Part 42

How to find performance and ultimate oil recovery of a depletion-type pool

using the Tracy material-balance approach

GIVEN: Table 1 shows the production history from an initially undersaturated lense-type oil reservoir shaled out on all sides. Fluid properties for this pool are given in columns 2 through 5 of Table 2. Fig. 1 shows the variation of k_g/k_o with total liquid saturation. Other data are as follows:

Interstitial water saturation, S_w = 25.0% of pore volume.

Reservoir temperature, T = 97° F.

Bubble-point pressure, p_b = 1,700 psia.

Average expansibility factors between original and bubble-point pressures:

$c_f = 4.0 \times 10^{-6}$ vol./pore vol./psi.

$c_o = 8.2 \times 10^{-6}$ vol./vol./psi.

$c_w = 3.1 \times 10^{-6}$ vol./vol./psi.

Original reservoir pressure = 2,110 psia.

Gas viscosity at reservoir conditions remained about 0.02 cp. for pressures between the bubble point and 100 psia.

Initial oil in place = 500×10^6 st.-tk. bbl.

No water production or influx ($W_e = 0$, $W_p = 0$).

Original oil formation-volume factor = B_{oi} = 1.256.

FIND: Performance and ultimate oil recovery for the reservoir using the Tracy material-balance approach.[1]

METHOD OF SOLUTION: Tracy's form of the material-balance equation for depletion-type pool is:

$$1 = N_{pn}\phi_{on} + G_{pn}\phi_{gn} \qquad (1)$$

and on an incremental basis

$$\Delta N_p = \frac{1 - N_{p(n-1)}\phi_o - G_{p(n-1)}\phi_g}{\phi_o + \phi_g[R_I + R_{I(n-1)}]/2} \qquad (2)$$

Where

$$\phi_o = [(B_o/B_g) - R_s]/X \qquad (3)$$

$$\phi_g = 1/X \qquad (4)$$

$$X = \{[(B_o/B_g) - R_s] - [(B_{oi}/B_g) - R_{si}]\} \qquad (5)$$

Other equations required in the solution are the liquid saturation equation

$$S_L = S_w + (1 - S_w)[(1 - N_p)/B_{oi}]B_o \qquad (6)$$

and the instantaneous gas-oil-ratio equation

$$R_I = R_s + (k_g/k_o)(\mu_o/\mu_g)(B_o/B_g) \qquad (7)$$

Symbols N_p, G_p, R_I, B_o, B_g, R_s, S_L, S_w, k_g, k_o, μ_o, μ_g, c_f, c_o, c_w, and B_w are defined in Reference 2 and

B_{oi} = initial oil formation-volume factor.

n is subscript to designate any particular point.

i is subscript to designate initial conditions.

c_f, c_o, c_w are expansibility factors for formation, oil, and water, respectively.

In Equations 1, 2, and 6, N is set equal to unity and all production terms are expressed as a fraction of the actual N. Normally the solution is begun at the bubble-point pressure. But any lower pressure where cumulative gas and oil recoveries are known can be used. From the beginning point pressure decrements of 100 to 300 psi. are used. At every pressure decrement cumulative recoveries are known for the higher pressure end of the decrement from previous calculations or measured data.

Equation 2 involves two unknowns, ΔN_p and R_I. Once this equation is solved, Equations 6, 7, and 1 can be solved. For any selected pressure Equations 3 and 4 can be solved since they involve only fluid properties that are a function of pressure.

SOLUTION:

1. Select a pressure decrement, Δp, and for $p = p_{n-1} - \Delta p$ solve Equation 2 for ΔN_p by assuming R_I.

The size of the decrement, Δp, depends on the rate of change of the gas-oil ratio with pressure. The greater the gas-oil ratio change with pressure, the smaller the pressure decrement must be chosen.

2. Solve Equation 6 using $N_p = N_{p(n-1)} + \Delta N_p$.

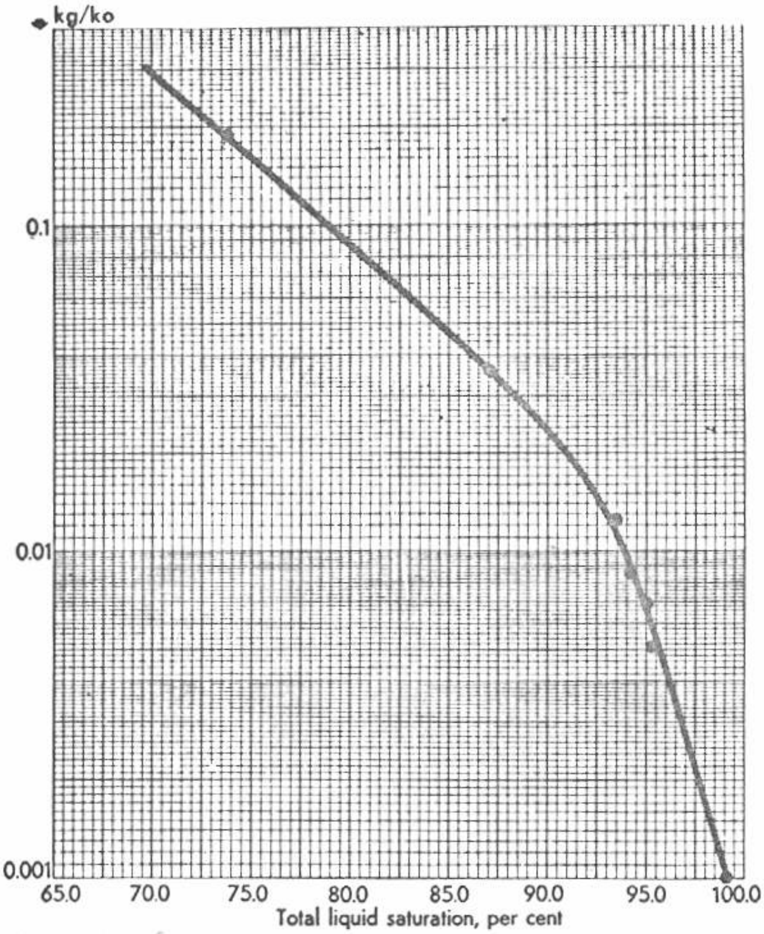
3. Solve Equation 7 for R_I after obtaining k_g/k_o at S_L.

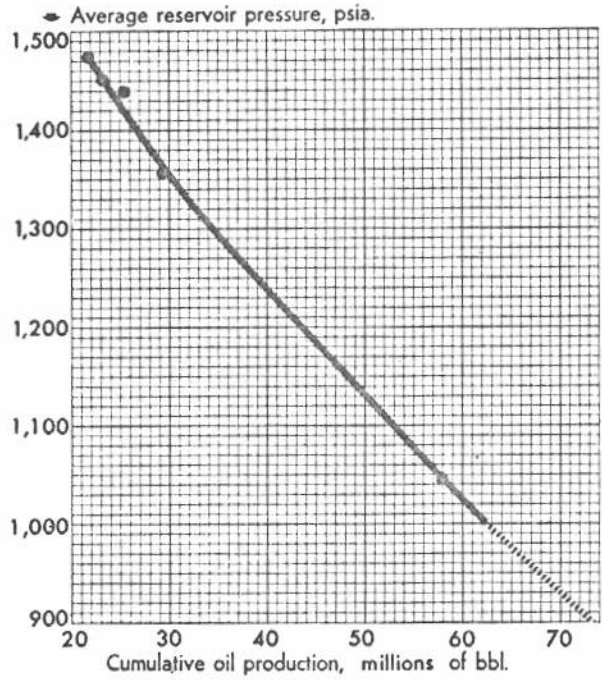
4. If the R_I computed is not approximately equal (to within 50 to 100 s.c.f./st.-tk. bbl. depending on the magnitude of R_I) to the R_I estimated in Equation 2, use the computed R_I and repeat steps 1, 2, and 3. The trial-and-error calculations are continued until R_I, computed, is about equal to R_I, estimated.

