore, by substituting Equations 1 and 3 in Equation 2 it is seen that

$$k_{avg} h_t = k_1h_1 + k_2h_2 + k_3h_3 + k_4h_4 + \ldots \qquad (4)$$

Thus, the average permeability is equivalent to the sum of the products of the individual permeabilities and their respective sand thicknesses divided by the entire thickness in question. The term kh is commonly referred to in terms of millidarcy-feet and is known as the "permeability capacity" of a section. It is convenient when dealing with cores to express flow in terms of permeability capacity because the addition of individual terms gives directly the quantity sought for average conditions, as shown by Equation 4.

For example, consider the graph of permeability versus depth shown in Fig. 2.* The average permeability of the interval between 6,185 and 6,212 ft. can be figured by multiplying each plotted permeability by the depth interval which it represents and then dividing the sum of all such products by 27 ft., the entire thickness. Referring to the graph, at a depth of 6,195.5 ft., the permeability of 110 md. represents the equivalent of approximately 0.75 ft. and the product kh for this point is therefore 82.5 md.-ft. This procedure is then followed for each permeability point given. A more expeditious method of carrying out the complete computation in place of making these individual multiplications is to calculate the area to the right of the graph line. This area is shown by the shaded portion of Fig. 2. In this particular case the area is equivalent to 1,886 md.-ft. Therefore, the average permeability of the 27-ft. interval is

$$k_{avg} = \frac{1{,}886}{27} = 69.9 \text{ md.}$$

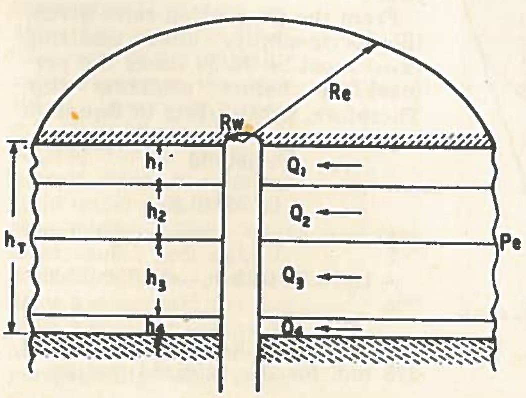
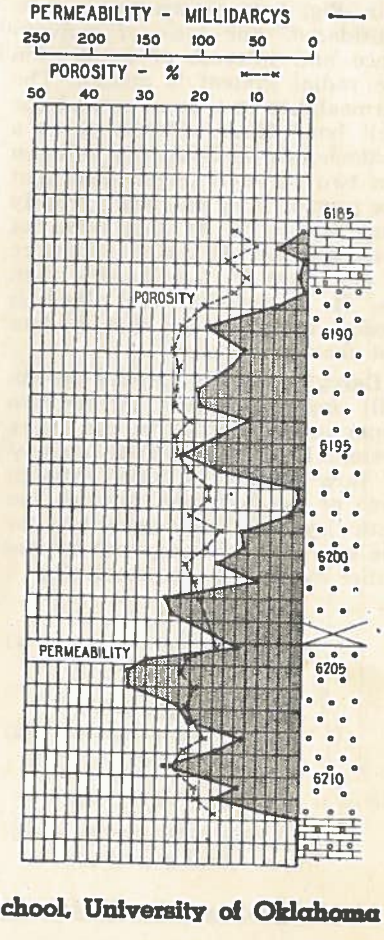


Fig. 1—(Left) Section showing several zones of different thickness lying in parallel. Fig. 2—(Right) Core graph of permeability and porosity (Rodessa field). Total flow capacity is 1,886 md.-ft., and the average permeability is 69.9 md.



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*R. I. 3715, U.S.B.M., Hill and Guthrie, An Engineering Study of Rodessa Oil Field, page 84.

No. 319

ENGINEERING *fundamentals*

Combination of Permeabilities—Series Beds

THE effect of a change in pay permeability as a result of well completion or other practices is an important consideration in production technique. The magnitude of such changes can be arrived at by the treatment of flow through two permeable beds in series. The problem also has significance in other situations, such as computing an average weighted reservoir permeability and in computing the advance of a fluid interface through a permeable bed. The present discussion will outline the method of attacking the problem of beds in series.

In Fig. 1 is shown the system considered. For sake of convenience and interest of application the radial system is chosen. The permeability in a zone around the well bore differs from that at a distance. At the boundary between the two zones it is assumed that the permeability changes abruptly from k_1 to k_2. At this point of change there will exist a pressure P less than P_e but greater than P_w. It is assumed that flow is steady with time so that P does not change in value.

Darcy's law for a radial system will apply to either of the two zones in question or to the entire system. However, the same amount of flow must take place through each of the beds and through the entire system. The expressions for the two separate zones and for the entire system are, respectively,

$$Q = \frac{2\pi k_1 h (P - P_w)}{\mu \ln R/R_w} \qquad (1)$$

$$Q = \frac{2\pi k_2 h (P_e - P)}{\mu \ln R_e/R} \qquad (2)$$

$$Q = \frac{2\pi k_{avg} h (P_e - P_w)}{\mu \ln R_e/R_w} \qquad (3)$$

The following relationship must also hold between the pressure differentials,

$$(P_e - P_w) = (P_e - P) + (P - P_w) \qquad (4)$$

Therefore, by solving for each of the pressure differentials from Equations 1, 2, and 3, and substituting them in Equation 4, the following relationship is found:

$$k_{avg} = \frac{\ln R_e/R_w}{\frac{1}{k_1}\ln R/R_w + \frac{1}{k_2}\ln R_e/R} \qquad (5)$$

This equation can be extended to the situation of any number of beds in series, in which case the equation is extended to read in the denominator the sum of terms for each permeable zone included. The relationship in Equation 5 can be used as a basis for estimating a number of useful quantities in production work. For example, the effects of mud invasion, acidizing, or well shooting can be estimated from it.

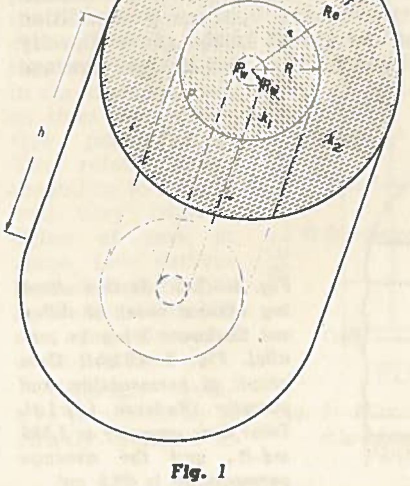

Fig. 1

First, consider the invasion of mud into a producing formation. Let it be supposed that a well of 6-in. bore is drilled into a pay of 500 md. on a spacing of 40 acres. Assume that mud penetrated for a distance of 1 ft. into the pay, and that experiment indicates that the pay will be reduced in permeability, where penetrated with mud, to a value of 10 per cent of its original. It is desired to know to what average permeability the well system is reduced by the mud penetration if it is not removed.

A 40-acre spacing gives a well about the equivalent of a 750-ft. drainage radius (R_e). The reduction by mud to 10 per cent permeability gives 50 md. for the invaded zone. Substituting the quantities given into Equation 5 yields, therefore,

$$k_{avg} = \frac{\ln 750/0.25}{\frac{1}{50}\ln 1.25/0.25 + \frac{1}{500}\ln 750/1.25} = 178 \text{ md.}$$

Due to the amount of mud invasion given, the well system flows as though it were from a pay of 178 md. rather than 500 md.

Second, consider the acidizing of a formation. Assume that a well on 40-acre spacing with a 6-in. bore produces 50 bbl. per day of fluid from a pay of 50 md. permeability before acidizing and 90 bbl. per day after acidizing, both rates being taken under identical producing conditions. Let it be said that sufficient acid had been injected to penetrate .15 ft. into the formation. From this data can be calculated the permeability increase which would have had to occur in the acidized section to produce the observed increase in production rate.

From the production rates given, the permeability after acidizing (k_{avg}) must be 90/50 times the permeability before acidizing (k_2). Therefore, substituting in Equation 5 gives,

$$(50)90/50 = \frac{\ln 750/0.25}{\frac{1}{k_1}\ln 15.25/0.25 + \frac{1}{50}\ln 750/15.25}$$

or, solving for k_1 yields a value of 373 md. for the acidized section.

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ENGINEERING *fundamentals*

No. 320

Relative Permeability Ratio

RELATIVE permeability has been defined previously (Engineering Fundamentals No. 317) as the ability of a reservoir material to conduct a given fluid at a partial saturation of that liquid referred to the ability of the same material to conduct the fluid at full saturation of the fluid. Thus, a relative permeability to oil of 0.5 denotes that at the oil saturation in question, the reservoir will conduct one-half of the quantity of oil that it would conduct under equal conditions of pressure and viscosity if it were completely saturated with oil.

It was also noted that there exists no analytical means of deducing what the relative permeability should be under given circumstances. Although general trends have been established to show qualitatively the variation of relative permeability with the degree of fluid saturation, there has not yet been established a method of deducing the quantitative variation of relative permeability with saturation.

Another useful concept which derives from the relative permeability concept is the relative (or effective) permeability ratio. This quantity lends itself more readily to analysis and to the correlation of flow performances than does relative permeability itself. The relative permeability ratio expresses the ability of a reservoir to permit flow of one fluid as related to its ability to permit flow of another fluid under the same circumstances. The two most useful permeability ratios are k_g/k_o, the relative permeability to gas with respect to that to oil and k_w/k_o, the relative permeability to water with respect to that to oil, it being understood that both quantities in the ratio are determined simultaneously on a given system. The relative permeability ratio may vary in magnitude from zero to infinity.

It has been noted in the literature that the relative permeability ratio lends itself to a simple relationship with oil saturation over a wide range of saturation values. Consider a system flowing gas and oil. At high oil saturations the flow of gas is zero, k_g is zero, and hence k_g/k_o is zero. As the gas saturation increases, k_g increases but k_o decreases and, therefore, k_g/k_o increases. When the oil saturation becomes sufficiently low, k_o approaches zero and the value of k_g/k_o approaches infinity. Fortunately, the form of the variation of k_g/k_o as saturation decreases is fairly regular in character over a wide range of conditions. Fig. 1 is a typical plot of k_g/k_o versus the liquid (or oil) saturation. Those who advanced the concept of relative permeability noted that k_g/k_o plotted against saturation on semilog paper as in Fig. 2 to give linearity over a wide range of saturations. The complete form of a typical semilog relationship is shown. It has become common usage to express the central straight-line portion of the relationship in the analytical form

$$k_g/k^o = a\ e^{-bS_o} \qquad (1)$$

or,

$$\log k_g/k_o = \log a\ ^{-bS_o} \qquad (2)$$

where S_o is the oil saturation (equal to one minus the gas saturation) and a and b are constants characteristic only of a given reservoir material and a given set of fluids. Slope of the linear plot in Fig. 2 is denoted by b.

The use of this analytical expression has been justified in view of agreement between theoretical relationships which may be deduced from it and actual observed data. Many examples of experimentally determined relationships which adhere to this basic concept are available in the literature. The range of saturations over which Equations 1 and 2 hold is the range of gas saturations within which a large share of petroleum production occurs. Their use gives expression to the quantity of gas which may be required to produce a given quantity of oil.

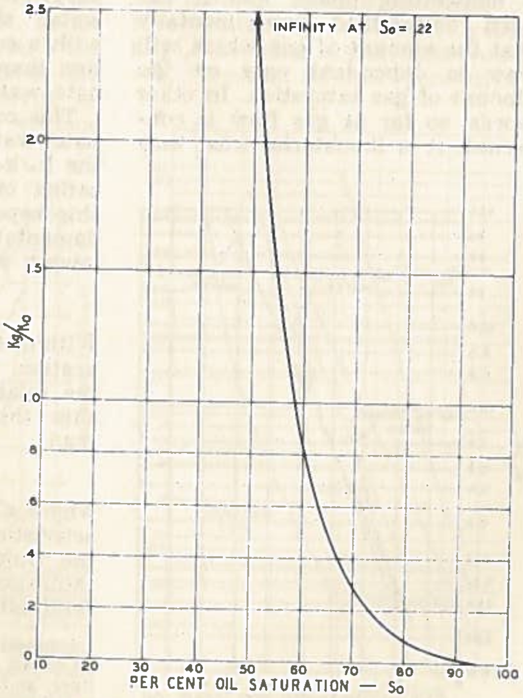

Fig. 1—Typical plot of k_g/k_o as a function of saturation

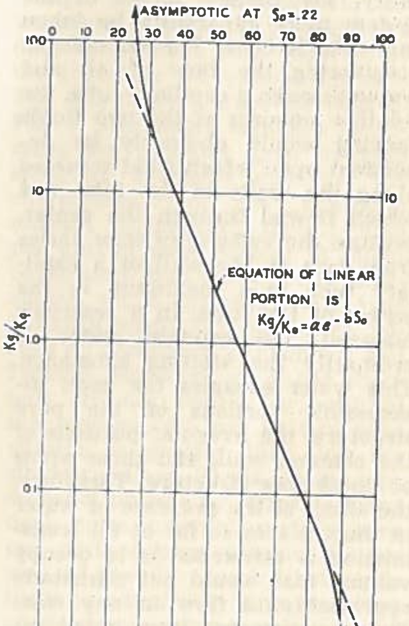

Fig. 2—Typical relationship between k_g/k_o and saturation

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No. 321

ENGINEERING
fundamentals

Effect of Connate Water on Relative Permeability Ratio

In previous instalments the concepts of relative permeability and relative permeability ratio have been described. The relative permeability ratio is a measure of the transmission of one saturating fluid with respect to the transmission of another saturating fluid at a given saturation condition. Since most naturally occurring reservoirs contain three saturating fluids—oil, water, and gas—it is pertinent to inquire into the effect of the presence of the third fluid, water, upon the relative permeability ratio between the oil and the gas. Sufficient experiments have been reported in the literature to allow certain observations to be drawn.

In discussing relative permeability, the surface forces of the system must apparently be taken into consideration. For example, in considering the flow of oil and water through a capillary tube, the relative amounts of the two fluids flowing would obviously be dependent upon which fluid traveled along the walls of the tube and which flowed through the center, because the velocity of flow varies from zero at the wall of a capillary tube to a maximum in the center of the tube. In a reservoir material, the connate water is ordinarily the wetting substance. This water occupies the most inaccessible portions of the pore structure, the irregular portions of the channel wall, and those areas of finest pore structure. Therefore, the effect of the presence of water in these places so far as oil transmission is concerned is to occupy volume that would not contribute appreciably to flow in any case. This function has been noted recently in some published relative permeability curves.*

Buildup of Saturation

It was previously noted that one characteristic in a porous system flowing oil and gas was the buildup of a definite oil saturation before any oil would flow. This volume occupied must be a volume which does not contribute very greatly to flow. In this respect, the amount of oil required can be regarded as being that necessary to fill up the same irregularities and small pore spaces that the connate water would fill. One might, therefore, anticipate the conclusion that an amount of wetting fluid, whether it be oil or water, up to an amount equal to the quantity of water which could be held in place without having water flow ensue, would affect very little the amount of oil flow at a specific gas saturation.

The gas phase, is in any instance, a nonwetting phase, and it has been determined experimentally that the amount of gas which will flow is dependent only on the amount of gas saturation. In other words, so far as gas flow is concerned, it is immaterial what may be the nature of the other fluids present so long as the gas continues nonwetting. Thus, the conclusion is reached that for a definite gas saturation, the flow of gas should be a constant, and if the water saturation is at a value below that value of saturation which initiates oil flow, the flow of oil should be essentially constant. Hence, the relative permeability ratio, k_g/k_o, should also be a constant under these conditions.

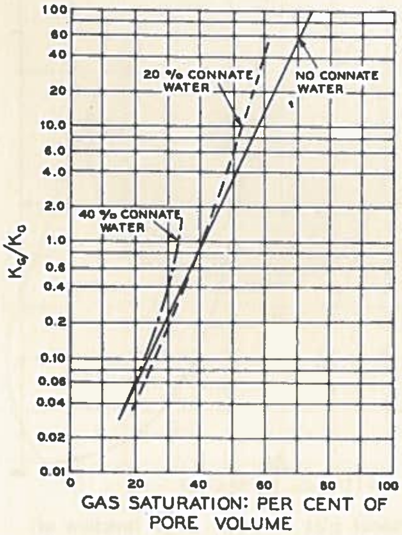

Fig. 1—Effect of connate water upon the k_g/k_o value

Reported experiments have verified this concept of behavior. Fig. 1 is a reproduction of data presented by Leverett.† In this instance when oil and gas were flowed in the absence of connate water, 20 per cent oil saturation was found necessary to initiate oil flow. With 20 per cent water in place of oil, Fig. 1 shows that these experimenters found that the k_g/k_o relationship is essentially unchanged in low-gas-saturation regions and deviates only slightly at higher gas saturations.