5. As a final check at each pressure, solve Equation 1, the right-hand side of which must give a value between 0.991 and 1.009. This will

Table 1—Production history for lense-type oil pool with depletion drive

Date	Avg. reservoir press., psia.	Cum. production Oil, M. bbl.	Cum. production Gas, M.M.c.f.	Average producing GOR, s.c.f./st.-tk. bbl.
4-1-50	....	12,150	......	......
4-1-51	....	15,830	......	......
10-1-51	....	18,270	......	......
4-1-52	1,472	21,730	13,005	830
10-1-52	1,450	23,505	......	860
10-1-53	1,435	25,330	......	925
4-1-54	1,357	29,300	20,205	1,020
3-1-57	1,043	57,900	59,410	1,765



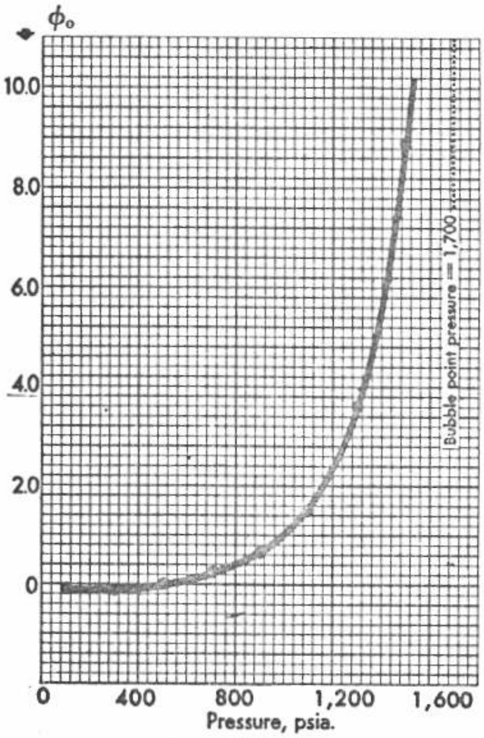


CUMULATIVE-OIL-RECOVERY VARIATION with reservoir pressure. Fig. 2.

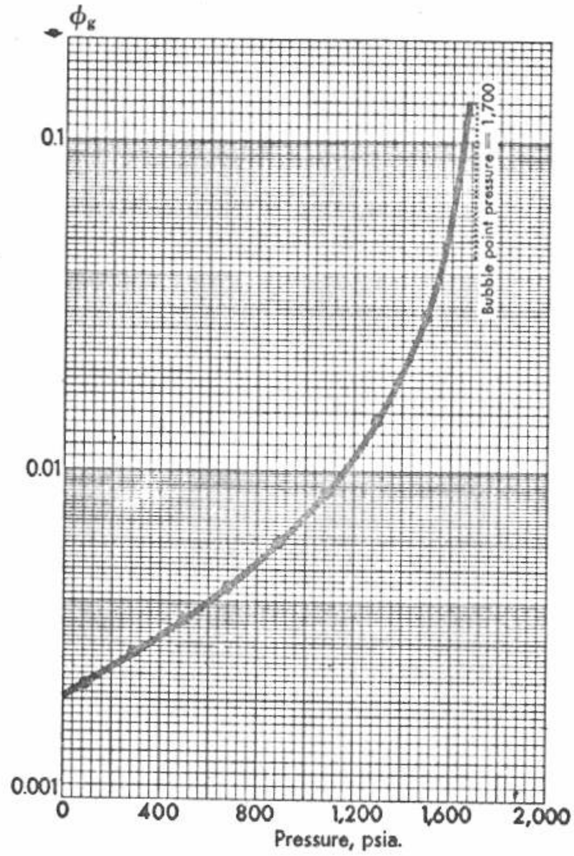
Table 2—Basic data and calculations

(1) P_I psia.	(2) B_o	(3) R_s s.c.f./ st.-tk. bbl.	(4) B_g Res. bbl./s.c.f.	(5) μ_o Cp	(6) B_o/B_g (2) ÷ (4)	(7) $(B_o/B_g)-R_s$ (6) − (3)	(8) B_{oi}/B_g 1.265 ÷ (4)	(9) $(B_{oi}/B_g)-R_{si}$ (8) − 540.	(10 (7) −
1,700	1.265	540	0.00132	1.19	958	418	958	418	(
1,500	1.241	490	0.00150	1.22	827	337	843	303	34
1,300	1.214	440	0.00175	1.25	694	254	723	183	71
1,100	1.191	387	0.00210	1.30	567	180	602	62	118
900	1.161	334	0.00262	1.35	443	109	483	− 57	166
700	1.147	278	0.00344	1.50	333	55	368	−172	227
500	1.117	220	0.00495	1.80	226	6	256	−284	290
300	1.093	160	0.00860	2.28	127	− 33	147	−393	360
100	1.058	84	0.02720	3.22	39	− 45	47	−493	448

(20) (11)+(19)	(21) ΔN_p (18)÷(20)	(22) N_p Σ (21)	(23) 1 − (22)	(24) (2)×(23)	(25) $\frac{(1-S_w)\times(24)}{B_{oi}} = \frac{(0.75)\times(24)}{1.265}$	(26) $S_L =$ (25) + $S_w =$ (25) + 0.25	(27) k_g/k_o from Fig. 1	(28) (μ_o/μ_g) (B_o/B_g) = $(\mu_o/0.02)\times$
......	......	......	......	......	.....	.....	......	......
28.293	0.0353	0.0353	0.9647	1.1972	0.710	0.960	0.0043	50,447
16.671	0.0338	0.0691	0.9309	1.1301	0.670	0.920	0.0165	43,375
12.621	0.0350	0.1041	0.8959	1.0670	0.633	0.883	0.0300	36,855
10.801	0.0306	0.1347	0.8653	1.0046	0.596	0.846	0.0495	29,903
8.842	0.0343	0.1690	0.8310	0.9532	0.565	0.815	0.0730	24,975
7.784	0.0315	0.2005	0.7995	0.8930	0.529	0.779	0.1130	20,340
6.997	0.0302	0.2307	0.7693	0.8408	0.498	0.748	0.1670	14,478
4.862	0.0419	0.2726	0.7274	0.7696	0.456	0.706	0.2800	6,279



OIL-PRESSURE FACTOR, ϕ_o, as function of reservoir pressure. Fig. 3.



GAS-PRESSURE FACTOR, ϕ_g, as function of reservoir pressure. Fig. 4.

ermine performance and ultimate oil recovery

(11) ϕ_o ÷ (10) l./bbl.	(12) ϕ_g 1 ÷ (10) bbl./s.c.f.	(13) $R_{I\ est}$	(14) $R_{avg}=$ $[R_I+R_{I\ n-1}]$ ÷ 2 $[(13)_n +$ $(13)_{n-1}]$ ÷ 2	(15) $Np_{n-1}\ \phi_o$ $(22)_{n-1}$ × $(11)_n$	(16) $Gp_{n-1}\ \phi_g$ $(33)_{n-1}$ × $(12)_n$	(17) (15)+(16)	(18) 1 − (17)	(19) $R_{avg}\phi_g$ (14) × (12)
∞	∞	540	.....	......	......	......	......	......
9.912	0.02941	710	625	......	......	......	1.0000	18.381
3.577	0.01408	1,150	930	0.1263	0.3101	0.4364	0.5636	13.094
1.525	0.00847	1,470	1,310	0.1054	0.4534	0.5588	0.4412	11.096
0.657	0.00602	1,900	1,685	0.0684	0.6014	0.6698	0.3302	10.144
0.242	0.00441	2,000	1,950	0.0326	0.6638	0.6964	0.3036	8.600
0.021	0.00345	2,500	2,250	0.0035	0.7510	0.7545	0.2455	7.763
0.092	0.00278	2,600	2,550	−0.0184	0.8074	0.7890	0.2110	7.089
0.100	0.00223	1,850	2,225	−0.0231	0.8193	0.7962	0.2038	4.962

9) × (28)	(30) R_I from Eq. 7	(31) Computed R_{avg} $[(30)_{n-1}+(30)_n]$ ÷ 2	(32) $\Delta G_p =$ (21) × (31)	(33) G_p Σ (32)	(34) $G_p\ \phi_g =$ (33) × (12)	(35) $N_p\ \phi_n =$ (22) × (11)	(36) N = 1.000 = (34) + (35)	(37) N_p, st.-tk. bbl. = (22) × (497,036,000)
..	540	.....	......	......	......	......	......	....
17	707	624	22.027	22.027	0.6478	0.3499	0.9977	17,545,000
16	1,156	932	31.502	53.529	0.7337	0.2472	1.0009	34,345,000
06	1,493	1,325	46.375	99.904	0.8462	0.1588	1.0050	51,741,000
80	1,814	1,654	50.612	150.516	0.9061	0.0885	0.9946	66,951,000
23	2,101	1,958	67.159	217.675	0.9599	0.0409	1.0008	83,999,000
98	2,518	2,310	72.765	290.440	1.0020	0.0042	1.0062	99,656,000
18	2,578	2,548	76.950	367.390	1.0213	−0.0212	1.0001	114,666,000
58	1,842	2,210	92.599	459.989	1.0258	−0.0273	0.9985	135,492,000

be obtained at each pressure if R_I, computed, and R_I, estimated, are about equal.

At 900 psi.

$$\phi_o = \frac{(1.161/0.00262) - 334}{[(1.161/0.00262) - 334 - (1.265/0.00262) + 540]} = 0.657$$

$$\phi_g = \frac{1}{[(1.161/0.00262) - 334 - (1.265/0.00262) + 540]}$$

$$= 0.00602 \text{ st.-tk. bbl./s.c.f.}$$

Assume R_I = 1,900 s.c.f./st.-tk. bbl.

$$\Delta N_p = \frac{1.0000 - (0.1041)(0.657) - (99.904)(0.00602)}{0.657 + [(1,900 + 1,470)/2](0.00602)}$$

$$= 0.0306$$

$$N_p = N_{p(n-1)} + \Delta N_p = 0.1041 + 0.0306 = 0.1347$$

$$S_L = 0.250 + (1.00 - 0.25)[(1.0000 - 0.1347)/1.265]\ 1.161 = 0.846$$

From Fig. 1, $k_g/k_o = 0.0495$.

$$R_I = 334 + (0.0495)$$

$$\times (1.35/0.02)(1.161/0.00262)$$

$$= 1,814 \text{ s.c.f./st.-tk. bbl.}$$

$$\approx 1,900$$

$$\Delta G_p = \{[R_{I(n-1)} + R_I]/2\}\ \Delta N_p$$

$$= [(1,493 + 1,814)/2](0.0306)$$

$$= 50.612$$

$$G_p = G_{p(n-1)} + \Delta G_p \mp 99.904$$

$$+ 50.612 = 150.516$$

Solving Equation 1

$$1.0000 = (0.1347)(0.657)$$

$$+ (150.516)(0.00602) = 0.9946$$

To convert the values of column 22, Table 2, to barrels, it is necessary to know the oil in place at bubble-point conditions. Using Equations 1 and 2 from Page 1 of this manual,

$$c_e = c_o + \frac{S_w}{1 - S_w} c_w$$

$$+ \frac{c_f}{1 - S_w} \quad (8)$$

Where:

c_e = composite compressibility, vol./pore vol./psi.

$$c_e = \{8.20 + [0.25/(1.00 - 0.25)]$$

$$\times 3.1 + [4.0/(1.00 - 0.25]\}\ 10^{-6}$$

$$= 14.56 \times 10^{-6}$$

$$N = \frac{N_p B_o - (W_p - W_e)}{B_{oi}\, c_e\, (p_i - p_b)} \quad (9)$$

$$500 \times 10^6 = \frac{(N_p)(1.265)}{(1.256)(14.56 \times 10^{-6})(2,110 - 1,700)}$$

$$N_p = \frac{(500)(1.256)(14.56)(410)}{1.265}$$

= 2,964,000 bbl. cumulative oil production to bubble-point pressure.

Therefore, oil in place at the bubble-point pressure is 500,000,000 − 2,964,000 = 497,036,000.

Stock-tank oil recovered to 900 psi. bbl. = (497,036,000) (0.1347) = 66,951,000. Extrapolation of the data in Table 1 gives an actual recovery of 73,000,000 − 2,964,000 or 70,036,000 st.-tk. bbl. to 900 psi. Fig. 2 shows that at 1,300 psia. the actual recovery was about 34,500,000 − 2,964,000 or 31,500,000 while the predicted recovery was 34,345,000 bbl., as shown in Table 2.

Discussion. The method used in this problem to predict recovery and performance of a depletion-type oil reservoir involves an equation that is a rearrangement of the Schilthuis[4] form of the material-balance equation.[1] Equation 2 can easily be obtained from the Schilthuis material-balance equation. Note that Equations 3 and 4 represent portions of Equation 2 that are a function of pressure and temperature. The factors ϕ_o and ϕ_g are called pressure factors and data for their calculation are obtained from bottom-hole fluid analyses. When a gas cap is initially present these factors also involve the ratio of initial gas-cap volume to oil volume in the reservoir.

To predict future performance of a reservoir using the material-balance equation it is common practice to estimate incremental oil production for each pressure decrement in trial-and-error calculations. Tracy's method, used in this problem, better lends itself to estimating the instantaneous gas-oil ratio. A solution is obtained with fewer trial-and-error steps since the instantaneous gas-oil ratio is less sensitive to small inaccuracies.[1]

Table 2 shows the basic data and calculations for determining performance and ultimate oil recovery.

The first five columns give basic data while columns 6 through 12 show the computation of the oil and gas pressure factors. The pressure factors given in columns 11 and 12 are plotted in Figs. 3 and 4. Note that each of the pressure factors is very sensitive to pressure for conditions near the bubble point. The rapidly changing characteristic of these factors near the bubble-point pressure and the fact that average reservoir pressure cannot be precisely determined point out that the material-balance equation cannot be reliably used for conditions near the bubble-point pressure. This is characteristic of all estimates of oil in place and other applications of the material-balance equation.[1]

In Table 2, columns 13 through 21 solve Equation 2, while columns 22 through 26 solve Equation 6, and columns 27 through 30 solve Equation 7. A comparison of columns 1 and 22 or 37 shows the performance of the reservoir or recovery as the pressure declines.

The sample calculation shows that cumulative oil recovery to the bubble-point pressure was estimated to be 2,964,000 bbl. This was subtracted from the initial oil in place to get oil in place at the bubble point and thus apply the material-balance equation at the bubble point and lower pressures. Table 1 and Fig. 2 show cumulative production and other history data for this reservoir down to a pressure of 1,043 psia. Extrapolation of these data and taking into account oil produced above the bubble point shows that the actual recovery to a pressure of 900 psia. was about 70,000,000 bbl. The value predicted from the material-balance calculations was about 67,000,000 bbl., which is in good agreement with the actual figures. Similarly at 1,300 psia. the actual recovery was about 31,500,000 bbl. while the predicted value was 34,345,000 bbl.

References

1. Tracy, G. W., "Simplified Form of the Material Balance Equation": Trans. AIME (1955), 204, 243.
2. "Standard Letter Symbols for Petroleum Reservoir Engineering and Electric Logging": Journal of Petroleum Technology, October 1956.
3. Guerrero, E. T., "How to Find Original Oil in Place by Material Balance Above the Bubble Point": The Oil and Gas Journal, Feb. 27, 1961, p. 104.
4. Schilthuis, R. J., "Active Oil and Reservoir Energy": Trans. AIME (1936), 118, 33.

Part 43

How to reduce the depletion-drive performance of a reservoir to a time basis

GIVEN: Predicted primary performance, fluid property and relative permeability data (Fig. 1) for a depletion-type reservoir as shown in the first seven columns of Table 1. Other data for this pool are as follows:

Allowable oil production rate, B/D/well	50
Abandonment oil production rate, B/D/well	5
Allowable producing days per month	15
Minimum sand face pressure, psia.	100
Penalty gas-oil ratio, cu. ft./bbl.	2,000
Bubble-point pressure, psia.	1,700

The field is divided into three productivity-index areas as follows:

Area	Productivity index, B/D/psi.	Number of wells	Oil in place at 1,700 psia., st.-tk. bbl.
A	0.10	100	83,300,000
B	0.50	350	291,700,000
C	0.75	150	125,000,000
			500,000,000

FIND: Convert the predicted primary performance to a time basis.