Oil Flow Would Decrease

At a connate-water saturation which is definitely above that oil saturation required to initiate oil flow, it could not be expected that the k_g/k_o relationship would remain unchanged. Although increased water content would not change the gas flow at a given gas saturation, it would decrease the amount of oil flow. Therefore, at a given gas saturation the ratio should be increased. In Fig. 1 the curve for 40 per cent connate water shows this characteristic with a considerably greater deviation than for the 20 per cent connate water.

This concept of no effect of connate water upon the position of the k_g/k_o curve allows a generalization of the analytical relationship reported in Engineering Fundamentals No. 320. There the following was noted:

$$k_g/k_o = ae^{-bS_o} \qquad (1)$$

Within that region of water saturation which has no effect upon the relative permeability relationship this can be generalized to read

$$k_g/k_o = ae^{-bS_t} \qquad (2)$$

where a and b are constants characteristic of the porous body and the fluids involved, S_o is the oil saturation, and S_t is the total fluid saturation.

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*"Relative Permeability Measurements on Small Core Samples," Morse, Terwilliger, and Yuster, The Oil and Gas Journal, August 23, 1947, p. 109.

†"Steady Flow of Gas-Oil-Water Mixtures Through Unconsolidated Sand," Leverett and Lewis; A.I.M.E. Transactions, 1941 (Vol. 142) pp. 107 ff.

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No. 322

ENGINEERING *fundamentals*

Relative Permeability in Various Types of Media

IT is apparent from the amount of work that has been performed upon relative permeability measurements that in any given instance the variation shown by the relative permeabilities and by the relative permeability ratio as a function of saturation is dependent upon the type reservoir material in question. The purpose of the present presentation will be to discuss some typical relative permeability curves from the standpoint of a qualitative variation with the type of media through which flow occurs.

In considering flow of oil and gas simultaneously, it has been noted that a certain definite saturation of the wetting phase, oil, is necessary before oil flow ensues. This required saturation should be some function of the internal pore structure of the material under consideration. The greater the number of dead pore spaces, the greater the pore irregularity, or the greater the internal surface area per unit of pore volume, the greater should be the saturation of oil necessary to initiate oil flow. Under the same circumstances which contribute to increased oil saturation for initiating oil flow, a lesser reduction in gas flow should occur with this oil saturation.

Relative Permeability Variations

Another characteristic of relative permeability to oil is that a small gas saturation usually suffices to markedly reduce the oil flow. Examining this effect of low gas saturation, one might logically deduce that the rate of decrease of oil flow as free gas increases would be a function of pore structure. Since the gas is considered to be always nonwetting, it would seem that its presence would give the greatest oil-flow decrease in those situations where it would have a blocking action. This would occur least in an open-pore structure and most in a pore structure having many irregularities and constructions. Furthermore, in a given porous material which has many pore irregularities so that large amounts of oil are required to initiate flow, a small gas saturation reduces by a greater percentage that portion of the pore space which is actually doing the conducting. From either viewpoint, a more rapid decrease of oil relative permeability should occur in the more complex pore structure.

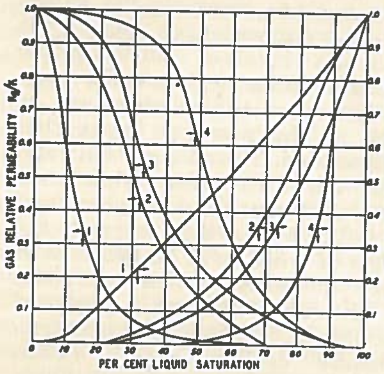

Fig. 1—Relative permeability for various types of media. (1) Capillary tube (Martinelli, Putnam and Lockhart, A.I.Ch.E. Trans., Aug. 25, 1946). (2) Dolomite (Bulnes and Fitting, A.I.M.E. Trans., Vol. 160, 1945). (3) Unconsolidated sand (Botset, A.I.M.E. Trans., Vol. 136, 1940). (4) Consolidated sand (Botset, A.I.M.E. Trans., Vol. 136, 1940)

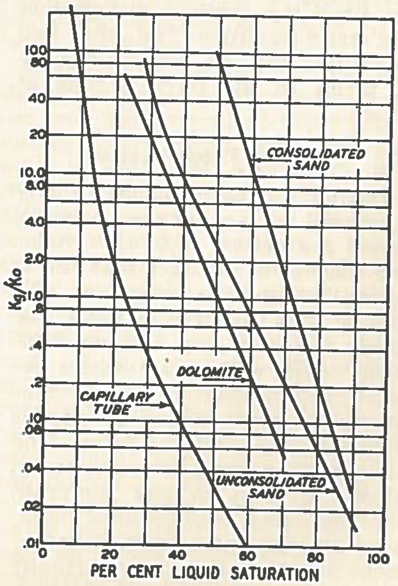

Fig. 2—Relative permeability ratios for various types of media (References same as for Fig. 1)

Unfortunately, we do not possess at present any defined variation of pore structure to enable specific correlations to be made. It would be very desirable to locate a parameter describing pore structure, the use of which would completely delineate a porous body for correlation purposes. Permeability, porosity, and internal surface area might enter into such a parameter. Since no parameter is presently available reservoir materials are classified broadly as to pore structure depending upon the type of material. It is commonly recognized that the pore patterns of limestones and dolomites differ from sands. Within each of these groups there are also differing classes of pore structure. Sands, broadly speaking, are classified as consolidated or unconsolidated. Limestones have been classified according to the manner in which the porosity was formed.

Figs. 1 and 2 demonstrate the variation of relative permeability data which have been reported in the literature for various systems. Fig. 2 is derived from the data in Fig. 1. The single capillary represents the simplest type of pore structure available and has the characteristic of the minimum amount of internal surface area per unit of pore volume. It might be deduced that dolomites or limestones would approach this type behavior since they are often porous by possession of channels and fissures. However, since the channels are not regular or uniform the surface area per unit of pore volume is greater than for the single capillary. Sands, on the other hand, possess by nature a great deal more internal surface area per unit of pore volume. In the case of heavily consolidated sands there are probably large numbers of dead-end pore spaces contributing to saturation but not to flow. Either of these conditions tends to increase the oil required to initiate oil flow and to increase the rate of oil-flow decrease at low gas saturations.

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No. 323

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Resume of Reservoir Forces And Controlling Factors

FOR a proper understanding of reservoir performance, it is necessary to have a knowledge of the forces which control or enter into the behavior of fluids within reservoirs. Once the basic forces have been enumerated, individual behaviors can be better analyzed in the light of the relative importance of each of the forces and correlated thereby.

It is generally accepted that three primary forces are to be considered. There are (1) forces of viscous resistance, (2) gravity, and (3) forces of capillary action. If there are other primary forces operative, due to any of the earth's natural force fields, or due to producing practices imposed, they are not at present defined or recognized.

The viscous resistance of fluids is a commonly understood term. These forces come into play in direct proportion to the pressure differentials present within a reservoir and are a function of the type of reservoir and type of fluid considered. They are retarding and not producing forces. Work must be performed against them in producing fluid from a reservoir.

Gravity can be considered as a driving force so far as reservoir performance is concerned. Its likeliest manifestations are through the segregation of fluids within the reservoir and through drainage down structure.

Forces of Capillary Action

Forces of capillary action are probably least understood of the group. These are the forces which allow a reservoir to retain an amount of connate water within the oil zone against the tendency of gravity to remove it to the bottom of the reservoir, and which act in a large measure to retain oil within the reservoir pore space against a producing differential. These forces are at times called surface forces and are in direct proportion to the amount of surface area involved. Since the surface area of reservoir material is relatively large, these forces assume importance. Capillary forces must most generally be considered as retaining forces so far as petroleum production is concerned but could under some situations act as expulsive forces.

The action of these three groups of forces to varying degrees give rise to the three basic engineering quantities which it is desirable to evaluate. They control (1) the original distribution of fluids within the reservoir, (2) the simultaneous movement of fluids through a reservoir, and (3) the displacement of one fluid by another within the reservoir. The first of these quantities can be described in terms of capillary forces and gravity since the original distribution is a static condition. The other two quantities can be described only in terms of all three groups of forces.

Primary Forces Active

During the performance of any reservoir all of these primary forces are active. It is an erroneous concept to consider that one is acting to the exclusion of the others. It is true that at times one group of forces may become dominent to the extent that others become insignificant. This is commonly assumed when gravity is discussed as a driving force. The effects of gravity are, however, always present in any reservoir. Viscous forces become most dominant at high rates of fluid movement and thus they may virtually overshadow effects of gravity. Capillary forces require time for adjustment of equilibrium. Therefore, they should be operative to a higher degree of effectiveness at low rates of fluid movement. At high rates of fluid movement their action may be negligible.

It is apparent, therefore, that the relative play enjoyed by each of the three groups of forces in any given case is dependent upon a number of factors. These factors could be given as a list of those variables necessary to consider in analyzing the behavior of a reservoir. For convenience these variables can be grouped naturally as follows: (1) the nature of the reservoir, including structure, internal pore arrangement, and chemical nature of the surface; (2) the nature and the relative amounts of the reservoir fluids; (3) the conditions of pressure and temperature, and (4) the operating conditions imposed upon the reservoir.

Must Consider Separate Quantities

Within each of these groups, moreover, it is necessary to consider many separate quantities. For example, the nature of reservoir fluids must consider gas solubility, viscosity, compressibility, density, etc. In any given reservoir when discovered the variables listed under (1), (2), and (3) are already fixed. It is only those variables in the fourth group which permit the operator any freedom in fixing himself the manner of performance of the reservoir. However, the relative role of each factor can be determined to a certain extent by operating procedure, and it is necessary to evaluate the role of all factors to know to what extent the effect of each is changed by a change in operating procedure.

In a given reservoir the values which are placed upon each of the variables involved suffice when taken together to define the magnitude or relative importance of each of the primary forces first enumerated. This in turn will evaluate the distribution, simultaneous movement, and displacement of fluids within the reservoir, the effect of which will be manifest in those quantities which are taken as criteria of reservoir performance, namely, rates of production, pressure decline, gas-oil ratios, water-oil ratios, and the ultimate recovery. At the present time we can follow this sequence of causes and effects through only partly because we understand how only a few of the variables enter in a quantitative sense. We understand most of them only in a qualitative manner.

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No. 324

ENGINEERING fundamentals

A Summary of Reservoir Energies

THE production of crude oil from a reservoir occurs by the action of some natural or secondarily imposed energy. This energy can be present in the form of the gas which is in solution in the oil and which will be evolved under a reduction in pressure, as free gas under compression, as water under compression, as a hydrostatic head of water behind the oil, or as compression of the oil itself. In any given reservoir, any or all of these forms of producing energies may be available. In addition, oil may be produced from a reservoir under the action of either gravitational or capillary forces as was outlined in the preceding installment.

For the production of crude oil from a reservoir two general conditions are necessary. First, it is necessary that a driving force exist, i.e., between the reservoir and the well bore there must be some pressure differential. It is also necessary that the oil be displaced by some other fluid. The displacing substance may be gas or water. The type of driving energy available determines what will be the nature of the displacing substance. Therefore, the effect of the various reservoir energies is considered in terms of displacing mechanism or efficiency.

Reservoirs are ordinarily classified according to the type reservoir energy that is available because the type of reservoir energy determines the recovery mechanism of the reservoir. We classify four distinct types of petroleum reservoirs, although it is not inferred that all can act independently. These are (1) **solution gas drive reservoirs**, (2) **gas expansion reservoirs**, (3) **water drive reservoirs**, and (4) **gravitational drainage reservoirs.**

By a solution-gas drive is meant the propulsion of oil to the well by means of the gas which comes from solution. This involves the concept of the simultaneous flow of the oil and the gas after it has formed. By free-gas expansion drive is meant the displacement of oil, not by gas flowing with it, but by gas pushing it in the manner of a piston. The differentation of behavior according to the expansion mechanism or to solution gas cannot be made definite, due to the fact that displacement of oil by gas in a piston-like fashion occurs only between zero gas saturation and a relatively low amount of gas saturation. Beyond this saturation range, the piston-like action breaks down to a simultaneous flow and the free gas present then behaves only as additional gas in solution.

Gravitational drainage must be differentiated from solution-gas drive and gas-expansion drive. It is not meant by this statement that the force of gravity is not present in a reservoir. What is meant is that the term gravitational drainage connotes a situation in which there is a countercurrent action. The reservoir liquids flow downstructure and simultaneously the gas moves upstructure. Thus, depending on conditions favorable to the separation of oil and gas, a solution-gas-drive reservoir may give rise to an expanding gas cap.

It should be pointed out that in the majority of known instances, not one, but several of these producing mechanisms may be at work or available. A reservoir is, strictly speaking, a solution-gas-drive reservoir only when the volume is definitely fixed at a constant value; when there is no water available; and when the reservoir permeability, amount of structure, and oil viscosity are such as to preclude absolutely any segregation of gas within the reservoir. On the other hand, a reservoir may be water driven or have water available for drive but not at a rapid enough rate to make the reservoir completely water displaced, in which case it may be substantially a solution-gas-drive reservoir.

It is axiomatic that gas will be produced from solution within the reservoir any time the pressure on the reservoir decreases blow the pressure of saturation. Thus, any reservoir which is subject to a decrease in pressure below the saturation point will have solution gas evolved, which gas will act as a producing and displacing agent. In this sense, very few reservoirs can escape partial control by solution gas.

The operating practices which are imposed on any given reservoir can control those producing energies which are used to the greatest advantage. In a few circumstances where some water drive is available, a gas cap is present, and a large amount of gas is in solution, the operating techniques which are imposed will determine which of these mechanisms or energies are used and to what extent.

It may never be possible to exclude any one mechanism entirely, but it is possible to make any one dominant or insignificant. Such possibilities must be taken into consideration when a reservoir is being analyzed. The factors which were previously enumerated as controlling reservoir forces contribute to the control of producing energy. For example, the internal pore structure of a reservoir and the characteristics of the fluids are important in determining the degree to which any mechanism can be made to operate.

It has been demonstrated by study of various types of reservoirs that it is common to expect the presence of certain types of reservoir energies in definite types of reservoirs. For example, water drive is not usually present to any great extent in a lenticular reservoir because such reservoirs are often stratigraphically sealed on all sides or are not continuous far enough to make any water large enough in volume to be important.

On the other hand, reservoirs which are of wide extent in sands which are uniform and which extend over a large area may very well be subject to water drive. Reservoirs which have much structure and are high in permeability are likely to be gravity controlled. It is furthermore usually anticipated that reservoirs within a given producing horizon will show a characteristic type of producing energy available. For example, the reservoirs in the Smackover lime of Arkansas are usually dominantly water-driven reservoirs. On the other hand, the producing energy of the Appalachian sand reservoirs is predominantly solution or gas-cap energy.

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ENGINEERING *fundamentals*

No. 325

Capillary Forces—Surface Tension

THE forces which express the molecular action between the various solid, liquid, and gas phases in a reservoir are called capillary forces. They are so named because one of the most obvious manifestations of their existence is the behavior of a liquid within a capillary tube. The present discussion will outline the basic terms necessary for an understanding of these forces.

At the boundary between any two phases (that is, between liquid and gas, liquid and liquid, or liquid and solid) there is an unbalance of molecular forces, the net result of which is to tend to contract the boundary area to a minimum value. Consider an amount of water in a vessel. An average molecule within the body of the water will be uniformly attracted in all directions by other molecules. However, a molecule in the surface has no water molecules above it, and thus the resultant force tends to attract the surface molecule into the liquid. Looking at this from another standpoint, every new molecule put into the surface area would require that work be done on it. To create a new unit of area of surface would require an amount of work done comparable to the number of molecules in the unit area. The amount of work in ergs which must be done to create 1 sq. cm. of a surface is termed the **surface energy** of the substance.

A term which is used more commonly than surface energy is **surface tension.** This is known as the force in dynes acting in the surface perpendicular to a line of 1 cm. length and for a distance of 1 cm. in order to produce the new uit area of surface. It is, therefore, numerically equal to the surface energy. The term surface tension is usually reserved for the tension in the surface of a liquid which is in contact with its vapor or with air. If the surface is that between two liquids or between a liquid and a solid, the term interfacial tension is used. The surface tension of water is 72.6 dynes per centimeter. Table 1 lists the surface tension of some hydrocarbon liquids. The interfacial tension between water and hydrocarbons will be of the order of magnitude of 30 dynes per centimeter, and will vary according to the nature of the liquids. The surface tension of a pure substance or the interfacial tension between two pure substances is a definite and constant characteristic of the substance or pair of substances.

Table 1—Surface Tensions at 20° C.

Substance	Surface Tension
Benzene	28.9 dynes/cm.
Cyclohexane	25.3 dynes/cm.
n-Hexane	18.4 dynes/cm.
n-Octane	21.8 dynes/cm.