METHOD OF SOLUTION: From the definition of productivity index and the steady-state radial flow equation it is possible to show in Equation 1 below that:

$$PI = (PI)_i \left(\frac{k_{ro}/B_o\mu_o}{(k_{ro}/B_o\mu_o)_i} \right) \qquad (1)$$

This gives the productivity index at any pressure in the history of a reservoir as a function of the initial productivity index, relative oil permeability and fluid properties. Other equations required are

$$Q_{om} = (p - p_{min})\ (PI) = (p - 100)\ (PI) \qquad (2)$$

The allowable producing rate is 50 bbl. per day for 15 days per month or 24.7 bbl. per day for 30.4 days per month. Hence

Q_{oa} = 24.7 B/D/well for Q_{om} equal to or greater than 24.7 and R_I less than 2,000
= 24.7 × (2,000/R_I) for Q_{om} equal to or greater than 24.7 and R_I greater than 2,000
= (24.7 × 2,000)/R_I or Q_{om} whichever is lower for Q_{om} less than 24.7 and R_I greater than 2,000
= Q_{om} for Q_{om} less than 24.7 and R_I less than 2,000 (3)

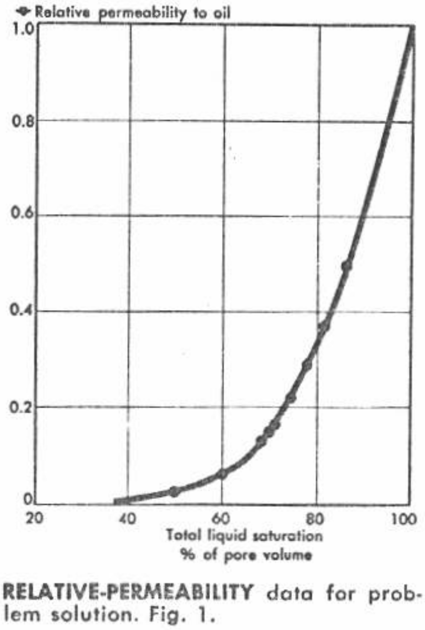
$$Q_{o\,(avg)} = \frac{Q_{oa\,(n)} + Q_{oa\,(n-1)}}{2} \qquad (4)$$

$$\Delta N_p = (\Delta N_p/N)\ N \qquad (5)$$

$$t = \sum_{i=1}^{n} \left[\frac{\Delta N_p}{(Q_{o(avg)})\ (30.4)\ (\text{number of wells})} \right]_i \qquad (6)$$

Where:

PI = productivity index at pressure p, BPD/psi.
K_{ro} = relative permeability to oil
B_o = oil formation volume factor
μ_o = viscosity of oil
i = subscript indicating initial conditions



RELATIVE-PERMEABILITY data for problem solution. Fig. 1.

p = reservoir pressure, psia.
p_{min} = minimum well flowing pressure, psia.
Q_o = rate of oil production, BPD
m = subscript for maximum
n = subscript for point under study.
a = subscript for actual
R_I = instantaneous or producing gas-oil ratio, SCF/STB
ΔN_p = incremental oil production, st.-tk. bbl.
N = initial oil in place, st.-tk. bbl.
t = time, months.

SOLUTION: A complete solution to this problem is shown in Table 1. Computations are similar for the three areas. At 900 psi. for Area A:

$$PI = (0.10)\left[\frac{0.460/(1.161)(1.35)}{1.000/(1.265)(1.19)}\right] = 0.0443$$

$$Q_{om} = (900 - 100)(0.0443) = 35.4 \text{ BPD}$$

At 900 psia—

$$R_I = 1{,}814 \text{ SCF/STB}$$

Therefore

$$Q_{oa} = 50/30.4 \times 15 = 24.7 \text{ BPD}$$

Also

$$Q_{o(avg)} = 24.7 \text{ BPD} = (24.7)(30.4)(100) = 75{,}088 \text{ bbl./mo. for area}$$

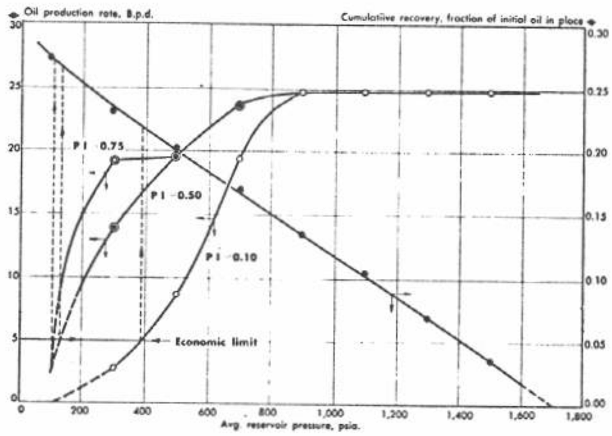
$$\Delta N_p = (0.0306)(83.3 \times 10^6) = 2.5490 \times 10^6 \text{ st.-tk. bbl.}$$

$$\Delta t = 2{,}549{,}000/75{,}088 = 33.9 \text{ months}$$

From the totals of columns 17, 25, and 33 ultimate oil recovery

$$= 132{,}810{,}000 \text{ bbl.}$$

Or using the initial oil in place for



GRAPHICAL DETERMINATION of cumulative oil recovery. Fig. 2.

Table 1—Basic data and calculations

(1) p, psia.	(2) B_o	(3) μ_o cp	(4) S_L % From depletion drive calc.	(5) k_{ro} From Fig. 1	(6) $\Delta N_p/N$ From depletion drive calc.	(7) R_I From depletion drive calc.	(8) $B_o\mu_o$ (2)×(3)	(9) $k_{ro}/B_o\mu$ (5)÷(8
1,700	1.265	1.19	100.0	1.000	......	540	1.505	0.664
1,500	1.241	1.22	96.0	0.835	0.0353	707	1.514	0.552
1,300	1.214	1.25	92.0	0.690	0.0338	1,176	1.518	0.455
1,100	1.191	1.30	88.3	0.565	0.0350	1,493	1.548	0.365
900	1.161	1.35	84.6	0.460	0.0306	1,814	1.567	0.294
700	1.147	1.50	81.5	0.370	0.0343	2,101	1.721	0.215
500	1.117	1.80	77.9	0.285	0.0315	2,518	2.011	0.142
300	1.093	2.28	74.8	0.230	0.0302	2,578	2.492	0.092
100	1.058	3.22	70.6	0.160	0.0419	1,842	3.407	0.047

Area B

(20) PI $(PI)_i\times(10)$ $(0.5)\times(10)$	(21) Q_{om} B/D/well (20)×(12)	(22) Q_{oa} B/D/well Considering allowable	(23) $Q_{o(avg)}$ B/D/well	(24) $Q_{o(avg)}$ Bbl./month for area (23)×(30.4)×(350)	(25) ΔN_p 10^6 bbl. (6)×291.7×10^6	(26) Δt months (25)÷(24)	(27) t month Σ (26
0.5000	800.0	24.7	24.7	262,808	....	....	....
0.4155	581.7	24.7	24.7	262,808	10.2970	39.2	39.2
0.3425	411.0	24.7	24.7	262,808	9.8595	37.5	76.7
0.2750	275.0	24.7	24.7	262,808	10.2095	38.8	115.5
0.2215	177.2	24.7	24.7	262,808	8.9260	34.0	149.5
0.1620	97.2	23.5	24.1	256,424	10.0053	39.0	188.5
0.1070	42.8	19.6	21.6	229,824	9.1886	40.0	228.5
0.0695	13.9	13.9	16.8	178,752	8.8093	49.3	277.8
0.0355	0	0	7.0	74,480	12.2222	164.1	441.9
					79.5174		

each area and recovery factors from Fig. 2, ultimate oil recovery

Because of proration require-

$$= [(83.3)(0.218) + (291.7)(0.266) + (125.0)(0.275)]\ 10^6 = 130{,}130{,}000 \text{ bbl.}$$

DISCUSSION: It is desirable to relate recovery with time to aid evaluation studies and determine rate of income. Although material-balance calculations are made assuming a reservoir as uniform and using average reservoir properties, the conversion of recovery to a time basis may be predicted better if attempts are made to recognize some of the variations in rock properties that occur over a reservoir.

In this problem the reservoir was divided into three areas having different productivity-index characteristics. The variation of cumulative oil recovery with pressure decline was assumed to be the same in each area. Although the areas are in communication, no oil migration was assumed. This assumption is satisfied if the different areas are depleted at about the same rate.

ments the different areas in this pool were depleted evenly except for pressures below 700 psi. Note in columns 19, 27, and 35 that areas A, B, and C will require 31, 37, and 33 years, respectively, to obtain ultimate recovery. Ultimate recovery for each layer can be obtained from columns 17, 25, and 33 of Table 1 or by using initial oil-in-place figures for each area and recovery factors obtained from Fig. 2. Variation of production rate for each area with decline in pressure is shown in Fig. 2. Also shown is variation of cumulative recovery with decline in average pressure. From these graphs cumulative recovery for each area is read at the economic limit. On this graph are shown plotted the rates of columns 14, 22, and 30 of Table 1. These rates were computed at the pressures indicated in column 1. Note in Table 1 that average rates were used for selected pressure increments in finding time. Ultimate recoveries from Table 1 and those computed from data of Fig. 2 are comparable.

Other methods to reduce depletion-drive performance to time basis involve average productivity index and other data for the entire reservoir. Also, it is possible to divide a reservoir into areas and plot production rate variation with pressure for each area and develop a composite productivity curve.[1,2] From this curve an average reservoir rate can be obtained, from which time is related to pressure and recovery.

References

1. Pirson, S. J., "Oil Reservoir Engineering": McGraw-Hill Book Co., Inc., second edition, 1958, pp. 509-532.

2. Muskat, M., "Physical Principles of Oil Production": McGraw-Hill Book Co., Inc., first edition, 1949, pp. 347-360.

duce performance to time basis

Area A									
(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
$_o/B_o\mu_o)$ $_{ro}/B_o\mu_o)_i$ ÷0.664	PI $(PI)_i \times (10)$ $(0.1)\times(10)$	$p - p_{min}$ (1) − 100	Q_{om} BPD/well (11)×(12)	Q_{oa} BPD/well Considering allowable	$Q_{o(avg)}$ B/D/well	$Q_{o(avg)}$ bbl./month for area (15)×(30.4) × 100	ΔN_p 10^6 bbl. (6) × 83.3 × 10^6	Δt months (17)÷(16)	t months Σ (18)
1.000	0.1000	1,600	160.0	24.7	24.7	75,088	......	.....	.....
0.831	0.0831	1,400	116.3	24.7	24.7	75,088	2.9405	39.2	39.2
0.685	0.0685	1,200	82.2	24.7	24.7	75,088	2.8155	37.5	76.7
0.550	0.0550	1,000	55.0	24.7	24.7	75,088	2.9155	38.8	115.5
0.443	0.0443	800	35.4	24.7	24.7	75,088	2.5490	33.9	149.4
0.324	0.0324	600	19.4	19.4	22.1	67,184	2.8572	42.5	191.9
0.214	0.0214	400	8.6	8.6	14.0	42,560	2.6240	61.7	253.6
0.139	0.0139	200	2.8	*5.0	6.8	20,672	2.5157	121.7	375.3
0.071	0.0071	....	0	....	....	......	......	....	....
							19.2174		

Area C								
(28)	(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)
PI ×(10))×(10)	Q_{om} B/D/well (28)×(12)	Q_{oa} B/D/well Considering allowable	$Q_{o(avg)}$ B/D/well	$Q_{o(avg)}$ Bbl./month for area (31)×(30.4) ×(150)	ΔN_p 10^6 bbl. (6)×125.0 ×10^6	Δt months (33)÷(32)	t months Σ (34)	N_p/N Σ (6)
7500	1,200.0	24.7	24.7	112,632	......	....	....	......
6233	872.6	24.7	24.7	112,632	4.4125	39.2	39.2	0.0353
5138	616.6	24.7	24.7	112,632	4.2250	37.5	76.7	0.0691
4125	412.5	24.7	24.7	112,632	4.3750	38.8	115.5	0.1041
3323	265.8	24.7	24.7	112,632	3.8250	34.0	149.5	0.1347
2430	145.8	23.5	24.1	109,896	4.2875	39.0	188.5	0.1690
1605	64.2	19.6	21.6	98,496	3.9375	40.0	228.5	0.2005
1043	20.9	19.2	19.4	88,464	3.7750	42.7	271.2	0.2307
0533	0	0	9.6	43,776	5.2375	119.6	390.8	0.2726
					34.0750			

*Rounded to economic limit.

Part 44

How to find performance and ultimate oil recovery of a combination solution gas, gas-cap drive reservoir

GIVEN: Well-test and log information show that a new reservoir has a gas cap with an initial size of one half that of the oil column ($m_i = 0.5$). The reservoir pinches out below the oil column so no water encroachment will take place. Initial reservoir pressure was 2,500 psia. and reservoir temperature is 180° F. Using the volumetric approach, initial oil in place was found to be 56 × 10^6 st.-tk. bbl. Average water saturation for the pool is 20.0% and the permeability ratio-total liquid saturation relationship (k_g/k_o vs. S_L) is that shown by Fig 1 on Page 18. Other basic data regarding the fluid properties are shown in the first seven columns of Table 1.

FIND: Performance and ultimate oil recovery for the reservoir if the rate of gas injection is one half of the gas produced at pressure levels below 1,500 psia. Also assume that the gas-oil contact remains stationary and that the gas resulting from expansion of the gas cap diffuses throughout the oil column.

METHOD OF SOLUTION: This problem can be solved using the material-balance, instantaneous-gas-oil-ratio, and total-liquid-saturation equations. The Schilthuis material-balance equation for incremental oil production in a pressure interval, taking into account injection into a gas cap, is

$$\Delta N_p = \frac{[(B_t - B_{oi})/B_g] + m_i B_{oi} [(1/B_{gi}) - (1/B_g)] + G_{i(n-1)} - N_{p(n-1)} [(B_t/B_g) - R_{si}] - G_{p(n-1)}}{(B_t/B_g) - R_{si} + (1 - I) R_{av}} \quad (1)$$

Table 1—Basic data and computation of gas cap-solution

(1) p, psia.	(2) B_o	(3) R_s s.c.f./STB	(4) $B_g \times 10^3$, Res. Bbl./SCF	(5) μ_o cp	(6) μ_g cp	(7) B_t	(8) $B_t - B_{oi}$ (7)−1.498	(9) $1/B_g$	(10) $\frac{B_t - B_{oi}}{B_g}$ (8)×(9)	(11) $\frac{1}{B_{gi}} - \frac{1}{B_g}$ 954.2−(9)
2,500	1.498	721	1.048	0.488	0.0170	1.498	0	954.2	0	0
2,300	1.463	669	1.155	0.539	0.0166	1.523	0.025	865.8	21.6	88.4
2,100	1.429	617	1.280	0.595	0.0162	1.562	0.064	781.3	50.0	172.9
1,900	1.395	565	1.440	0.658	0.0158	1.620	0.122	694.4	84.7	259.8
1,700	1.361	513	1.634	0.726	0.0154	1.701	0.203	612.0	124.2	342.2
1,500	1.327	461	1.884	0.802	0.0150	1.817	0.319	530.8	169.3	423.4
1,300	1.292	409	2.206	0.887	0.0146	1.967	0.469	453.3	212.6	500.9
1,100	1.258	357	2.654	0.981	0.0142	2.251	0.753	376.8	283.7	577.4
900	1.224	305	3.300	1.085	0.0138	2.597	1.099	303.0	333.0	651.2
700	1.190	253	4.315	1.199	0.0134	3.209	1.711	231.7	396.4	722.5
500	1.156	201	6.163	1.324	0.0130	4.361	2.863	162.3	464.7	791.9
300	1.121	149	10.469	1.464	0.0126	7.109	....	....	....	....

(23) ΔN_p (18)÷(22)	(24) N_{pn} Σ(23)	(25) B_o/B_{oi} (2)÷1.498	(26) $(1-N_{pn})\frac{B_o}{B_{oi}}$ [1−(24)] (25)	(27) $(1-S_w)$ (26) (0.8) (26)	(28) $S_{Ln} = S_w + (27)$ 0.200+(27)	(29) k_g/k_o	(30) μ_o/μ_g (5)÷(6)	(31) B_o/B_g (2)×(9)	(32) $\frac{\mu_o}{\mu_g} \times \frac{B_o}{B_g}$ (30)×(31)	(33) $k_g/k_o \times$(32) (29)×(32)
.....	.....	1.000	...	....	1.000	0	28.71	1,429.4	41,038	0
0.0668	0.0668	0.977	0.912	0.730	0.930	0.0010	32.47	1,266.7	41,130	41
0.0555	0.1223	0.954	0.837	0.670	0.870	0.0294	36.73	1,116.5	41,009	1,206
0.0415	0.1638	0.931	0.779	0.623	0.823	0.0535	41.65	968.7	40,346	2,159
0.0324	0.1962	0.909	0.731	0.585	0.785	0.0840	47.14	832.9	39,263	3,298
0.0255	0.2217	0.886	0.690	0.552	0.752	0.1250	53.47	704.4	37,664	4,708
0.0353	0.2570	0.862	0.640	0.512	0.712	0.1980	60.75	585.7	35,581	7,045
0.0313	0.2883	0.840	0.598	0.478	0.678	0.2900	69.08	474.0	32,744	9,496
0.0220	0.3103	0.817	0.563	0.450	0.650	0.4050	78.62	370.9	29,160	11,810
0.0201	0.3304	0.794	0.532	0.426	0.626	0.5400	89.48	275.7	24,670	13,322
0.0195	0.3499	0.772	0.502	0.402	0.602	0.7100	101.85	187.6	19,107	13,566