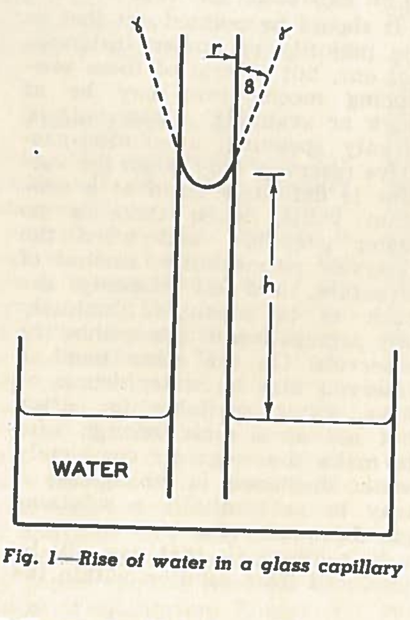

Fig. 1—Rise of water in a glass capillary

One of the simplest manifestations of surface tension is the tendency of free liquid volumes to, assume the minimum possible form, as, for example, a sphere in the case of a free drop of liquid, or in the case of one liquid dispersed within another.

Surface tension can be measured in a number of ways. One of most interest is the rise of a liquid in a capillary tube, a matter of common experience. Fig. 1 illustrates the static condition of liquid held in a capillary. The angle θ that the liquid surface makes with the solid surface is called the **contact angle.** Since surface tension acts in the surface of the liquid, it acts at the angle θ to the solid surface.

The total force which holds the liquid up is equal to the force per unit length of surface times the total length of surface, or

$$F_{up} = \gamma \cos\theta \, 2\pi r \qquad (1)$$

where r is the radius of the tube, γ is the surface tension of the liquid, and $2\pi r$ is the circumference of tube. This force is counteracted by the weight of the liquid, which is equivalent to a downward force of mass times acceleration, or

$$F_{down} = \pi r^2 h \rho g \qquad (2)$$

where ρ is the density of the liquid and h is the height to which the liquid is held. When conditions are static, the two forces must be equal. Equating them and solving for γ gives,

$$\gamma = \frac{r h \rho g}{2 \cos\theta} \qquad (3)$$

This expresses the familiar result that a given liquid (γ = constant) will rise higher in a tube of smaller radius. Different investigators have used this equation to measure the radii of capillary tubes, surface tensions, contact angles, and the height of rise of liquids in tubes.

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ENGINEERING *fundamentals*

Capillary Forces—Wettability

THE concept of surface tension was discussed in Engineering Fundamentals No. 325. This quantity alone is insufficient completely to define capillary forces because the surface or interfacial tension characterizes only the contact between two fluids. The knowledge desired is the manner in which the two fluids behave when in contact with a solid surface. The extra variable necessary to describe this behavior is the contact angle that the fluids make at the solid surface. This term is an evidence of a general property termed wettability.

In Fig. 1 are shown two opposite cases of capillary action or wettability. The rise of water in a capillary tube (Fig. 1) was discussed previously, and it was stated that the relationship between the variables would be:

$$\gamma = \frac{rh\rho g}{2\cos\theta} \qquad (1)$$

In this particular case, it is said that the water wets the glass capillary. In Fig. 1B, the opposite effect is shown. Here is considered the depression of mercury in a glass capillary. Equation 1 relating the variables is still a valid equation for this situation although it is apparent that h will have a negative value. In this case, it is said that the mercury does not wet the glass capillary.

Whether a given liquid will rise or fall in a capillary can be described analytically in Equation 1 by the contact angle, θ. This angle is defined as that measured through the liquid between the plane of the solid surface and the plane of the liquid surface at the point of contact. Thus, for water rising in the capillary, the value of θ is less than 90° and the mercury depressing in the capillary, the value is greater than 90°. Since the cosines of angles between 90° and 180° are negative, h must be negative in Equation 1 when it is applied to Fig. 1B. For a value of θ exactly 90°, the liquid would neither rise nor fall. The point should be stressed, therefore, that surface tension and the characteristic called wettability express different properties of a capillary system. A high surface tension alone will not permit a fluid to rise to a great height in a capillary. The property which determines whether the fluid will rise or fall is that property which gives the contact angle. The contact angle which a given system enjoys will be determined by the nature of the two fluids and the solid surface of the system.

The generality of these considerations is not lost by applying them to the behavior of two liquids within a capillary. Figs. 2A and 2B show the counterparts of Figs. 1A and 1B when oil and water are considered within a capillary. If the situation of Fig. 2A occurs, water is said to wet preferentially the solid surface. If the situation of Fig. 2B occurs, oil is said to wet preferentially the solid surface. In either case, the wettability can be defined by the contact angle, that is, by the fact that its value is less than or greater than 90°.

Equation 1 will not express the relationship of variables in Fig. 2 because the density of the oil is not negligible. The equation becomes:

$$\gamma_{o-w} = \frac{rh(\rho_w - \rho_o)g}{2\cos\theta} \qquad (2)$$

Nomenclature:

r = radius of capillary tube, cm.

θ = contact angle

h = height of capillary rise, cm.

γ = surface tension, dynes/cm.

γ_{o-w} = interfacial tension between oil and water, dynes/cm.

ρ = density, g./cm.^{3}

g = acceleration due to gravity

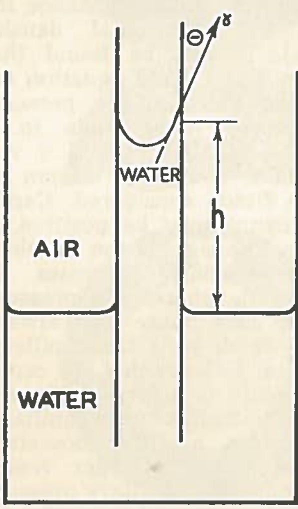

Fig. 1A

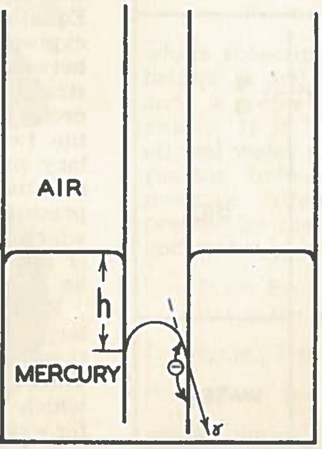

Fig. 1B

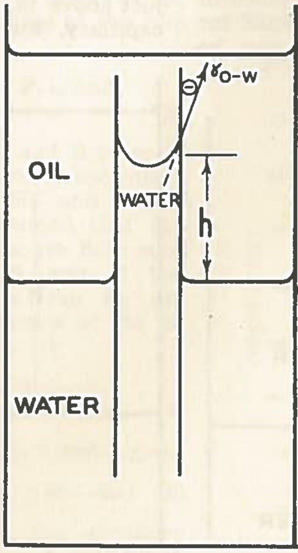

Fig. 2A

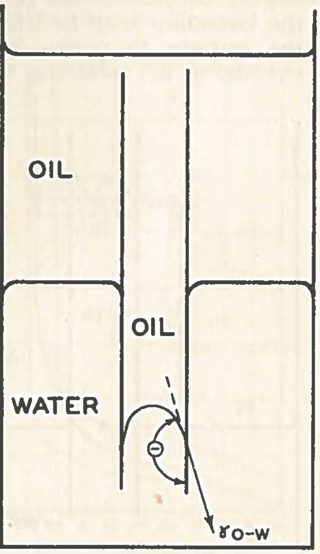

Fig. 2B

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Capillary Pressure—The Straight Capillary

WHEN a discontinuity of phases exists within a capillary system there will be a pressure drop at the phase boundary. This pressure discontinuity is known as the capillary pressure, a term which has become fairly common recently in petroleum engineering. The present discussion will define and outline some of the properties of the term.

Capillary pressure is defined as the pressure drop existing between any two liquid phases, both of which are static, in a capillary system. Thus, in a reservoir we may be concerned with the capillary pressure between oil and water, between oil and gas, or between water and gas. The magnitude of the pressure drop expresses the magnitude of the capillary forces. The usual designation of capillary pressure is P_c, it being understood that a definite pair of phases is considered.

The idea of a pressure drop existing across a phase boundary merely because of the presence of the boundary may be difficult for the initiate to grasp. It can be elucidated by referring to Fig. 1, in which is represented the rise of water in a capillary tube. In this case there are two phases, water and air, within the capillary system. The capillary pressure is $P_c = P_a - P_w$, where P_a and P_w are the pressures on either side of the boundary in the air phase and in the water phase respectively. It can be shown that a pressure drop does exist and its quantitative expression can be determined as follows.

At the point A within the capillary the pressure is the same as that at the point A′ outside the capillary, which pressure is atmospheric, P_a. At the point B, just under the meniscus within the capillary, the pressure is equal to that at A within the capillary minus the head of water. The pressure at B is, therefore,

$$P_w = P_a - h \rho g \qquad (1)$$

where h is the height to which the water has risen and ρ is the water density. Now at the point B′, just above the meniscus within the capillary, the pressure is P_a, the same as that at A′. This statement can be made because the density of the air can be neglected. The pressure across the meniscus (or phase boundary) is, therefore, $P_a - P_w$, or,

$$P_a - P_w = h \rho g \qquad (2)$$

which quantity is the capillary pressure, P_c.

In Engineering Fundamentals No. 325 it was noted that the quantity h could be expressed by the surface tension, γ, and the contact angle, θ, in the relationship

$$h \rho g = \frac{2 \gamma \cos \theta}{r} \qquad (3)$$

Therefore, the capillary pressure for the situation under discussion is

$$P_c = \frac{2 \gamma \cos \theta}{r} \qquad (4)$$

It will be noted that the pressure is smaller in the water phase and water is the fluid which wets the capillary. This is a general consideration. In this particular case the magnitude of P_c could have been determined by evaluating the force causing the fluid to rise and expressing it as a force per unit area, i.e., a pressure.

These considerations are still valid when applied to the capillary containing oil and water as in Fig. 2. However, the pressure at B′ is now equal to that at A′ minus the head of oil, because the oil density is not negligible compared to the water density. The resulting expression for pressure drop across the interface is,

$$P_o - P_w = h (\rho_w - \rho_o) g \qquad (5)$$

but the resulting evaluation for capillary pressure is still as given by equation 4 with γ_{o-w}, because the fluid rise in this instance involves the differential density $(\rho_w - \rho_o)$. It will be found that Equation 4 is a valid equation for expressing the capillary pressure between any two fluids in a straight capillary tube if γ expresses the boundary tension of the two fluids considered. Capillary pressure may be positive or negative. The sign of the capillary pressure merely expresses in which of the phases the pressure is lower. This phase will always be that which wets the capillary.

Equation 4 shows that the capillary pressure is inversely proportional to the radius of the capillary. Thus, across a fluid boundary which is within a larger vessel, for example, the capillary pressure will be zero, or substantially so, because r becomes infinitely large.

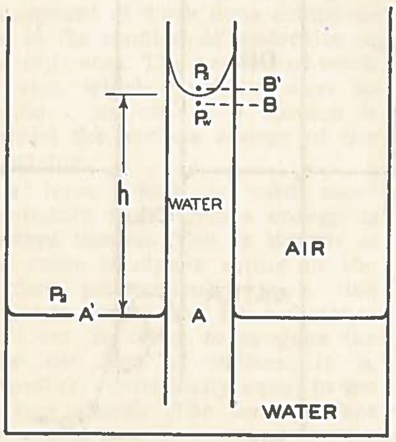

Fig. 1

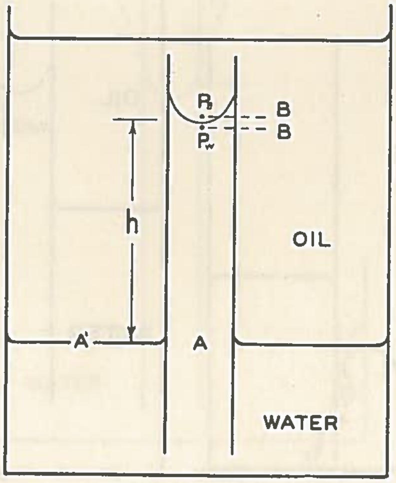

Fig. 2

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ENGINEERING *fundamentals*

No. 328

Capillary Pressure—Unconsolidated Sand

THE concept of capillary pressure as the pressure drop between fluid phases in a capillary system, and the expression for capillary pressure in a single straight capillary was presented in Engineering Fundamentals No. 327. The straight-bore capillary, is, however, an ideal and simple system, and although useful for explaining basic concepts, it does not give the quantitative expression sought for the capillary behavior in porous materials. Reservoir materials also are varying in complexity of pore structure, and a simple or ideal porous structure is required as a starting point to explain their capillary behavior. The ideal pore configuration usually chosen is that made up of spherically uniform particles of definite size, i.e., an unconsolidated sand. The capillary pressure for this system has been presented in the literature* and will be reviewed here.

Consider two spherical grains in contact as shown in Fig. 1 with a wetting fluid at the point of contact. A contact angle of zero will be assumed in order to have the condition of a continuous film of the wetting phase around the sand grain. In this system the capillary pressure is given by

$$P_c = 2\gamma\,(1/R_1 + 1/R_2) \qquad (1)$$

where R_1 and R_2 are the radii defining the curved fluid interface as shown.

Now the values of R_1 and R_2 express the amount of fluid that is contained at the contact, or the saturation of that fluid within the porous body if a number of such contacts are considered. Therefore, Equation 1 states that the capillary pressure is dependent upon the saturation of the fluid phase which wets the system, although the exact nature of the dependence may not be simply stated because the variation of the saturation with R_1 and R_2 may not be simply stated.

Capillary pressure is that pressure across the phase boundary. If water is considered to be the wetting fluid and oil the non-wetting fluid, then the capillary pressure across any interface is

$$P_c = P_o - P_w \qquad (2)$$

Now consider two such contacts as are shown in Fig. 1, at two different heights in a porous section, one at h_A, the other at h_B. The capillary pressures at the two heights are

$$P_{cA} = P_{oA} - P_{wA}, \text{ and}$$
$$P_{cB} = P_{oB} - P_{wB} \qquad (3)$$

where subscripts A and B refer to heights A and B, and subscripts o and w refer to oil and water phases. If it is assumed that the oil and water phases are both continuous between A and B the pressure differences can be expressed by the densities of the oil and water, ρ_w and ρ_o, or

$$P_{oA} = P_{oB} - \rho_o g\,(h_A - h_B)$$
$$P_{wA} = P_{wB} - \rho_w g\,(h_A - h_B) \qquad (4)$$

Combining Equations 3 and 4 gives

$$P_{cA} - P_{cB} = (\rho_w - \rho_o)\,g\,(h_A - h_B) \qquad (5)$$

which states that the capillary pressure is greater at A than at B, or that capillary pressure increases with height in a reservoir.

From Equation 1 it is noted that capillary pressure increases as the radii, R_1 and R_2, decrease. The latter occurs when a smaller amount of water exists at the grain contact. In other words, a smaller water saturation gives a greater capillary pressure. Thus, Equation 5 demands that the water saturation must decrease with height in a reservoir. The exact rate of change of water saturation with height is not readily apparent because the variation of saturation with R_1 and R_2 is not readily apparent.

Although it is not so apparent, Equations 1 and 5 together also demand the result that at a given height within a reservoir, the amount of water that is held by capillary forces will increase as the permeability decreases.

The experimental determination of water or oil saturation held by capillarity as a function of height in a sand column has been performed by a number of investigators, and the results have shown a characteristic type of curve. Fig. 2 shows the curves for oil, water, and normal heptane saturation versus height in an unconsolidated sand column where the fluids were retained due to capillary forces. As shown by Equation 5, these curves could just as well represent capillary pressure instead of height if h_B be taken as zero where P_{cB} is zero, which point will be that of 100 per cent fluid saturation at the bottom of the column. In this particular instance, the amount of fluid that is contained at a given height will vary since Equation 1 includes the factor γ. Furthermore, since these investigators had to repack their column to obtain each curve, the pore structure was probably not exactly duplicated.

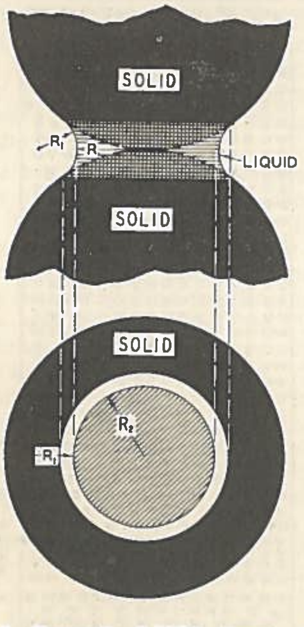

Fig. 1—(Leverett, A.I.M.E. Trans., 1941, Vol. 142, p. 153)

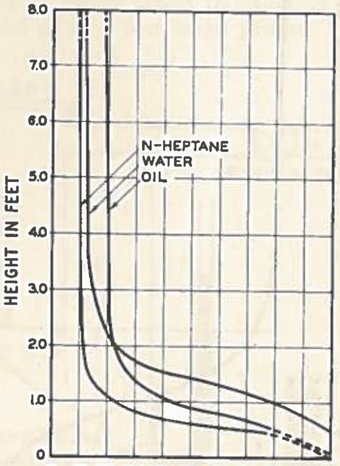

Fig. 2—(Stahl, Martin, and Huntington, A.I.M.E. Trans., 1943, p. 140)

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*Leverett, Capillary Behavior in Porous Solids, A.I.M.E. Trans. 1941.