Where:

ΔN_p = oil production in pressure interval p_{n-1} to p_n, fraction of N

B_t = two-phase formation-volume factor at p_n

B_{oi} = initial oil formation-volume factor

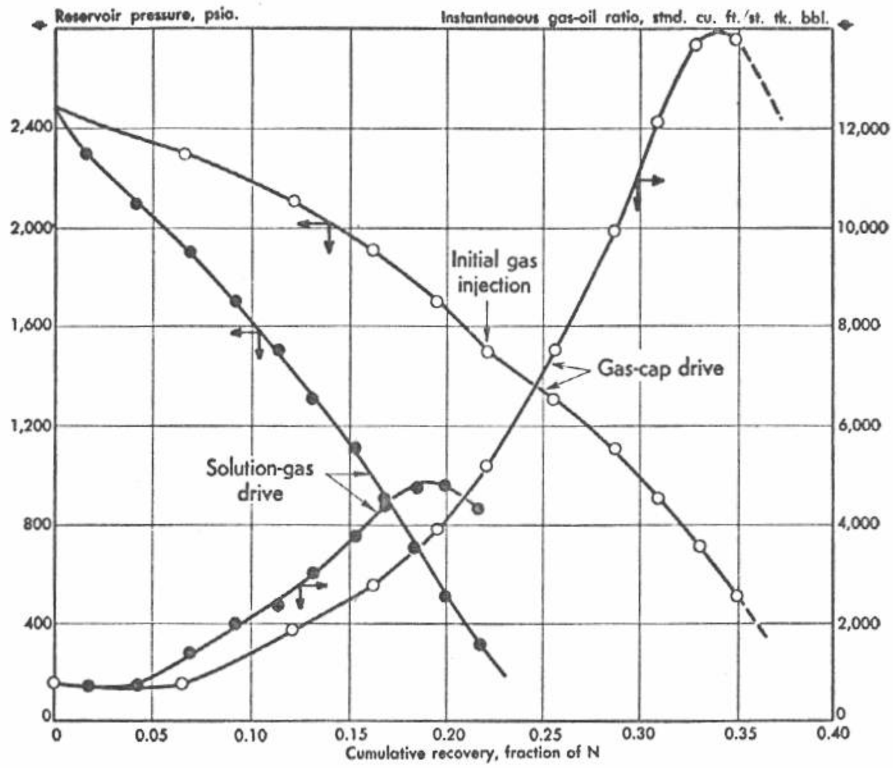
B_g = gas formation-volume factor at p_n, res. bbl./s.c.f.

m_i = ratio of initial reservoir pore space occupied by gas to that occupied by oil

B_{gi} = initial gas formation-volume factor, res. bbl./s.c.f.

I = fraction of produced gas injected into gas cap

$G_{p(n-1)}$ = cumulative gas production to p_{n-1}, fraction of N



COMPARISON of solution-gas drive and gas-cap drive (with injection) performances. Fig. 1.

...as drive performance with gas injection into gas cap.

(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
...B_{oi} (11) ...749) (11)	$G_{p(n-1)}-G_{p0}$ $(37)_{(n-1)}-(37)_6$	$G_{I(n-1)}=$ (I) (13)	$B_t/B_g=$ (7)×(9)	$\frac{B_t}{B_g}-R_{si}$ (15)−721	$N_{p(n-1)}\times(16)$ $(24)_{(n-1)}\times(16)$	(10)+(12)+ (14)−(17)− $(37)_{(n-1)}$	R_{In} Estimated	R_{av} $\frac{(19)_{(n-1)}+(19)_n}{2}$	(1−I)R_{av}	(16)+(21)
0	....	....	1,429.4	508.4	0	0	721	....	....	....
66.2	....	....	1,318.6	597.6	0	87.8	711	716	716	1,314
129.5	....	....	1,220.4	499.4	33.4	98.3	1,830	1,271	1,271	1,770
194.6	....	....	1,124.9	403.9	49.4	111.8	2,750	2,290	2,290	2,694
256.3	....	....	1,041.0	320.0	52.4	115.6	3,750	3,250	3,250	3,570
317.1	....	....	964.5	243.5	47.8	120.2	5,200	4,475	4,475	4,719
375.2	....	....	891.6	170.6	37.8	117.1	7,380	6,290	3,145	3,316
432.5	222.8	111.4	848.2	127.2	32.7	139.2	9,920	8,650	4,325	4,452
487.7	493.7	246.9	786.9	65.9	19.0	122.0	12,000	10,960	5,480	5,546
541.2	734.8	367.4	743.5	22.5	7.0	129.8	13,690	12,845	6,423	6,446
593.1	994.5	497.3	707.8	−13.2	− 4.4	133.1	13,650	13,670	6,835	6,822

(34)	(35)	(36)	(37)	(38)	(39)	(40)	(41)	(42)	(43)	(44)
...R_{In}= ...+(33) ...)+(33)	$R_{Iav}=\frac{R_{In}+R_{I(n-1)}}{2}=\frac{(34)_n+(34)_{(n-1)}}{2}$	$\Delta G_p=$ $R_{Iav}\Delta N_p=$ (35)×(23)	$G_{pn}=$ Σ (36)	(16)×(24)	(38)+(37)	SDI= (10)÷(39)	$G_{In}=$ (14)+(I) (36)	(12)+(41)	GDI= (42)÷(39)	SDI+GDI= 1.000= (40)+(43)
721	......	....	0	0	0	0.0	....	......	.....	.....
710	716	47.8	47.8	39.9	87.7	0.2463	....	66.2	0.7548	1.0011
1,823	1,267	70.3	118.1	61.1	179.2	0.2790	....	129.5	0.7227	1.0017
2,724	2,274	94.4	212.5	66.2	278.7	0.3029	....	194.6	0.6982	1.0021
3,811	3,268	105.9	318.4	62.8	381.2	0.3258	....	256.3	0.6724	0.9982
5,169	4,490	114.5	432.9	54.0	486.9	0.3477	....	317.1	0.6513	0.9990
7,454	6,312	222.8	655.7	43.8	699.5	0.3039	111.4	486.6	0.6956	0.9995
9,853	8,654	270.9	926.6	36.7	963.3	0.2945	246.9	679.4	0.7053	0.9998
...2,115	10,984	241.6	1,168.2	20.4	1,188.6	0.2802	367.7	855.4	0.7197	0.9999
...3,575	12,845	258.2	1,426.4	7.4	1,433.8	0.2765	496.5	1,037.7	0.7237	1.0002
...3,767	13,671	266.6	1,693.0	− 4.6	1,688.4	0.2752	630.6	1,223.7	0.7248	1.0000

$N_{p(n-1)}$ = cumulative oil production to p_{n-1}, fraction of N

R_{si} = initial solution-gas-oil ratio, s.c.f./st.-tk. bbl.

$G_{i(n-1)}$ = cumulative gas injected to p_{n-1}, fraction of N

$R_{av} = (R_{I(n-1)} + R_{In}) \div 2$ or average instantaneous gas-oil ratio, s.c.f./st.-tk. bbl.

N = initial oil in place, st.-tk. bbl.

below p_{n-1}; (2) compute ΔN_p; (3) solve for N_{pn} ($N_{pn} = N_{p(n-1)} + \Delta N_p$); (4) compute S_{Ln}; (5) obtain k_g/k_o at S_{Ln}; and (6) solve Equation 3 for R_{In}. This should check to within about 5.0% or less with the estimated R_{In} or else the procedure is repeated for another estimated R_{In} value. As a final check solve Equation 4.

$$\times \left[\frac{(1.0000 - 0.3103)(1.224)}{1.498}\right]$$

$$= 0.20 + (0.80)(0.6897)(0.817)$$

$$S_L = 0.20 + 0.45 = 0.65 \text{ or } 65.0\%$$

From Fig. 1 of Reference 1, k_g/k_o = 0.4050.

$$SDI + GDI = 1.000 = \frac{(B_t - B_{oi})/B_g}{N_{pn}[(B_t/B_g) - R_{si}] + G_{pn}} + \frac{m_i B_{oi}[(1/B_{gi}) - (1/B_g)] + G_{in}}{N_{pn}[(B_t/B_g) - R_{si}] + G_{pn}} \quad (4)$$

Since it is assumed that the gas-oil contact does not move, the oil-column size remains constant and thus the total liquid saturation is given by

$$S_{Ln} = S_w + (1 - S_w) \times [(1 - N_{pn})/B_{oi}] B_o \quad (2)$$

Where:

S_{Ln} = total liquid saturation at p_n

S_w = interstitial water saturation (a constant)

N_{pn} = cumulative oil production to p_n, fraction of N

The instantaneous-gas-oil-ratio equation is

$$R_{In} = R_{sn} + [(k_g/k_o)(\mu_o/\mu_g)(B_o/B_g)]_n \quad (3)$$

Where:

SDI = solution-gas-drive index

GDI = gas-cap-drive index

The trial-and-error calculations are continued until Equation 4 is satisfied to within 1% accuracy (SDI + GDI = 0.99 to 1.01). Once the correct ΔN_p is computed for a pressure increment p_{n-1} to p_n, a new increment is taken and the calculations are repeated. This procedure is continued to the abandonment pressure to obtain the variation of recovery (N_p) with pressure (p) and ultimate oil recovery.

SAMPLE CALCULATION: The computations at 900 psi. are as follows:

Assume R_{In} = 12,000

$$R_{av} = (R_{I(n-1)} + R_{In}) \div 2 = (9{,}920 + 12{,}000) \div 2 = 10{,}960$$

$$R_{In} = 305 + (0.4050) \times \frac{1.085}{0.0138} \times \frac{1.224}{0.003300}$$

$$= 305 + (0.4050)(78.62)(370.9)$$

$$= 305 + (0.4050)(29{,}160)$$

$$= 305 + 11{,}810 = 12{,}115$$

$$R_{av} = (9{,}853 + 12{,}115) \div 2$$

$$= 10{,}984 \approx 10{,}960$$

$$G_{pn} = G_{p(n-1)} + \Delta N_{pn} R_{av}$$

$$= 926.6 + (0.0220)(10{,}984)$$

$$\Delta N_p = \frac{\frac{(2.597 - 1.498)}{0.003300} + (0.5)(1.498)\left(\frac{1}{0.001048} - \frac{1}{0.003300}\right) + 246.9 - 0.2883\left(\frac{2.597}{0.003300} - 721\right) - 926.6}{\left(\frac{2.597}{0.003300} - 721\right) + (1.0 - 0.5)(10{,}960)}$$

Where:

R_{In} = instantaneous-gas-oil ratio at p_n, s.c.f./st.-tk. bbl.

R_{sn} = solution-gas-oil ratio at p_n, s.c.f./st.-tk. bbl.

k_g/k_o = permeability ratio, gas to oil at S_{Ln}

μ_o, μ_g = oil and gas viscosities at p_n, cp.

B_o = oil formation-volume factor at p_n

PROCEDURE: In Equation 1 all factors are either known or can be determined independently except R_{In} in R_{av} and ΔN_p. The recommended procedure is: (1) estimate R_{In} at a pressure p_n, about 200 psi.

$$= \frac{333.0 + (0.749)(651.2) + 246.9 - (0.2883)(65.9) - 926.6}{65.9 + 5{,}480}$$

$$= \frac{333.0 + 487.7 + 246.9 - 19.0 - 926.6}{5{,}546}$$

$$= \frac{122.0}{5{,}546} = 0.0220$$

$$N_{pn} = 0.2883 + 0.0220$$

$$= 0.3103$$

$$S_L = 0.20 + (1.00 - 0.20)$$

$$= 926.6 + 241.6 = 1{,}168.2$$

$$G_{in} = G_{i(n-1)} + I\,\Delta G_p$$

$$= 246.9 + (0.5)(241.6)$$

$$= 246.9 + 120.8 = 367.7$$

$$SDI + GDI = 1.0000 = \frac{(2.597 - 1.498) \div 0.003300}{(0.3103)\left(\frac{2.597}{0.003300} - 721\right) + 1{,}168.2}$$

$$+ \frac{(0.5)(1.498)\left(\frac{1}{0.001048} - \frac{1}{0.003300}\right) + 367.7}{(0.3103)\left(\frac{2.597}{0.003300} - 721\right) + 1{,}168.2}$$

$$= \frac{333.0}{(0.3103)(65.9) + 1{,}168.2} + \frac{487.7 + 367.7}{(0.3103)(65.9) + 1{,}168.2}$$

$$= \frac{333.0}{1{,}188.6} + \frac{855.4}{1{,}188.6} = 0.2802 + 0.7197 = 0.9999$$

DISCUSSION: The Schilthuis material - balance approach used earlier[1] to solve depletion-drive performance can be readily generalized to cover gas-drive reservoirs initially overlain by gas caps, provided the downdip gravity drainage of the oil does not play a significant role in the production mechanism.[1 2 3 4]

This assumes that the gas-oil contact will not move into the oil zone but rather that the gas expansion from the gas cap will merely be diffused through the oil column. Although in practice some degree of gravity drainage will be present, except when there is oil migration into the gas cap due to excessive depletion of the gas-cap gas, this problem considers only the case where the effect of the gas cap as a simple gas reservoir is evaluated.[3] This is believed to closely assimilate many gas-cap drives, particularly those with low formation permeability (less than 25 md. absolute permeability).

Equation 1 is a modified version of the Schilthuis material-balance equation. Equations 2 and 3 are the liquid-saturation and instantaneous-gas-oil-ratio equations. Equation 4 is the material-balance equation expressed as the solution-gas-drive and gas-cap-drive indexes.

In Equations 1 and 4 it is assumed that no gas is produced from the gas cap (these equations could be easily modified to take this into account) but that the gas-cap gas diffuses into the oil column and is eventually produced in the oil wells. Such a procedure further assumes that the gas-oil contact remains fixed and, thus, the oil column does not shrink and has a constant pore volume. The calculation procedure used in these problems is similar to that recommended by Tracy.[5]

Table 1 shows the basic data and computations for this problem. The first eight columns contain basic data. Columns 8 through 23 are a solution of Equation 1, while columns 24 through 28 are a solution of Equation 2. Columns 29 through 34 are a solution of Equation 3 and columns 35 through 44 are a solution of Equation 4.

It is well to note that two checks are used in these calculation procedures. In column 19 the instantaneous-gas-oil ratio at the end of the pressure increment is estimated and subsequently computed in column 34. The estimated and computed values should agree within about 5.0%. The subsequent calculation of Equation 4 serves as an additional and more accurate check. The sum of the solution-gas-drive and gas-cap-drive indexes should be within ±1% or less of 1.000.

In Table 1, columns 13, 14, and 41 take into consideration the reinjection of gas into the gas cap. No values are shown in these columns until the pressure falls below 1,500 psia. since injection was initiated at this pressure. Columns 20 and 21 remain identical from initial reservoir pressure to 1,500 psia. for the same reason.

The same data were used in this problem as were used to solve a solution-gas-drive problem earlier. In addition, however, a gas cap was assumed one-half the size of the initial oil column and injection of one-half of the produced gas was assumed to take place starting at 1,500 psia. Fig. 1 shows a comparison of the performance reported for the solution-gas drive and that obtained in this problem. The presence of a gas cap and the injection of gas resulted in more than a 50% increase in recovery over that of a solution gas drive. Also, the peak gas-oil ratios were much higher for the former. The more rapid initial decline in reservoir pressure caused higher initial gas-oil ratios in the solution-gas-drive case than in the gas-cap-drive case. It can be seen in Fig. 1 that the initiation of gas injection caused a decrease in the rate of decline of cumulative recovery with reservoir pressure.

Calculations similar to those presented in this problem can be performed for gas injection at other points in the history of a reservoir or using larger or smaller fractions of gas injection. Such computations can be made and compared with each other to select the most advantageous operating procedure. Usually such studies also include a thorough consideration of economics.

Proper exploitation of a reservoir with an initial gas cap requires that the pressure in the gas cap always remain higher than that in the oil column. Otherwise migration of oil into the gas cap will occur and the ultimate oil recovery could be less under such conditions than that by solution-gas drive.

References

1. Guerrero, E. T., Reservoir Engineering 40: The Oil and Gas Journal, Sept. 11, 1961, p. 111.
2. Schilthuis, R. D., "Active Oil and Reservoir Energy": Trans. AIME (1936) 118, 33.
3. Muskat, M., "Physical Principles of Oil Production": McGraw-Hill Book Co., Inc., first edition, 1949, pp. 427-431.
4. Pirson, S. J., "Oil Reservoir Engineering": McGraw-Hill Book Co., second edition, 1958, pp. 658-681.
5. Tracy, G. W., "Simplified Form of the Material-Balance Equation": Trans. AIME (1955), 204, 243.