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Use of Capillary Pressure Curves —Original Field Distribution

THE relationship between water saturation at any point in a porous body and the capillary pressure at that point is known most frequently as the capillary-pressure curve. Since the capillary pressure must vary with height above the free water table in a porous section, the capillary-pressure curve expresses also the relationship of water saturation to height above the free water table. Fig. 1 shows a typical curve. As discussed in previous Engineering Fundamentals, any point on this curve represents an equilibrium condition.

The principal uses for the capillary-pressure curve depend upon the concept of the following sequence of events having occurred during the past history of a reservoir. First, it is assumed that the reservoir was originally filled with water. Second, this water was displaced by the oil which accumulated in the reservoir. Third, displacement of the water was not complete, but rather sufficient water must have been retained to satisfy the capillary forces. Fourth, today, upon discovery, the reservoir exists with amounts of water and oil equal to those required by an equilibrium between gravitational and capillary forces, geologic times having sufficed to insure equilibrium between phases in the reservoir. In other words, the distribution of original connate water in a reservoir is said to be equivalent to that shown by the capillary-pressure curve.

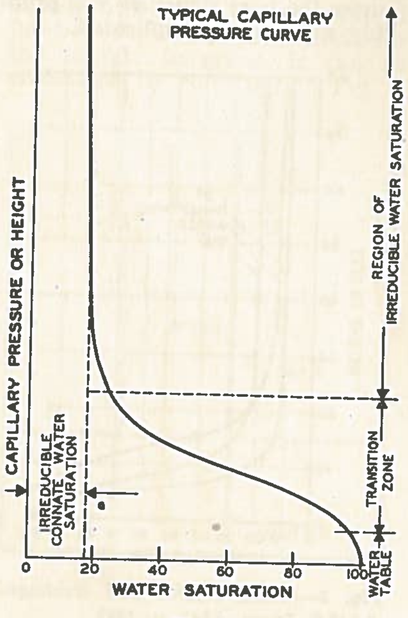

Fig. 1

This concept of connate water being distributed in a reservoir at discovery according to the condition of capillary equilibrium is used in petroleum technology to explain several practical observations long recognized as a matter of common experience. One example was the observed condition that no sharp line of demarcation existed between oil and water levels or between oil and gas levels in a reservoir. Reference to Fig. 1 shows a depth interval within which the saturation changes from 100 per cent water to 20 per cent water gradually. This is referred to as the "transition" zone, which zone must exist in any reservoir where there is a bottom water table. The water-oil contact cannot, therefore, be said to exist at a definite depth, but rather within a range of depth. The thickness of the transition zone depends on the individual fluid and porosity characteristics of each reservoir. In general, less-permeable sands are expected to have a greater transition zone, although the thickness of the zone is more a function of the internal pore-structure arrangement rather than of the permeability constant.

The reservoir section above the oil-water transition zone is said to be the region of irreducible water saturation. It is so called because the water saturation has been reduced to an amount below which it cannot be taken without excessive pressure gradient. The value of irreducible water will not be strictly a constant because the curve is not exactly vertical. It is deduced that this value of water saturation represents the connate water in the pay section of the reservoir. The observed fact that connate water does exist within some reservoirs without being produced is explained by this concept of irreducible water saturation. That is, capillarity holds it in place and flow differentials will not remove it. Connate water in transition zone can be removed by flow.

Just as a transition zone exists between oil and water, so a transition zone exists between oil and gas. The quantitative definition of the oil-gas zone cannot be made so readily as the oil-water because it requires a knowledge of capillary pressures when three phases (oil, water, and gas) are present, and very few data have been collected to define this behavior. It has been suggested* that the transition zone between oil and gas will be more abrupt than that between oil and water. Fig. 2 is a representation of a theoretical distribution of oil, water, and gas according to these concepts. This picture, based on capillary pressure, agrees well with observed reservoir conditions.

It is not suggested that in all reservoirs there exists a transition zone or an irreducible water zone. This would depend on reservoir pay thickness and the presence of a free water table.

*Leverett, A.I.M.E., 1941.

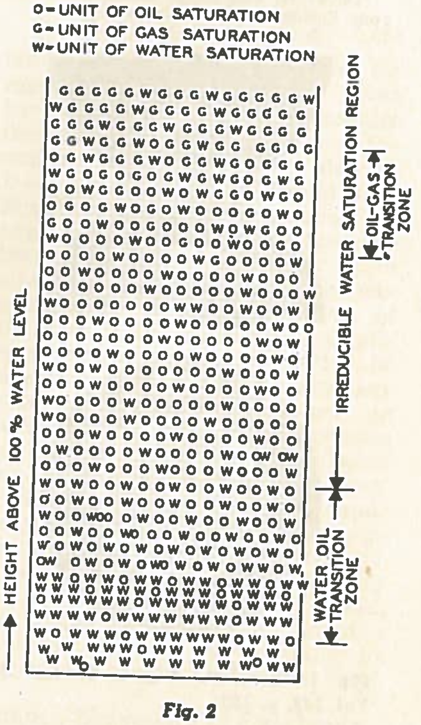

Fig. 2

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Laboratory Determination of Capillary Pressure Curve

THE use of capillary pressure curves, as discussed previously, is based on the assumption that the sequence of events in the development of a reservoir were (a) presence of water, (b) invasion of oils, (c) retention and distribution of water according to capillary forces. Thus, the amount of connate water found in a reservoir should be that which can be held by capillary forces. This quantity can be measured in the laboratory by a similar sequence of events, and gives a possible method of obtaining a value for reservoir connate water.

The present discussion will outline the laboratory determination of this connate-water value. Although the natural event was displacement of water by oil, in the laboratory it is simpler to displace water by air and the following discussion is based on this practice.

A typical apparatus which is used for the determination of a capillary pressure curve and consequently for determination of connate water is shown in Fig. 1. Many variations of the apparatus are used but the basic principles are the same.

The crux of the entire apparatus is the porous diaphragm, so chosen that it will permit water to flow though it, but will not permit air to flow through it under the pressures necessary. The fulfillment of these conditions means that the diaphragm must have very small pores and must be water wet. Various grades of porcelain are most frequently used for this purpose.

To determine the capillary pressure curve, two quantities must be measured—(1) the water saturation in the core at any time, and (2) the capillary pressure at that same time. By definition, the capillary pressure is the difference in pressure between the water and gas phases in the core. The water saturation can be determined either by weighing the core sample from time to time, or by measuring the volume of water that has been removed from the core sample from time to time. The measurement of the pressure difference between the gas and water phases is accomplished by the presence of the porous diaphragm, as will be explained.

The procedure followed can be explained by reference to Fig. 1. The porous diaphragm must first be completely saturated with water. This is usually best done by filling with water after complete evacuation. On the diaphragm is placed a piece of absorbent tissue or a layer of finely divided powder, the purpose of which is to insure good contact between the diaphragm and the core. The core itself is completely saturated with water and placed on the layer of powder. The apparatus is then closed so that an air pressure can be applied. This pressure is set at some definite and constant value, P_{air}.

Because the diaphragm has been chosen with such a fine pore structure that air cannot penetrate it, there will be no air flow except into the core. Air flow into the core will displace water which itself can pass through the diaphragm. However, air will displace water from the core only to the extent that sufficient water remains to satisfy capillary forces. Therefore, after the pressure P_{air}, is applied, water will be displaced through the diaphragm for a while but eventually all flow will stop and the system will be static.

When the system has reached this point of no flow, the pressure in both the water and air phases within the core is known. The pressure in the air phase is that which has been applied. The pressure in the water phase is atmospheric (the pressure below the diaphragm) minus the head, **Δh,** from the bottom of the diaphragm to the center of the core. The capillary pressure (defined as pressure in the air minus that in the water) is therefore determined. At this point, the core can be removed from its holder and the weight determined. Weight of the core minus the dry weight gives the weight of water present and consequently the saturation.

The core is replaced in the holder and the process is repeated, a new air pressure higher than the first being used to determine a new capillary pressure and a new saturation. By making a succession of such determinations the entire capillary-pressure curve is obtained and plotted. As explained in a previous discussion on the capillary-pressure curve, the connate-water value is represented by that value of water saturation below which the core cannot be taken.

In procuring such laboratory data, it is usually found that a waiting period of many hours or several days may be necessary before all the water that can be displaced is displaced at a given air pressure setting. It is also found that more consistent results are obtained by increasing the pressure in many small increments instead of in a few large jumps.

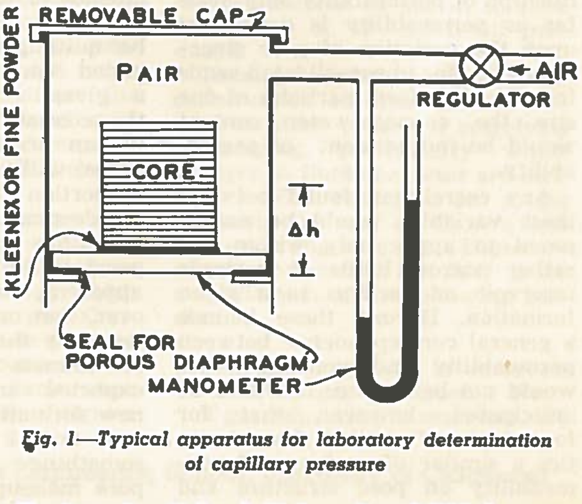

Fig. 1—Typical apparatus for laboratory determination of capillary pressure

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ENGINEERING *fundamentals*

Correlation of Connate Water With Permeability

THE determination of connate water by capillary-pressure measurement (sometimes called the restored-state method) has allowed the evaluation of connate-water values on samples of varying permeability and within a given reservoir to a wider extent and to a greater accuracy than was beforehand possible. These measurements have accumulated to the point where it is possible to correlate connate-water content with the permeability of the sample in a given reservoir and to a certain extent between reservoirs.

In an ideal pore configuration of uniform structure the irreducible connate water would be independent of permeability, lower permeabilities being obtained merely by a scaled reduction in particle size. In an actual porous system formed by deposition of graded particles or by some other natural means, the connate water might be expected to increase as permeability decreases. This conclusion results from the thought that lower permeabilities result from increases nonuniformity of pore structure by a gradation of particles rather than by a scaled reduction of particles. In this sense connate-water content is a function of permeability only insofar as permeability is dependent upon the variation of pore structure. Thus for unconsolidated sands formed of uniform particles of one size the connate-water content would be independent of permeability.

Any correlation found between these variables would be anticipated to apply only within the rather narrow limits of a single reservoir or perhaps to a given formation. Beyond these bounds a general correspondence between permeability and pore structure would not be known. It would be anticipated, however, that for formations of similar characteristics a similar dependence of permeability on pore structure and consequently similar correlation of connate water and permeability would be found.

It has been generally considered for many years that connate water reached higher values in lower permeabilities. This observation amounted to nothing more than a trend. The data from capillary pressure measurements have indicated that the relationship is semilogarithmic, although it is not yet certain from published data that this is the exact relationship. Some authors describing this type of measurement prefer to give a curvature to the semilogarithmic plot. Three references were used to bring together published correlations (references appended). Actually, such published data represent a small percentage of the measurements that have been made by the industry, but they suffice to demonstrate the possibilities.

The connate-water versus permeability trends have been plotted as straight lines in Fig. 1. Twelve separate formations are represented. No generalizations are apparent from this amount of data although it can now be quite generally stated that within a given reservoir the connate water (if an irreducible value) will increase proportionally to the decrease in the logarithm of the permeability. It is apparent, moreover, that one cannot state the value of connate water expected in any new formation unless one knows something of its pore makeup.

It should be noted that the permeabilities referred to in Fig. 1 are air permeabilities. In Reference 3 one group of data was given with water permeability. These had no correlation. This results from the consideration that air permeability gives a measure of pore configuration alone whereas water permeability may combine pore structure and surface effects.

It could be noted that the correlation lines of Fig. 1 possess similarity of slope. Numerical values of the slopes are as follows:

Formation	Slope	Formation	Slope
1	28.2	7	11.4
2	53.0	8	8.4
3	13.0	9	11.5
4	13.0	10	18.4
5	9.0	11	18.3
6	10.3	12	17.6
		13	15.0

With the exception of 1 and 2, these give an average value of 13.3, or in other words the average connate-water increase is 5.2 per cent each time the permeability is cut in half.

References

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2. Estimating Interstitial Water by the Capillary Pressure Method; Thornton and Marshall, A.I.M.E. Transactions, 1947 (170); (Abs.) The Oil and Gas Journal, October 5, 1946, p. 97.

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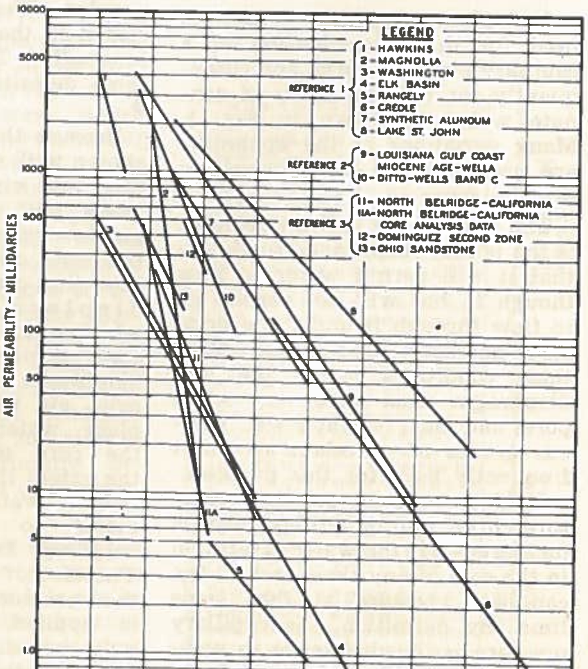

Fig. 1

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ENGINEERING fundamentals

Material Balance Principles

TWO of the basic principles utilized in all engineering work are the law of conservation of mass and the law of conservation of energy. Every engineer is familiar with these principles in one form or another. Using an elementary case, for example, we know that if two separate weights of a substance are combined, the total weight is the sum of the individual weights. Or again, the principles of thermodynamics require that the total energy change in a process will show up either as a heat change or a work change.

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Most natural processes can be broken down into separate unit steps within each of which steps a balance can be made, either upon the amounts of material concerned or upon the amounts of energy concerned. Thus, in the processing of a wet gas, a liquid gasoline fraction and a dry gas are the end products. For this process, a balance can be made merely by stating that the weight of wet gas supplied to the unit in an interval of time must equal the weight of the dry-gas residue plus the weight of the liquid gasoline removed from the unit in the same time interval. Since any of these weights can be expressed as a product of a volume and a density, the expression for such a balance becomes

Wt. wet gas into process = wt. dry gas residue + wt. gasoline produced (1)

Volume wet gas × density wet gas = volume dry gas × density dry gas + volume gasoline × density of gasoline (1a)

In stating the above balance, it is inherent that the units of measurement are the same. Pounds are equated to pounds or grams to grams. Therefore, the density units must be such as to produce the desired weight units. It is immaterial whether volume of gasoline is expressed as cubic feet or gallons so long as densities are correspondingly pounds per cubic foot or pounds per gallon.

Another characteristic required when striking a balance is that any definable fraction of the total must also be balanced. Thus, considering the same process as above, the wet gas, dry gas, and gasoline each include many individual components, i.e. methane, ethane, etc. A balance must hold for each of these. Taking methane, for example, as a definable fraction of the total, the following must be true:

Wt. of methane in wet gas = wt. of methane in residue gas + wt. methane in gasoline produced (2)

Vol. methane in wet gas × density methane = Vol. methane in a residue gas × density of methane + Vol. methane in gasoline × density of methane (2a)

Again it is immaterial how the volumes or densities are expressed so long as the product gives the same unit of weight in each case.

Complexity of Definition

Although all balances may be simply stated in such general terms as "the weight of material into the process equals the weight of the material leaving the process," the definition of each of these weights in terms of measurable quantities may become quite complex. This depends upon the number of variables that affect the process and the number of subprocesses that may be occurring and the manner in which they are interconnected.

A more detailed balance than the one given above would be written for the lifting of oil from a well by means of a gas which is introduced as a dry gas. Consider that an oil which is pumped as a 42° A.P.I. oil becomes a 40° A.P.I. oil in the stock tank when lifted by dry gas because of lighter components taken up by the dry gas during the lifting process. For every barrel of 42° oil which might have been pumped, there will be less than a barrel of 40° oil lifted by gas because of the material lost to the gas. In this case, the balance on weights of material is represented by the two equations:

Wt. of 42° A.P.I. oil = Wt. of 40° A.P.I. oil + wt. of components added to dry gas (3)

Wt. of gas after lift = Wt. of dry gas before lift + wt. of components added to dry gas (4)

In these equations, it is understood that the weights of gas and oil all represent the same interval of the process. In this case, perhaps the simplest reference point to choose so that the quantities are consistent is to choose everything standard to 1 bbl. of 40° A.P.I. oil produced. Thus, the volume of 42° A.P.I. oil involved will be more than 1 bbl. by the liquid volume represented by the components added to the gas, and the quantities of gas will be those injected and produced with 1 bbl. of 40° oil.

Weight Calculation

In this instance any one of the two oil weights or gas weights can be calculated if the other three are known. Furthermore, the problem can be put on a volume basis if the densities are included. It would become more complex to state a balance for this problem if it were considered that the weight of gas after lifing included not only that introduced as dry gas plus those components removed in reducing the oil from 42° to 40° gravity but also that gas which would ordinarily be in solution and be produced along with either the 42° oil or the 40° oil. To include this, one would need to introduce a third equation expressing the amount of gas in solution per barrel of 42° A.P.I. oil produced.

These principles of applying the conservation of mass can be utilized in many phases of petroleum engineering, particularly where changes in fluid character are concerned. For example, on a gas reservoir the use of Boyle's law for predicting reserves *(Engineering Fundamentals No. 304)* is an amplification of the following balance:

Wt. of gas originally in reservoir = Wt. of gas produced + Wt. of gas remaining in reservoir. (5)

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ENGINEERING *fundamentals*

No. 333

Reservoir Material-Balance Equation

THE production of petroleum from an underground reservoir as a unit may be considered as a basic operation to which the law of conservation of mass must apply. As with other similar engineering processes, a balance can be made between the materials in the reservoir and those produced. Balances have been derived and explained by many authors.[1 2 3 4] Most of these balances are complexly stated and seem formidable to the initiate. Perhaps the simplest way of stating that the law of conservation of mass must be observed is by the equation:

> Weight of hydrocarbons in reservoir originally = weight of hydrocarbons produced + weight of hydrocarbons remaining in reservoir (1)

The term hydrocarbons includes all oil and gas, which quantities can be measured only as residual quantities, i.e., stock-tank oil and standard cubic feet of gas produced. Furthermore, the standard cubic feet of gas produced must be expressed in terms of the stock-tank oil in order to account for the amount of gas in solution. The equation becomes upon expansion:

> Wt. S.T. oil and S.C.F. gas originally in reservoir = Wt. S.T. oil and S.C.F. gas produced + Wt. S.T. oil and S.C.F. gas remaining in reservoir (2)

The following is also true:

> Wt. S.T. oil originally in reservoir = Wt. S.T. oil produced + Wt. S.T. oil remaining in reservoir (3)

This separate equation is of no value because the S.T. oil produced is the only one of the three terms that can be measured directly. Subtracting this equation from Equation 2 and realizing that volume can be substituted for weights because the gas is stated always as S.C.F., the balance becomes:

> S.C.F. of gas originally in reservoir = S.C.F. gas produced + S.C.F. gas remaining in reservoir (4)

This is the basic equation for evaluating the reservoir material balance.

The simplest reservoir for which Equation 4 can be expanded is one which has no free gas cap and in which the pore volume available to hydrocarbons remains constant, i.e., no water encroaches and no water is produced. The definition of certain necessary terms have been made in previous Engineering Fundamentals. These are:

N = the number of barrels of S.T. oil which were originally in the reservoir

ΔN = the number of barrels of S.T. oil which have been produced up to the time the balance is made.

r = the number of S.C.F. of gas which will dissolve in 1 bbl. of S.T. oil under the pressure P which exists at the time the balance is made.

r_o = the number of S.C.F. of gas which will dissolve in 1 bbl. of S.T. oil under the original reservoir pressure P_o.

β = the formation volume factor at P, i.e., the volume occupied in the reservoir at P by 1 bbl. of S.T. oil plus the amount of gas, r, in solution.

β_o = the formation volume factor at the original pressure P_o, i.e., the volume occupied in the reservoir at P_o by 1 bbl. of S.T. oil plus the amount of gas, r_o, in solution.

v = the space in barrels occupied in the reservoir at P by one S.C.F. of gas when not in solution, equal to $\dfrac{Z\ 14.7\ T_f}{5.62\ P\ 520}$ (Eng. Fund. No. 305).

Also, let V_g be the number of S.C.F. of gas produced from the reservoir at the time the balance is made. Then $V_g/\Delta N$ is the gas-oil ratio on a cumulative basis. Let this be represented by R_c.

The expansion of Equation 4 in these terms subject to the conditions of no gas cap, no water production and no water encroachment is outlined in Fig. 1. The resulting expression after collecting terms is:

$$N\,[\beta + (r_o - r)\,v - \beta_o] = \Delta N\,[\beta - rv + R_c v] \quad (5)$$

It is convenient to add and subtract the term $\Delta N r_o v$ from the right side of this equation and to introduce the terminology (Engineering Fundamentals No. 309):

$$u = \beta + (r_o - r)\,v$$
$$u_o = \beta_o$$

Equation 5 then becomes:

$$N\,[u - u_o] = \Delta N\,[u + (R_c - r_o)v] \quad (6)$$

which equation expresses the oil (ΔN) and gas (R_c) production in terms of the original oil content (N) and the reservoir fluid properties (β, r, and v).

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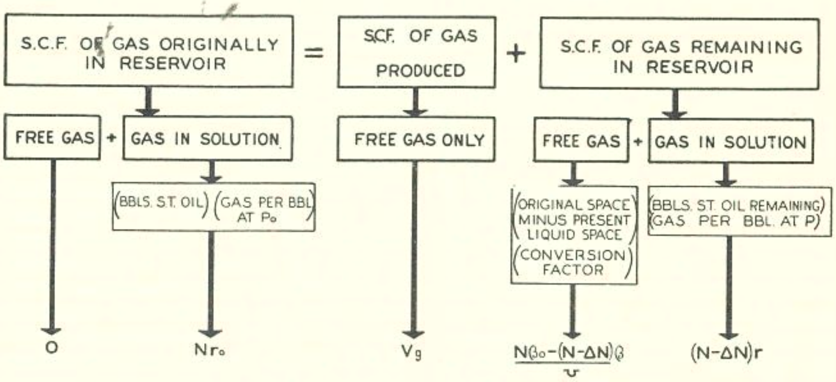

Fig. 1—Material-balance derivation

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ENGINEERING fundamentals

Material Balance Equations to Include

Gas-Cap Volumes, Water Encroachment

IN Engineering Fundamentals No. 333 a material balance equation was developed for a reservoir having no gas cap, no water production, or water encroachment and which was of a definite constant pore volume. Although such an ideal case is not met within practical operations, the equation is a good beginning point for a theoretical analysis. In the present discussion the material balance idea will be extended to include a gas cap and water encroachment.

As before, the reservoir volumes are considered first at the original state when the pressure was P_o and then at some future time when the cumulative oil withdrawal has been ΔN barrels of stock tank oil, the cumulative gas production has been $\Delta N \times R_c$ standard cubic feet, the water production has been w barrels, the water that has encroached into the space originally occupied by oil has been W barrels and the reservoir pressure has fallen to P. The reservoir initially is assumed to have contained free gas over and above that which was in solution and the reservoir volume occupied by this gas is defined as $mN\beta_o$. Since the original oil volume is by definition $N\beta_o$, the term m represents the ratio of the original reservoir gas cap volume to the original reservoir oil volume. Fig. 1 shows an idealized cross-section representing the relative reservoir volumes of oil and gas originally and the relative reservoir volumes of oil, water, and gas when the pressure has fallen to P.

The balance between the two situations shown in the figure can be expanded from the basic balance given previously (Engineering Fundamentals No. 333) as (1) S.C.F. of gas originally in reservoir = S.C.F. gas produced + S.C.F. gas remaining in reservoir. The expansion of these terms is shown diagrammatically in Fig. 2 using the terminology given below.

The equation which results after collecting the terms shown and after substituting $u = [\beta + (r_o - r)v]$ and $u_o = \beta_o$ is:

$$N\left[(u - u_o) + mu_o\left(\frac{v - v_o}{v_o}\right)\right]$$
$$= \Delta N\,[u + (R_c - r_o)v] - W + w \quad (2)$$

which equation relates the amount of cumulative oil production to the properties of the reservoir fluids, to the pressure decline, to the amount of gas and water production, to the magnitude of gas cap and to the amount of water encroachment.

Terminology

N = the number of barrels of S.T. oil which were originally in the reservoir

ΔN = the number of barrels of S.T. oil which have been produced up to the time the balance is made

r = the number of S.C.F. of gas which will dissolve in 1 bbl. of S.T. oil under the pressure P which exists at the time the balance is made

β = the formation volume factor at P. i.e., the volume occupied in the reservoir at P by 1 bbl. of S.T. oil plus the amount of gas, r, held in solution

β_o = the formation volume factor at the original pressure P_o, i.e., the volume occupied in the reservoir at P_o by 1 bbl. of S.T. oil plus the amount of gas, r_o, held in solution

v = the space in barrels occupied in the reservoir at P by one S.C.F. of gas when not in solution, equal to $\frac{Z}{5.62}\,\frac{14.7}{P}\,\frac{T_f}{520}$ (Eng. Fund. No. 305)

v_o = the space in barrels occupied in the gas cap at P_o by one S.C.F. of gas

T_f = the reservoir temperature

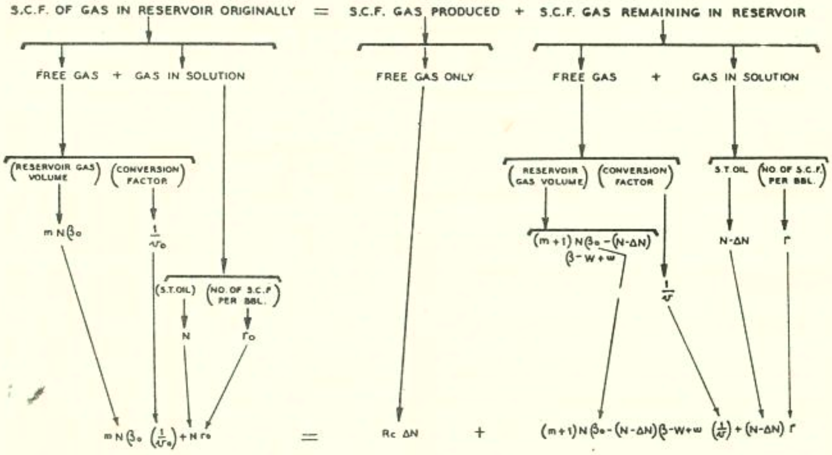
R_c = the gas-oil ratio on a cumulative basis, equal to total gas production divided by ΔN

$u = \beta + (r_o - r)\,v$, or the space in barrels occupied in the reservoir at P by 1 bbl. of S.T. oil plus all the gas which was dissolved in that oil at the original pressure P_o

$u_o = \beta_o$

m = the ratio of the original reservoir gas cap volume to the original reservoir oil volume

Fig. 2—Expansion of material balance equation



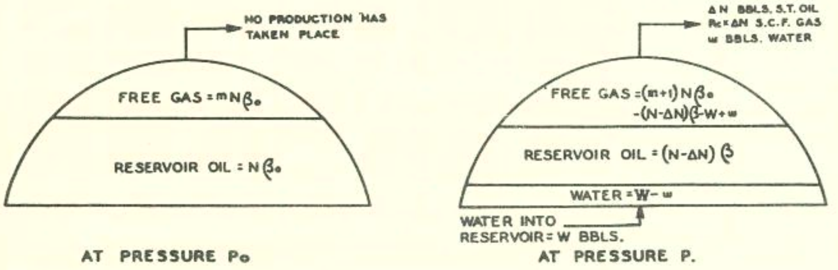


Fig. 1—Relative reservoir volumes of oil, gas, and water at original time and later

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ENGINEERING *Fundamentals*

Dependence of Production on Gas Withdrawals

THE amount of pressure decline which accompanies a certain fractional withdrawal of oil from a reservoir will depend upon the physical properties of the reservoir fluids (gas in solution, compressibility, formation volume factor), upon the magnitude of the gas cap that was present, upon the amount of water production and encroachment, and upon the amount of gas produced along with the oil. The physical properties of the fluids and the amount of gas cap are fixed quantities. It is of interest to inquire into the changes due to the amount of water production and encroachment or due to the amount of gas produced with the oil. The present discussion will be concerned with the latter.

In order to simplify the approach, it will be assumed that the water production and encroachment and the initial gas cap are zero. The quantitative evaluation of the desired change is given by the material balance equation (Engineering Fundamentals No. 333). This equation was written as:

$$N\,(u - u_o) = \Delta N\,[u + (R_c - r_o)v] \quad (1)$$

Rearranging this equation gives:

$$\frac{\Delta N}{N} = \frac{u - u_o}{u + (R_c - r_o)\,v} \quad (2)$$

Now $\Delta N/N$ is the desired fractional production of original oil in place, R_c is the total amount of gas produced divided by the total amount of oil produced, and u_o, u, r_o, and v are physical properties of the reservoir fluids dependent in value upon the original pressure and the pressure at a given time. Thus, from Equation 2 it is possible to evaluate the fractional production of oil at a given pressure value if a value of R_c is assumed.

It would appear that Equation 2 might yield a value of $\Delta N/N$ equal to one (all oil produced) by having R_c equal to zero or a small number. Although the equation would actually yield this result, it does not mean that such can be achieved. The material balance equation does not reflect the mechanism of production and any calculation therefrom must be carried only as far as the production mechanism will allow. In other words, the assumed calculation of $\Delta N/N$ must terminate at that value in agreement with the residual oil content that results from the type recovery being considered. Furthermore the value of R_c does not reflect the magnitude to which the current gas-oil ratio has risen. It measures only the **cumulative** gas divided by **cumulative** oil.

Fig. 1 illustrates the variation obtained in cumulative recovery at a given pressure for varying amounts of gas withdrawn. This calculation is not meant to apply quantitatively to any specific reservoir but it does show the general effect of producing with various quantities of gas. The data used for this calculation assumed an initial reservoir pressure of 3,500 psia. and saturated crude oil with no initial gas cap. The original gas in solution was assumed to be 1,040 cu. ft. per barrel and the original f.v.f. to be 1.58. For convenience, the amount of the produced gas is put in terms of a multiple of the original gas in solution. $R_c = 0$ means all gas was reinjected. $R_c = r_o$ means that reinjection was carried on to the extent that only the gas originally in the produced oil remained as produced gas. All calculations were terminated at 60 per cent $\Delta N/N$, it being assumed that the gas-recovery mechanism would not produce beyond this point.

Such a chart as this could be prepared for any reservoir fluid and the predicted cumulative at a given pressure be picked knowing the cumulative gas-oil ratio. Or, conversely if a certain cumulative were desired by the time pressure had fallen to a definite value, the quantity R_c indicated by such a chart would allow the prediction of the amount of gas which would have to be reinjected. This, of course, assumes no bypassing.

If an oil of different physical properties were under consideration, the actual values plotted would differ from those given, but relative positions of lines indicating varying amounts of produced gas would be the same as shown.

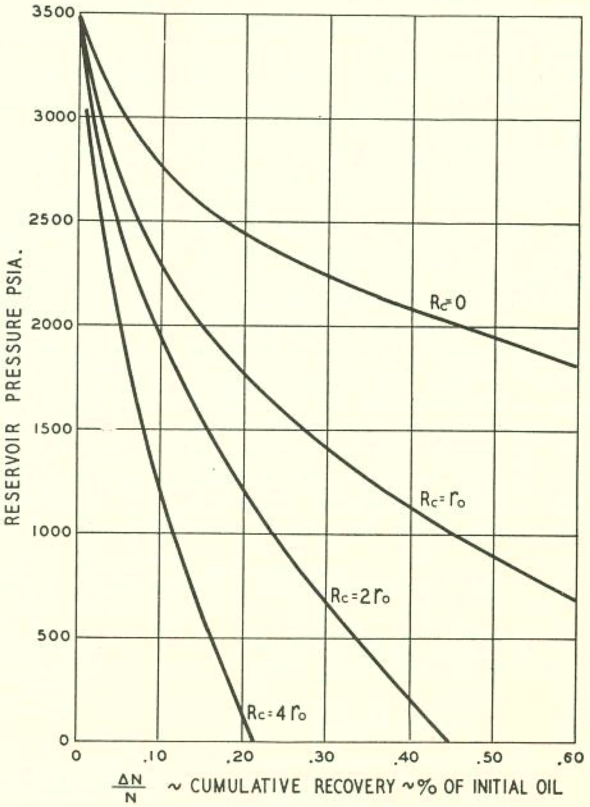

Fig. 1—Calculated effect of gas withdrawals on ultimate recovery. R_c = Cumulative gas-oil ratio. r_o = Original gas in solution

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ENGINEERING fundamentals

No. 336

Effect of Net Water Encroachment On Reservoir Pressure Decline

MAINTENANCE of reservoir pressure can be attained under continued withdrawal of production by the control of the net quantities of gas produced or by the control of the net water encroachment. The effect of produced gas in terms of net cumulative gas-oil ratio was outlined in a previous Engineering Fundamentals (No. 335). The present discussion will outline the variation of pressure decline under differing amounts of net water encroachment with an assumed constant net cumulative gas-oil ratio.

The material balance equation expresses the quantitative variations sought. This equation is written, after expressing the gas cap as being equivalent to zero, as (Engineering Fundamentals No. 334):

$$N(u - u_o) = \Delta N[u + (R_c - r_o)v] - (W - w) \quad (1)$$

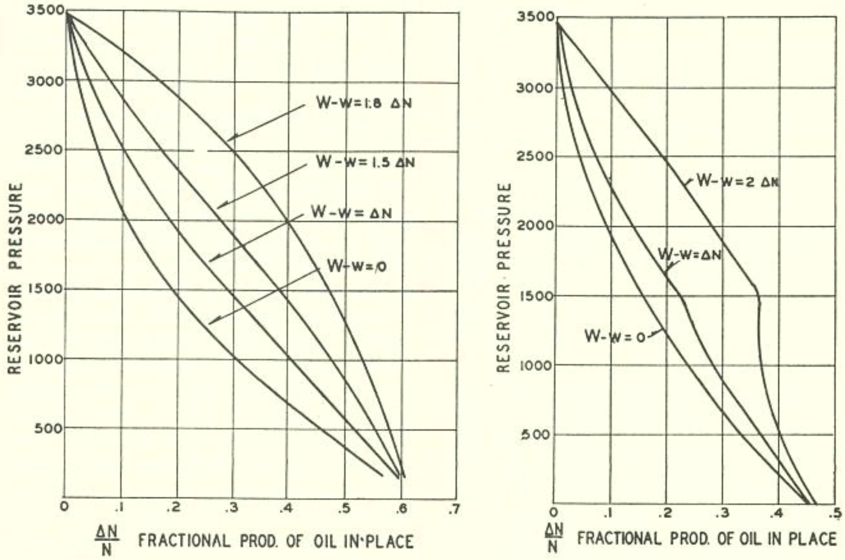

Fig. 2—(Left) Calculated effect of net water encroachment. (Net gas-oil ratio = 1½ times original gas in solution). Fig. 3—(Right) Calculated effect of net water encroachment. (Net gas-oil ratio = twice original gas in solution)

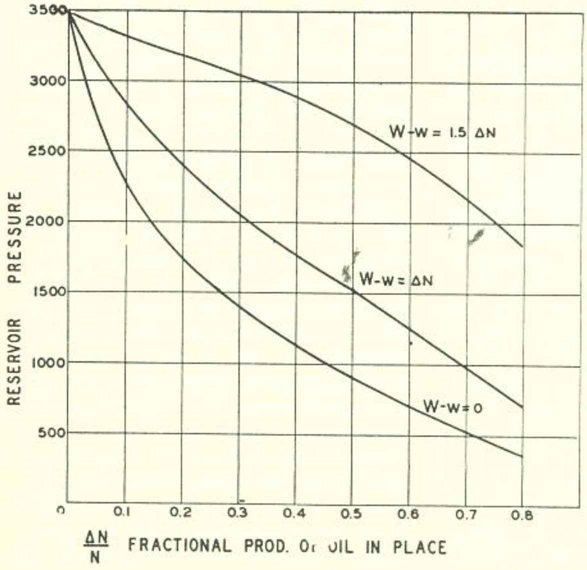

Fig. 1—Calculated effect of net water encroachment. (Net gas-oil ratio = original gas in solution)

The term (W — w) is termed the net water encroachment and is equal to water entering the pore space originally occupied by oil minus the water produced. ΔN/N is the fractional production desired and all quantities excepting R_c are dependent upon the reservoir pressure at any given time.

For convenience (W — w) will be expressed as a fraction f_w of the oil produced. For example, values of (W — w) chosen as 0.1 ΔN, 0.5 ΔN and ΔN mean that the net water encroachment is one-tenth as rapid as the withdrawal of oil, half as rapid as the withdrawal of oil, and equal to the withdrawal of oil. The equation then becomes after substitution:

$$\frac{\Delta N}{N} = \frac{u - u_o}{u + (R_c - r_o)v - f_w} \quad (2)$$

It is now possible to compute the fractional withdrawal ΔN/N accompanying a certain decline in pressure with definite values of R_c and (W — w) known. Graphs from calculations of this type are shown in Figs. 1, 2, and 3 for various water encroachment values and various gas-oil ratio values.

These graphs indicate the nature of the pressure decline for fixed pairs of gas control and water control values. In any producing reservoir these quantities are not fixed throughout entire field history. Hence, the graphs drawn do not indicate the path taken in actual production. They illustrate, however, the relative qualitative effect of various amounts of water encroachment on pressure decline and can be interpreted only in this sense. The graphs were carried only to 80 per cent production of oil in place, this being chosen as the maximum recovery the water drive mechanism would yield even though the pressure might still be at a substantial value.

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ENGINEERING *fundamentals*

No. 337

Estimation of Reservoir Saturation

THE number of barrels of stock-tank oil which remain in a reservoir at any time is readily calculated by subtracting that which has been produced from the amount which was originally present within the reservoir. The quantity so found, however, does not suffice to define the fluids in the reservoir for flow or displacement calculations, because both flow and displacement are measured in terms of the oil saturation and not in terms of stock-tank barrels. A knowledge of the amount of reservoir pore space that is filled with reservoir liquid at any time is the desired quantity, and it will differ from the barrels of stock-tank oil that remain.

The present discussion will concern itself with outlining a means used to estimate reservoir oil saturation at any time during the life of the reservoir. In order to make this estimation, it is required that numerous simplifying assumptions be made. The results, although limited by such assumptions, are useful at least in a qualitative and sometimes quantitative sense.

The assumptions made are primarily those which limit analysis to an internal gas-driven reservoir. These are (1) that the amount of reservoir pore volume remains constant during the production history, and (2) that the oil which remains within the reservoir (and the gas also) be distributed evenly throughout the pore space. It will be assumed at first also that no water is produced from the reservoir, although the calculation can be made without this assumption.

The pore volume of the oil zone of a reservoir may of course be calculated directly from a knowledge of the areal extent, thickness of pay, and the porosity of the formation. It can also be calculated indirectly from a knowledge of the reservoir oil volume in place and the per cent oil saturation. By definition, oil saturation equals the reservoir oil volume divided by pore volume, so if the first two quantities are known, the latter can be found. This will be the procedure used below.

Let the initial oil saturation be designated as S_{oi} and the initial water saturation be designated by S_w and let it be assumed that no free gas is present originally (only the oil zone is being discussed). Then the sum of the oil and water saturations must be unity, or:

$$S_{oi} = 1 - S_w \qquad (1)$$

By definition, however, the oil saturation is:

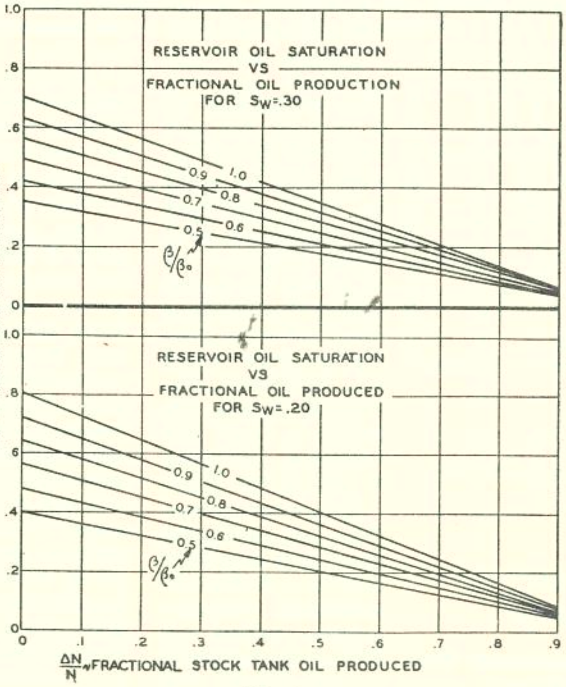

Fig. 1

$$S_{oi} = \frac{\text{Reservoir oil volume}}{\text{Pore volume}} \qquad (2)$$

The reservoir oil volume is equal to $N\beta_o$ if there are N barrels of stock-tank oil in place originally, with a formation volume of β_o. Therefore, the pore volume of the oil zone is:

$$\text{Pore volume} = \frac{N\beta_o}{1 - S_w} \qquad (3)$$

According to the first assumption made, this will always be the pore volume of the oil zone. Therefore, at a subsequent time, the new oil saturation can be found by dividing the new reservoir oil volume by this pore volume.

When ΔN barrels of stock-tank oil have been produced and the pressure has fallen to a value P, the stock-tank oil remaining is $(N - \Delta N)$ barrels, and the reservoir volume it occupies will be $(N - \Delta N)\,\beta$, where β is the formation volume factor at P. The oil saturation S_o, in the reservoir is, from the above quantities, found to be:

$$S_o = \frac{(N - \Delta N)\,\beta}{\text{Pore volume}} = \frac{(N - \Delta N)\,\beta}{N\beta_o/(1 - S_w)} \qquad (4)$$

By rearrangement this becomes:

$$S_o = \left(1 - \frac{\Delta N}{N}\right) \frac{\beta}{\beta_o} (1 - S_w) \qquad (5)$$

In the event that the reservoir is not one of constant volume, Equation 4 would have the pore volume modified by the amount of water encroachment, and the resulting expression for oil saturation would not be so simple. For many calculations, Equation 5 is useful and yields results which are in agreement with observed conditions. Its primary use has been for the evaluation of saturation conditions at which a given relative permeability applies.

Since, for a given reservoir, N, S_w, and β_o are fixed constants, Equation 5 can be considered to give oil saturation as a function of produced oil and pressure, the latter being the quantity which determines β. For a given reservoir the calculation can be made and plotted to show the separate variations.

Fig. 1 is a graph of Equation 5 which can be used for any calculation of S_o at the water saturations indicated. Equation 5 is, of course, linear between S_o and $\Delta N/N$ for fixed values of β/β_o and S_w. Such a graph could be made for any chosen value of S_w.

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ENGINEERING fundamentals

No. 338

Discussion of Material Balance Equation

THE material-balance approach to the analysis of reservoir problems is a useful and fruitful one. It must be used, however, with discretion and under circumstances where information is sufficient to justify it. Its main usefulness, moreover, is not as a tool in itself but as a tool in conjunction with other techniques.

Consider the material-balance equation, written as:

$$N\left[(u - u_o) + mu_o\left(\frac{v - v_o}{v_o}\right)\right] = \Delta N\left[u + (R_c - r_o)\,v\right] - W + w \qquad (1)$$

The produced oil, ΔN, the produced water, w, and the net cumulative gas-oil ratio, R_c, are all measured production quantities. The two-phase formation-volume factors, u_o and u, the gas-conversion factors, v_o and v, and the original gas in solution, r_o, are all measurable fluid properties fixed in value by the original pressure, P_o, and the present pressure, P. The original stock-tank oil in place, N, and the original gas-cap volume expressed by the term, m, are both constant but undetermined values. The water encroachment, W, is a variable quantity with time and with pressure drop and is also an undetermined value.

The undetermined values of N, m, and W are the chief objectives for which the material-balance equation is used. Obviously, the one equation cannot be used to solve for each of three unknowns. Two must be fixed before the third can be found. Thus, the material balance equation is of no value in itself. It must be supplemented by other information, either core data or operating data, which will allow two of the three values to be estimated or determined independently.

Case 1 can be considered to be that where m and W are known and N is to be solved for. The simplest situation occurs when both m and W are zero, or when W only is zero. This is often **assumed** to be the true condition in order that the equation can be used. If a constant value of N cannot be found at successive time intervals on the basis of such assumptions, it may be taken as evidence that W or m is not zero as assumed. It has been shown by Woods and Muskat,[1] however, that a constant value of N can be calculated for successive time intervals on some reservoirs not only with both m and W assumed as zero, but also with m and W assumed as finite values of considerable magnitude. Therefore, the assignment of definite values to m and W to allow the calculation of N cannot be arbitrary. The choice must be backed up by supporting evidence. In the case of the m value, supporting evidence might be the fact that the reservoir oil was undersaturated originally or it might be log measurements of the relative magnitudes of the oil and gas zones. For a value of W the supporting evidence might be in the form of water-oil ratios, water-table rise, or similar observations made during operation of the reservoir.

Case 2 will be that where a value of N can be estimated by other means more definitely that W can be estimated by outside data. In the event the assigned values of N and m can be used to solve for W. Cases where N might be known accurately enough for this are reservoirs where good core recovery is obtained and where analysis of the cores can be relied upon.

Case 3 would involve solution for an m value when N and W are known. Usually the value of m is more definitely determinable by outside means than are either N or W, and this type calculation is not often used.

In any of the above uses of the equation, the resultant answer is, of course, only as precise as the accuracy to which the basic production data, pressures, and fluid characteristics are known. Of these quantities, the amount of gas produced and the pressure measurements are most subject to error. Usually, however, more doubt can be cast upon the estimation of reasonable values for two of the three unknowns, N, m, or W, as the case may be, than upon the accuracy of the pressures, production, or fluid characteristics. That is to say, u, u_o, r_o, v, etc., are usually more definitely known than are m, N, or W.

The material-balance equation has been written in many forms or rearranged in many ways. The manner of rearrangement, however, cannot remove the basic fact that three unknowns are involved and none of the three can be solved for uniquely without a knowledge of the other two. Independent means of arriving at two of the other unknowns may not always be possible. N may be as hard to determine by volumetric calculations, or W may not be ascertainable because of meager data. In such instances, however, both material balance calculations and volumetric calculations taken together will suffice to define the limits of the value of N more closely than either one alone.

In using the equation, it is well to note that there is no regularity that can be expected of the apparent values of N calculated. In general, if a water drive is active, and it is assumed in calculations that $W = O$, the values of N arrived at will increase with successive time intervals. Also, higher values of N will be calculated with m assumed as zero rather than as some finite number.

Although the material-balance equation can be used to estimate a value of stock-tank oil in place before the locations in a reservoir are entirely drilled up, it will generally not give consistent or reliable results in early stages of field development. As a rough rule of thumb, it may be considered that 3 to 5 per cent of the initial oil in place should have been produced to be beyond the initial period of uncertainty. In the early life of a reservoir, moreover, special care must be taken that reliable average pressures are chosen, and that reliable gas-production figures are available because the differences in terms of Equation 1 is small during the early period when reservoir pressure is near its original value.

Reference

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ENGINEERING fundamentals

Average Gas-Oil Ratios

FOR material-balance and other calculations, it is necessary to know amounts of gas produced. In the material-balance equation, the produced gas shows up as a cumulative gas-oil ratio rather than as a gas volume. The expression of gas production by gas-oil ratios is more common than by gas cumulative figures. Very often current producing gas-oil ratios are available and it is desirable to convert them to a cumulative gas-oil-ratio basis. The present discussion is concerned with the handling of such gas-measurement quantities.

Producing, or instantaneous, gas-oil ratio signifies a current rate. It is by definition the current rate of gas production divided by the current rate of oil production. It is expressed usually as standard cubic feet of gas per barrel of stock-tank oil. Such ratios are given as a result of a few hours' testing or as a result of several days of testing. The symbol R will be used to designate current producing gas-oil ratio over whatever testing period may be applicable.

Method of Computing

To obtain an average producing gas-oil ratio for several wells or for all wells in a field, one cannot take an arithmetic average value of the ratios. For example, two wells with gas-oil ratios of 2,000 and 8,000 would not necessarily have an average ratio of 5,000. They would only if both wells were producing the same amounts of oil. An average producing gas-oil ratio must be obtained by dividing total current gas production from all wells involved by total current oil production from all wells involved. Thus, if the 2,000-ratio well produced at the rate of 100 bbl. per day and the 8,000 ratio well at the rate of 50 bbl. per day, the average value of R for the two wells would be:

$$R = \frac{2{,}000 \times 100 + 8{,}000 \times 50}{150}$$

$$= 4{,}000 \text{ cu. ft. per bbl.}$$

For a large number of wells the average ratio is figured as:

$$R_{ave} = \frac{\Sigma R_i \times Q_{oi}}{\Sigma Q_{oi}} \qquad (1)$$

where R_i and Q_{oi} signify the individual ratios and stock-tank oil production rates and the sign Σ means the addition of such quantities for all the wells in question.

The total gas produced in an interval of time is equal to the producing gas-oil ratio during that period of time multiplied by the oil-production rate during the interval. To find the cumulative gas produced up to a certain time on a reservoir, therefore, one needs the current ratio at various periods and the amount of production during each period.

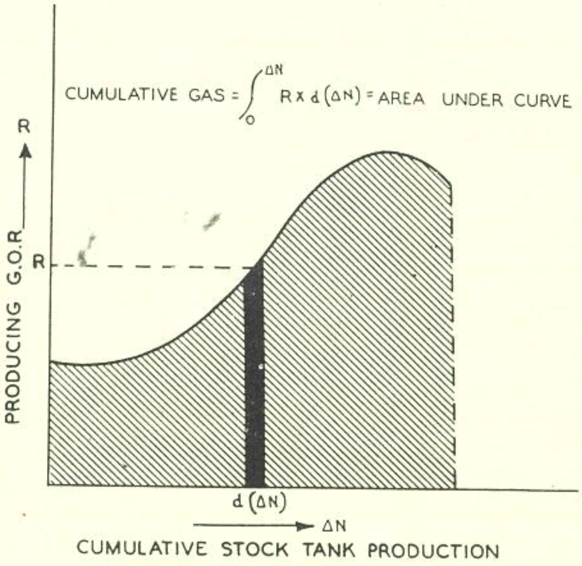

Fig. 1

If ΔN represents cumulative stock-tank production, then the production over a short interval of time is its derivative, or $d(\Delta N)$. The product of the gas-oil ratio at this interval and the production is equal to the gas produced in the interval, or:

$$\text{Gas produced in interval} = R \times d(\Delta N) \qquad (2)$$

The cumulative gas produced from zero time up to a certain time t is the integral of Equation 2 or:

$$\text{Cumulative gas} = \int_0^t R \times d(\Delta N) \qquad (3)$$

Expressed another way, this is the area under the curve of R plotted against the cumulative production ΔN. This is shown in Fig. 1.

The cumulative gas-oil ratio, expressed by the symbol R_c, is defined as all the gas produced and kept from the reservoir up to a certain time divided by the cumulative oil produced at that same time. Therefore:

$$R_c = \frac{\int_0^t R \times d(\Delta N) - \text{gas reinjected}}{\Delta N} \qquad (4)$$

In the case where no gas is reinjected, the cumulative gas-oil ratio is simply the area shown in Fig. 1 divided by ΔN.

In averaging individual gas-oil ratios, therefore, at a specific time, the important thing to remember is that they must be weighted according to the production which they represent. In averaging current producing gas-oil ratios at successive times to get a cumulative ratio, the important thing to remember is that the current ratios cannot be weighted on the basis of the time they represent but on the basis of the cumulative production they represent. In other words a graph of the gas-oil ratio, R, versus time does not give average or cumulative gas-oil ratio. A knowledge of R versus cumulative production is necessary as indicated by Equation 4.

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No. 340

ENGINEERING *fundamentals*

Producing Gas-Oil-Ratio Equation

AN equation frequently used in reservoir analysis and in the prediction of future behavior is one which gives the producing gas-oil ratio in terms of the pressure and saturation of the reservoir at the desired time. The equation is written as:

$$R = \frac{k_g}{k_o}\,\frac{\mu_o}{\mu_g}\,\frac{\beta}{v} + r \qquad (1)$$

where

R = current producing gas-oil ratio, standard cubic feet per barrel of stock tank oil.
μ_o = reservoir oil viscosity at the current pressure in centipoises.
μ_g = reservoir gas viscosity at the current pressure in centipoises.
β = formation volume factor at the current pressure.
v = factor to convert standard cubic feet of gas to barrels of reservoir gas at the current pressure.
r = gas in solution at the current pressure expressed as standard cubic feet per stock-tank barrel.
k_g/k_o = relative permeability ratio of gas to oil at the current reservoir oil saturation, S_o.

According to the above equation the producing gas-oil ratio is seen to be a function of reservoir pressure and reservoir saturation.

It is of interest to note the derivation of this equation in order better to understand the limitations and assumptions involved. Consider a radial flow system, and an increment of dr thickness within the system (refer to Fig. 1). At this radius, the pressure of the reservoir is P and the pressure gradient is dp/dr. It will furthermore be assumed that the pressure gradient is the same through the gas phase and through the oil phase. There are q_o barrels of reservoir oil per day flowing past the radius r and q_g barrels of reservoir gas per day flowing past the radius r.

Darcy's law can be used to express the flow of either the oil or the gas. For the former:

$$v_o = \frac{q_o}{2\pi rh} = \frac{-7.07\,k_o\,dp}{2\pi\ \mu_o\,dr} \qquad (2)$$

and for the latter:

$$v_g = \frac{q_g}{2\pi rh} = \frac{-7.07\,k_g\,dp}{2\pi\ \mu\ dr} \qquad (3)$$

where v_o and v_g are the apparent velocities of reservoir oil and gas at r, and $7.07/2\pi$ is a conversion factor to allow rates to be expressed in barrels per day and dp/dr in pounds per square inch per foot.

Now the stock-tank oil produced per day (Q_o) will be equivalent to the rate of reservoir oil movement divided by the formation volume factor, or:

$$Q_o = \frac{q_o}{\beta} \qquad (4)$$

The standard cubic feet of gas produced per day (Q_g) will be equal to the rate of movement of reservoir gas converted to surface conditions plus the gas which is evolved from the oil produced, or:

$$Q_g = \frac{q_g}{v} + rQ_o \qquad (5)$$

The producing gas-oil ratio is by definition, however, the quotient Q_g/Q_o. Dividing Equation 5 by Equation 4, therefore, and substituting Equations 2 and 3 gives:

$$R = \frac{q_g\,\beta}{q_o\,v} + r = \frac{k_g\,\mu_o\,\beta}{k_o\,\mu_g\,v} + r$$

which is the equation first given.

The quantities in this equation were evaluated at the conditions of pressure and temperature existent at radius r. This is not a measurable quantity. Only an average pressure for such a well system can be measured by shutting in the well. However, if the value of ΔP across the system shown in Fig. 1 approached zero as a limit, one value of P would suffice to define the whole system. This would be the average reservoir pressure. In using Equation 1, therefore, as a reservoir equation in which β, v, r, μ_o/μ_g are defined by the average reservoir pressure, it is assumed that production is occurring at zero pressure differential.

To summarize, the assumptions that are tacitly involved in the use of Equation 1 are that (1) the pressure drawdown is zero, (2) the gas and oil are uniformly distributed and are flowing according to the equilibrium relative permeability concept, and (3) the pressure gradients in the gas are the same as those through the oil phase.

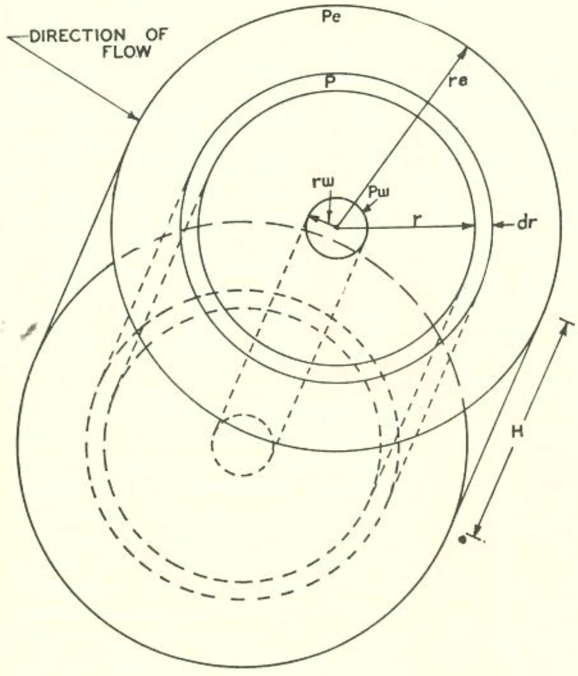

Fig. 1

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ENGINEERING *fundamentals*

Use of Production Data to Construct Relative Permeability Curves

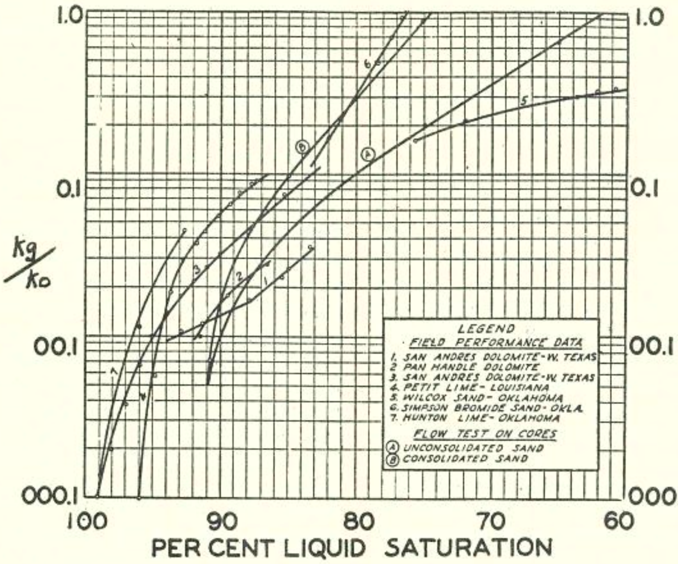

Fig. 1

THE production data obtained in operating a reservoir can be used to calculate curves giving the relative permeability ratio as a function of the oil saturation of the reservoir rock. Such curves, constructed from operating data alone, reflect not only the flow characteristics of the reservoir fluid through the particular reservoir rock but also the operating practices imposed on the reservoir.

Two separate equations are used to calculate k_g/k_o and oil saturation, S_o, at a particular time in a reservoir's history. These equations are:

$$\frac{k_g}{k_o} = (R - r)\,\frac{\mu_g}{\mu_o}\,\frac{v}{\beta} \qquad (1)$$

and

$$S_o = (1 - S_w)\left(1 - \frac{\Delta N}{N}\right)\frac{\beta}{\beta_o} \qquad (2)$$

(These equations were discussed in Engineering Fundamentals Nos. 340 and 337 respectively.)

The equations involve only the pressure measured at the given time, the cumulative production measured at the given time, and the producing gas-oil ratio at the given time. The water saturation is assumed to be a given, constant value.

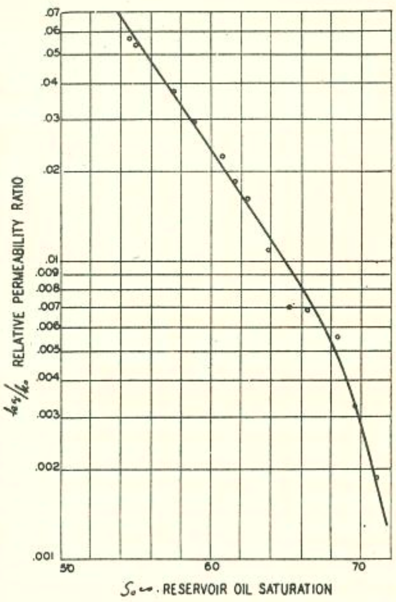

Fig. 2

Calculations of k_g/k_o and S_o made at a number of times in a reservoir's history can be plotted to give the expected relationship on semilog paper. A number of such curves, calculated from observed data, are available in the literature.[1, 2] One such group of calculations is duplicated here as Fig. 1.

To illustrate the actual calculation, the above equations are applied to data taken from a literature source.[3] The same source gives N for this reservoir as 116.5 × 10^6 bbl., S_w as 28.5 per cent, and β_o as 1.45. Tabulation of the data showing the steps in calculation follow. Results are plotted in Fig. 2.

Average pressure	R	ΔN	$\Delta N/N$	β	v	r	μ_o/μ_g	S_o	k_g/k_o
3,448	850	.476 × 10^6	.0041	1.443	.000840	752	30.4	.710	.00187
3,303	920	1.743 × 10^6	.0150	1.432	.000875	725	32.1	.696	.00321
3,153	990	2.818 × 10^6	.0242	1.420	.000910	695	34.0	.684	.00556
2,938	1,020	4.652 × 10^6	.0399	1.403	.000970	657	36.8	.664	.00682
2,813	1,000	6.030 × 10^6	.0517	1.393	.001010	632	38.4	.652	.00694
2,678	1,180	7.360 × 10^6	.0632	1.382	.001062	608	40.5	.638	.01085
2,533	1,420	8.751 × 10^6	.0752	1.371	.001122	580	42.4	.625	.01620
2,453	1,510	9.873 × 10^6	.0847	1.364	.001162	565	43.6	.615	.01848
2,318	1,660	11.259 × 10^6	.0965	1.354	.001230	540	45.5	.609	.0224
2,153	1,920	12.619 × 10^6	.1083	1.340	.001330	509	48.0	.589	.0292
1,978	2,220	13.998 × 10^6	.1202	1.326	.001453	476	50.8	.575	.0377
1,818	2,480	15.321 × 10^6	.1318	1.313	.001590	446	53.8	.563	.0456
1,658	2,710	16.552 × 10^6	.1420	1.301	.001758	416	57.4	.550	.0540
1,625	2,800	16.929 × 10^6	.1453	1.298	.001795	410	58.2	.546	.0567

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1. The Importance of Injected Gas As a Driving Medium in Limestone Reservoirs As Indicated by Recent Gas Injection Experiments and Reservoir Performance History; L. E. Elkins, A.P.I. Drilling and Production Practice, 1946, p. 160.
2. Evaluation of Pressure Maintenance by Internal Gas Injection in Volumetrically Controlled Reservoirs; Patton, A.I.M.E. Trans. 1947 (Vol. 170) Fig. 6.
3. Engineering Features of the Schuler Field and Unit Operation; H. H. Kaveler, A.I.M.E. Trans. (Vol. 155) 1944.

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No. 342

ENGINEERING fundamentals

Productivity Index

IT is desirable to be able to assign to a producing well a quantity which indicates its ability to produce or which gives some indication of its longevity. In the early days of proration where such a quantity was desired, the well potential was a commonplace measurement. This potential was a measure of the well's capacity to produce under wide-open flow. The productivity index has taken the place of such a potential measurement and presumably gives the same information.

By definition, **productivity index** is equal to the barrels per day of stock-tank oil production per pound of pressure differential between the well bore opposite the producing horizon and the static reservoir pressure, which is presumably the pressure at the well's radius of drainage. According to accepted concepts of flow, the rate of flow in a system containing a single fluid under steady state conditions should be directly proportional to the pressure drop. Therefore, it was anticipated that the productivity index would be the slope of the straight line resulting from plotting rate of flow against pressure drop. On such a plot the wide-open-flow quantity or well potential would be measured at the maximum pressure drop obtainable. Such an ideal case is shown in Fig. 1.

The first attempts at explaining or correlating productivity index, therefore, were on the basis of Darcy's Law. For a radial system, steady state, the equation giving the relationship between variables is:

$$Q = \frac{7.07\,k\quad h\quad (P_e - P_w)}{\mu\quad \ln\quad R_e/R_w} \qquad (1)$$

Therefore, the term equivalent to productivity index is:

$$\text{P.I.} = \frac{Q}{\Delta P} = \frac{7.07\,k\,h}{\mu \ln R_e/R_w} \qquad (2)$$

This elementary analysis indicates that the productivity index should be a function of the formation characteristic, k, the fluid characteristic, μ, and the system characteristics, h, R_e and R_w.

The term **specific productivity index** is frequently used and means usually the productivity index per foot of pay. In other words it is made specific by eliminating one of the variables of the system. The productivity index could just as well be made specific to any other of the variables on the right side of Equation 2.

Of course, well systems do not operate under steady state conditions nor with a single fluid flowing. Therefore Equation 2 cannot be expected to yield exact correlation. Early attempts at correlation were by this equation with factors introduced to consider the departure of k from the air value to the relative permeability value, to account for the shrinkage of the oil between the reservoir and the stock tank, and to account for variations in reservoir oil viscosity and in drainage areas. The primary correlation sought has been that with permeability in order that productivity index values could be predicted from core analysis. One such early correlation is given by Fig. 2.

Observed values of production rates and the pressure differentials do not give ideal straight lines as shown in Fig. 1. It is quite common to obtain a curved line as shown by Fig. 3. This is a result of the fact that the well system is not under steady state flow nor is it completely filled with liquid. It may also result from the fact that relatively short intervals of time are consumed in taking such measurements and the pressure differentials measured are not always effective back to the same value of R_e in the well system. Productivity-index data on older wells are usually linear more often than data taken on flowing wells.

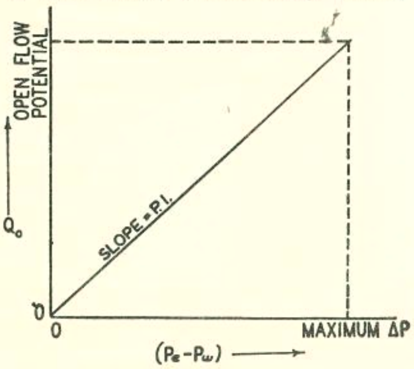

Fig. 1

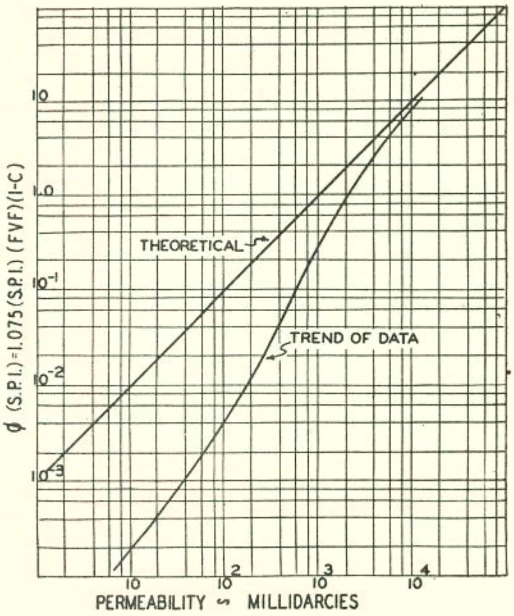

Fig. 2

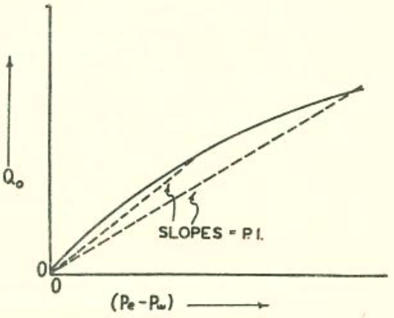

Fig. 3

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ENGINEERING *fundamentals*

No. 343

Steady State Flow in Radial System

CONSIDER the radial system of Fig. 1. Consider that pressures P_e and P_w are constant with time, that the ratio of oil to gas entering the system at R_e and the ratio of oil to gas leaving the system at R_w are constant with time. Further, since flow is steady, the mass of gas and oil entering per unit of time equals the mass of oil and gas leaving per unit of time. Under such conditions, there will be a definite pressure gradient from P_e to P_w and a definite oil saturation distribution from R_e to R_w.

The rate of reservoir oil flow at the radius, r, is by Darcy's law:

$$q_o = \frac{v_o}{A} = 7.07\, r\, h \frac{k_o\, dp}{\mu_o\, dr} \qquad (1)$$

where subscripts o refer to the the oil phase and the terms have their usual meaning. This differential equation, expressing dependence of pressure and saturation with radius, cannot be integrated between the limits desired unless further information concerning the variables is at hand.

q_o can be eliminated as a variable because q_o divided by the formation volume factor at the pressure existing at the radius, r, must equal a constant volume of stock-tank oil. That is:

$$q_o = \beta\, Q_o \qquad (2)$$

However, this introduces a new variable β. Equation 1 can now be rearranged to read:

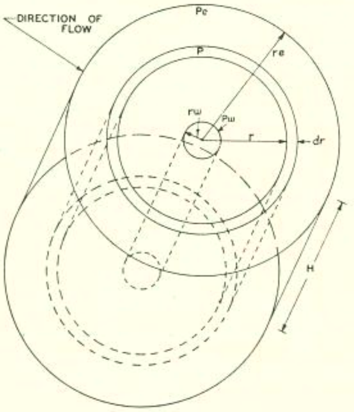

Fig. 1

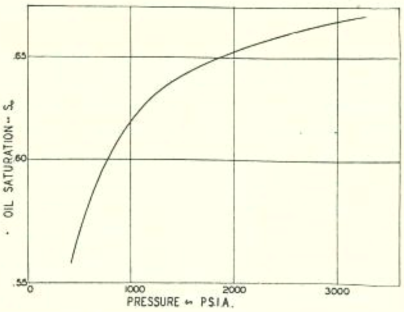

Fig. 2.—Typical variation of saturation with pressure in a steady state radial system. Gas-oil ratio is constant

$$\frac{Q_o}{7.07\, hk} \int_{R_w}^{R_e} \frac{dr}{r} = \int_{P_w}^{P_e} \frac{k_o/k}{\mu_o\, \beta}\, dp \qquad (3)$$

The term dr/r can be integrated between limits. The term $\frac{k_o/k}{\mu_o\, \beta}$ can be evaluated as a function of pressure and integrated between limits in the following manner.

The system is at a steady and constant ratio of gas to oil. Let this be expressed at the outflow area by R, the producing gas-oil ratio. It can be shown (Engineering Fundamentals No. 340) that at any radius r in the system, the following relationship between R and the variables exists:

$$R = \frac{k_g}{k_o}\, \frac{\mu_o}{\mu_g}\, \frac{\beta}{v} + r_s \qquad (4)$$

Since R is a constant for the system, pressure and saturation determine Equation 4. For a given R value, therefore, a plot of pressure versus saturation can be made, saturation giving k_g/k_o and pressure giving μ_g/μ_o, β, v, and r_s. Fig. 2 is such a plot for an oil with defined characteristics and a reservoir rock with a defined relative permeability curve.

Having determined dependence of pressure and saturation within the system, k_o/k values can be determined for the saturation at a given pressure and the term $\frac{k_o/k}{\mu_o\, \beta}$ can be evaluated and plotted against pressure. Such a plot is shown in Fig. 3. The right-hand side of Equation 3 can now be integrated graphically by taking the area between values of P_e and P_w under the curve as shown, which numerical area is the desired value of the integral. Thus, the following is obtained:

$$Q_o = (\text{area under curve}) \frac{7.07\, kh}{\ln \frac{R_e}{R_w}} \qquad (5)$$

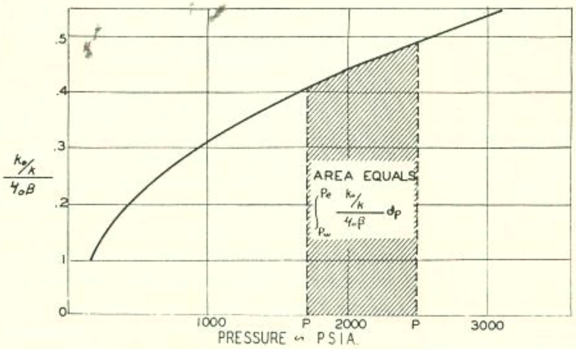

Fig. 3.—Variation of $\frac{k_o/k}{\mu_o\beta}$ with pressure for radial system of Fig. 2

Nomenclature

P_e = pressure at radius of drainage, psia.
P_w = pressure at sand face, psia.
R_e = radius of drainage, ft.
R_w = radius of well bore, ft.
q_o = volume rate of oil flow at radius r, bbl. per day.
v_o = apparent oil velocity at radius r, ft. per day.
h = height of radial system, ft.
k = permeability of formation, darcies.
k_o = specific permeability to oil at saturation of radius r, darcies.
k_g = specific permeability to oil at saturation of radius r, darcies.
β = formation volume factor at radius r.
Q_o = stock-tank volume rate of oil flow, bbl. per day.
μ = viscosity of reservoir oil at radius r, centipoises.
μ_g = viscosity of reservoir gas at radius r, centipoises
v = conversion factor standard cubic feet of gas to barrels of reservoir gas.
r_s = gas in solution at radius r, standard cubic feet per barrel of stocktank oil.
ln = logarithm to base e.
A = cross-sectional area of flow at radius r, square feet.

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ENGINEERING fundamentals

No. 344

Theoretical Productivity Index

THE term productivity index has been defined as the barrels per day of stock-tank oil production per pound pressure drop between the flowing sand face pressure and the static reservoir pressure. An expression for this quantity can be worked out on a theoretical basis using the steady state formula as developed for a radial system flowing oil and gas. By such an approach, it can be shown that productivity index for a given well system can be expressed in terms of three parameters, (1) the producing gas-oil ratio, (2) the absolute reservoir pressure, and (3) the pressure gradient in the well system. The original development of this idea is due to Muskat and Evinger.[1]

For the steady state flow of oil and gas in a radial system, the following equation expresses the rate of oil flow (Engineering Fundamentals No. 343):

$$Q_o = \frac{7.07\ kh}{\ln R_e/R_w}\int_{P_w}^{P_e}\frac{k_o/k}{\mu_o\,\beta}\,dp \qquad (1)$$

The integral is evaluated as shown in Fig. 1 by finding the area under a plot of $\frac{k_o/k}{\mu_o\,\beta}$ versus pressure between the limits P_e and P_w. Now, if the term Q_o is divided by the pressure drop $(P_e - P_w)$, then a quantity equivalent to the productivity index is obtained. In other words:

$$\text{P.I.} = \frac{Q_o}{(P_e - P_w)}$$

$$= \frac{7.07\ kh.\ (\text{area under curve})}{\ln R_e/R_w\ (P_e - P_w)} \qquad (2)$$

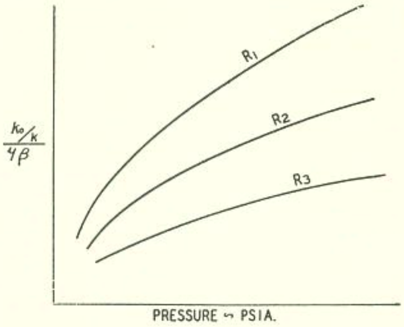

Fig. 2—Change of function $\frac{k_o/k}{\mu_o\,\beta}$ *for a given well system for various gas-oil ratios* $R_1 < R_2 < R_3$

It is well to inspect this result to determine what factors are important. In the first place, it will be noted that P.I. from Equation 2 will not double if $(P_e - P_w)$ is doubled because the area under the curve will not double. Second, it will be noted that of $(P_e - P_w)$ equal to a definite value be taken at a high absolute pressure, the area will be greater than if taken at a lower absolute pressure. All other things being constant, therefore, the P.I. value will be greater at higher reservoir pressures and low drawdowns.

The third thing to note is that the P.I. value will depend on the producing gas-oil ratio, R. This is not readily apparent unless one remembers how the relationship drawn in Fig. 1 was obtained. It is a result of steady state flow of an oil and gas mixture. To define such steady flow the ratio of oil flowing to gas flowing must be stated. Therefore, each value of R means a different steady state system for which a different curve will apply. Fig. 2 demonstrates how the function changes, with R values on a given well system.

Hence, the comparison of P.I. values on two well systems must not only include permeability, sand thickness, well radius, drainage radius, fluid characteristics, and flow relationships. A comparison must also be made on the basis of equal reservoir pressures, equal pressure drawdowns, and equal gas-oil ratios. Using the curve of Fig. 1, the variation shown in Fig. 3 has been prepared. This demonstrates the dependence of P.I. on reservoir pressure and pressure drawdown for one gas-oil ratio only. A similar figure would be necessary for each and every gas-oil ratio chosen. These curves will vary, of course, depending on the physical characteristics of the reservoir fluid (μ_o/μ_g, β, v, and r) but the general variation is as indicated by the diagrams.

Reference

1. Evinger and Muskat, Theoretical Productivity Factor, A.I.M.E., 1942 (Vol. 146).

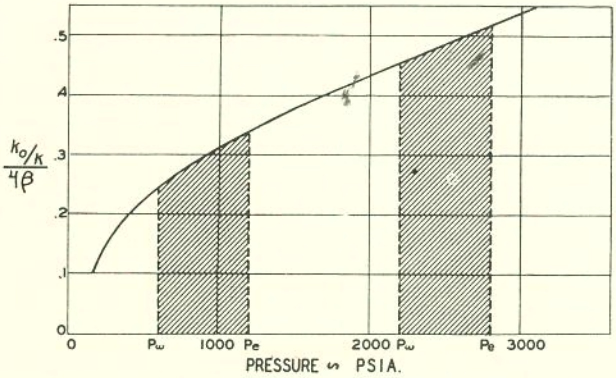

Fig. 1—Plot of function to obtain $\int_{P_w}^{P_e}\frac{k_o/k}{\mu_o\,\beta}\,dp$

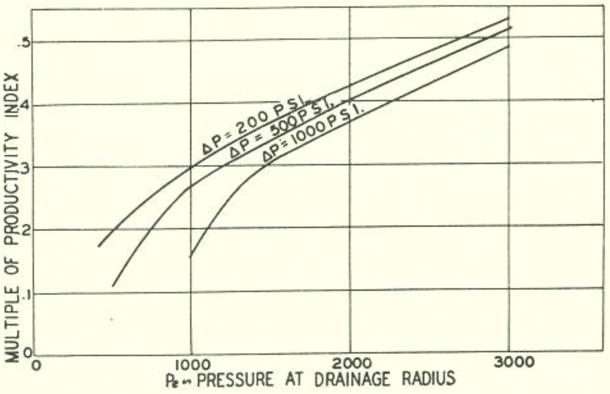

Fig. 3—Variation of P. I. on a steady-state system with gas-oil ratio constant

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ENGINEERING fundamentals

No. 345

Decline of Productivity Index on an Internal Gas-Drive Reservoir

THE theoretical productivity index of a radial system under steady state flow can be expressed as:

$$\text{P.I.} = \frac{7.07\ kh}{(P_e - P_w)\ \ln R_e/R_w} \int_{P_w}^{P_e} \frac{k_o/k}{\mu_o \beta}\, dp \qquad (1)$$

and it is inherent that the value obtained will depend on both P_e and P_w for a given gas-oil ratio of the system because the quantity under the integral sign does not vary linearly with pressure. In order to use the P.I. value as a comparison, therefore, some standard conditions of measurement must be used. It has been suggested[1] that the comparison be made for $(P_e - P_w)$ equal to zero, that is, for the condition that P_w approaches P_e as a limit. If $(P_e - P_w)$ approaches zero as a limit then the P.I. becomes:

$$\text{P.I.} = \frac{7.07\ kh}{\ln R_e/R_w} \left(\frac{k_o/k}{\mu_o \beta}\right)_{P_e} \qquad (2)$$

where P_e is the static reservoir pressure. To compare P.I. values at two different times when the static reservoir pressures are P_{e_1} and P_{e_2} one has the following (if it is assumed that at both time k, h, R_e, and R_w are constant):

$$\frac{(\text{P.I.})_1}{(\text{P.I.})_2} = \frac{\left(\dfrac{k_o/k}{\mu_o \beta}\right)_{P_{e_1}}}{\left(\dfrac{k_o/k}{\mu_o \beta}\right)_{P_{e_2}}} \qquad (3)$$

This relationship has been used[2 3] to evaluate the productivity index on an internal gas drive reservoir. To do so it is first necessary to establish the reservoir oil saturation at some future pressure on the reservoir. From the pressure, μ_o and β are defined, and from the saturation, k_o/k is defined. A typical decline in productivity index is shown in Fig. 1, where the subscript i represents initial pressure conditions.

Such a productivity index de-

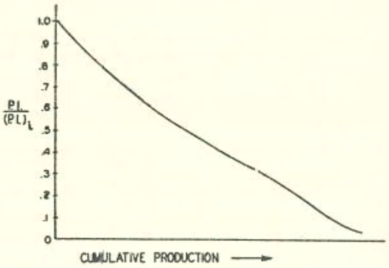

Fig. 1—Typical theoretical decline of productivity index for an internal gas-drive reservoir

cline applies to an ideal case, of course, and strictly speaking, to a reservoir rather than to a well. However, the relative drop in P.I. as cumulative increases can be used for estimations even on well systems. The usefulness of this theoretical approach to practical calculations lies more in the relative decline indicated than it does in the absolute value of the productivity index. Thus, it may be useful to plot actual productivity index values for a well divided by some initial P.I. base, versus the cumulative production from the well in order to correlate with the theoretical productivity index decline calculated.

The theoretical drop indicated for the reservoir system would apply to a well system provided withdrawals and pressure decline for the well system were equivalent to the reservoir average. Since such is not ordinarily the case, one does not have, for a well system, the pressure and saturation values in the same pairs that give the graph of Fig. 1. However, a group of theoretical declines can be calculated for various pressure and saturation pairs that might exist in the well system. The observed P.I. drop versus cumulative can then be fitted to one such curve to indicate the performance of the well system compared to the reservoir as an average.

Fig. 2 indicates the type variation of the quantity $\text{P.I.}/(\text{P.I.})_i$ at various pressures for constant saturations. Thus, if pressure on a particular well system and saturation were known such a graph would give a value of $\text{P.I.}/(\text{P.I.})_i$, or if the latter and pressure were known, a value of saturation might be estimated. Fig. 2 is, of course, prepared for a particular system. In this case the original oil saturation was 0.8 and the original pressure chosen was 3,500 psia.

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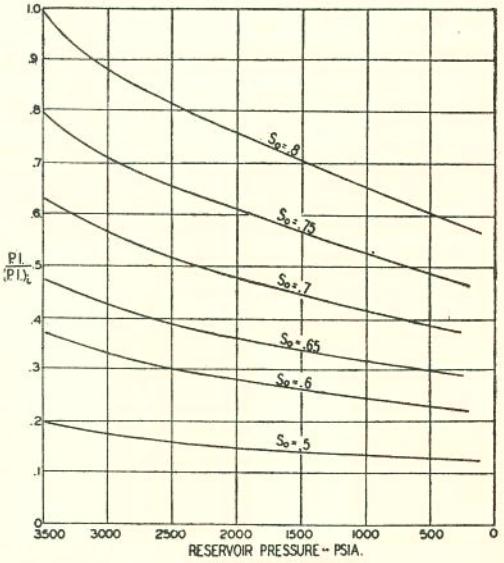

Fig. 2—Theoretical variation of productivity index decline with pressure and saturation of a particular well system

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