Part 45

How to find cumulative water influx by material balance

. . . by using production, fluid property, and related data

GIVEN: Production and pressure histories for a limestone reservoir are given in the first five columns of Table 1. Columns 6, 7, and 8 give fluid-property data for this pool. Other data are:

Initial reservoir pressure, p_i = 3,640 psia.

Initial oil in place, $N = 39.5 \times 10^6$ st.-tk. bbl.

Ratio of initial reservoir free-gas volume to initial reservoir oil volume, $m_i = 0.644$.

Reservoir temperature, T_r = 211° F.

Initial oil formation volume factor, $B_{oi} = 1.464$.

FIND: Water - influx history of this pool.

METHOD OF SOLUTION: This problem involves a pool that has an initial gas cap and water drive. It can be solved with the generalized material-balance equation, which is commonly expressed as[1]

$$N\{B_t - B_{oi} + m_i B_{oi}[(B_g/B_{gi}) - 1]\} = N_p[B_t + B_g(R_p - R_{si})] - (W_e - W_p) \quad (1)$$

or solving for W_e

$$W_e = N_p[B_t + B_g(R_p - R_{si})] + W_p - N\{B_t - B_{oi} + m_i B_{oi} \times [(B_g/B_{gi}) - 1]\} \quad (2)$$

Where:

R_{si} = initial solution gas-oil ratio, s.c.f. per st.-tk. bbl.

B_{oi} = initial oil formation volume factor.

B_{gi} = initial gas formation volume factor, reservoir bbl. per s.c.f.

W_e, N_p, B_t, B_g, R_p, W_p, N, and B_g are standard AIME symbols.[2]

STB = stock tank barrels.

SCF = standard cubic feet

SAMPLE CALCULATION: Solving for the cumulative water influx for 1956 using Equation 2 gives

$$W_e = 1{,}210{,}700 \times [1.476 + (0.000918)(4{,}688 - 888)] + 261{,}260 - 39.5 \times 10^6 \times [1.476 - 1.464 + (0.644)(1.464) \times (0.000918/0.000892) - 1]$$
$$= 6{,}009{,}915 + 261{,}260 - 1{,}556{,}300$$
$$= 4{,}714{,}875 \text{ or } 4{,}715{,}000 \text{ bbl.}$$

DISCUSSION: Performance of an oil reservoir is controlled by the producing mechanism which depends on the nature of the energy available for oil expulsion and the manner in which this energy is used. One of the major types of reservoir energy is the energy of compression of contiguous waters in mobile communication with the oil column.

Within the oil column, energy of compression of oil and water is of minor importance. An exception to this is found during the early phases of production from reservoirs containing highly undersaturated crudes and before water encroachment develops enough to be effective in decreasing the rate of pressure decline.

A reservoir is controlled by water drive if a large part of the volumetric withdrawals are replaced by entry of water into the oil column. Water intrusion may be predominantly lateral (along the bedding plane), as an edge-water encroachment, or water may invade the oil zone as a bottom-water drive. In such reservoirs, rate of pressure decline may be higher at first than later when a pressure differential is established between the oil zone and aquifer that induces water intrusion sufficient to replace a large portion of the fluid withdrawals.[3]

The ratio of fluid-withdrawal rate to inflow capacity of the aquifer affects the quantitative aspects of this behavior. Furthermore, pressures will rise if the field is shut in.

Under an effective water drive, producing gas-oil ratios will not

Table 1—basic data and calculations for

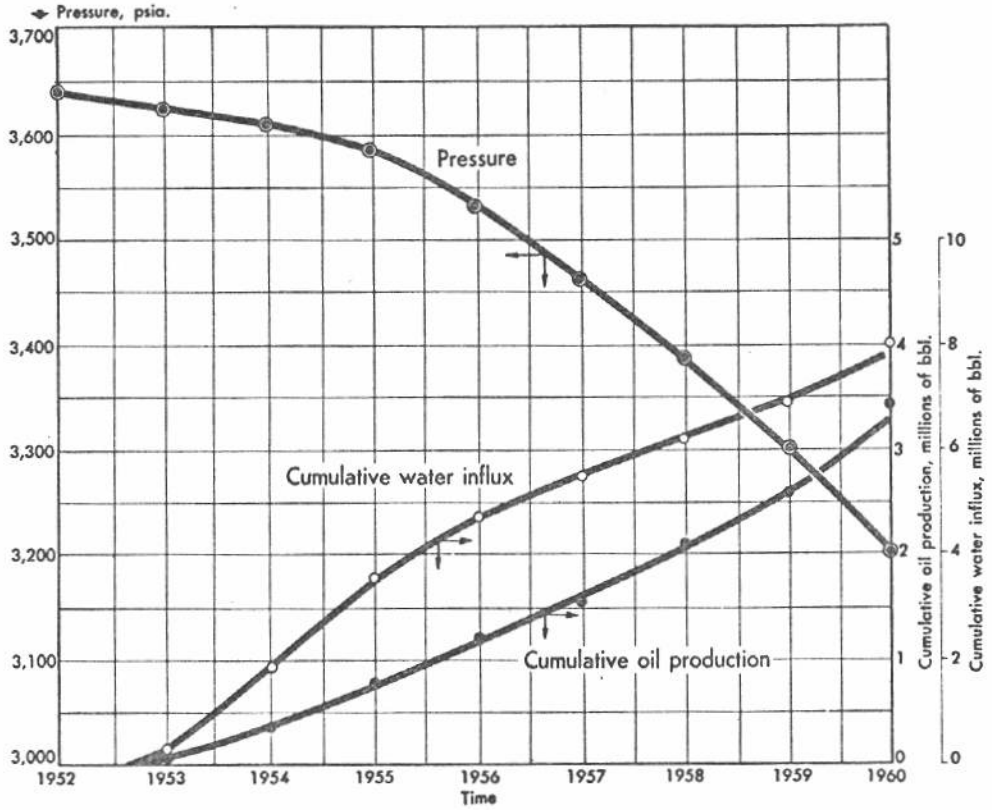
(1) Date	(2) p, psia.	(3) N_p, STB	(4) G_p, M.c.f.	(5) W_p, bbl.	(6) $B_g \times 10^3$ res. bbl./s.c.f.	(7) R_s, s.c.f./STB	(8) B_t	(9) R_p s.c.f./STB (4)÷(3)	(10) $R_p - R_{si}$ (9)−888	(11) $B_g \times (10)$
1952	3,640	0	0	0	0.892	888	1.464	....	....	....
1953	3,625	65,850	491,200	0	0.895	884	1.466	7,459	6,571	5.881
1954	3,610	364,700	2,311,450	990	0.899	880	1.468	6,338	5,450	4.900
1955	3,585	791,810	4,115,260	83,110	0.905	874	1.469	5,197	4,309	3.900
1956	3,530	1,210,700	5,675,800	261,260	0.918	860	1.476	4,688	3,800	3.488
1957	3,460	1,541,670	7,004,920	412,175	0.936	846	1.482	4,544	3,656	3.422
1958	3,385	2,080,400	8,405,270	601,360	0.957	825	1.491	4,040	3,152	3.016
1959	3,300	2,575,710	9,706,110	924,675	0.982	804	1.501	3,768	2,880	2.828
1960	3,200	3,401,420	11,615,300	1,375,705	1.014	779	1.519	3,415	2,527	2.562

vary greatly and productive capacities of the producing wells will remain substantially constant, or decline slowly. Such conditions retard the growth of the free-gas phase in the oil zone.[3]

It is characteristic for reservoir pressures to be higher near the region of water encroachment and to fall off toward the interior of the pool if the reservoir is uniformly produced.[3]

When the initial oil and free gas in place are known, application of Equation 2 in future reservoir-performance predictions requires knowledge of future oil and water production rates, and producing gas-oil ratios in addition to the behavior of the fluid properties. Its use requires the assumption of future producing gas-oil and water-oil ratios. However, when sufficient past production-performance data are available, this equation affords a direct means for determining the magnitude of water encroachment. It is also a valuable aid in establishing the production mechanism.[3]

These applications are demonstrated with this problem. Table 1 gives the basic data and calculations for water influx using Equation 2. In this case the initial oil and free gas in place are known along with 9 years of pressure and performance data. In addition, the variation of fluid properties with pressure is also known. Thus all the factors in Equation 2 are known except the cumulative water influx. Under such conditions the solution of Equation 2 is readily performed and does not require any trial-and-error calculations. In Table 1 the solution of Equation 2 has been performed in columns 8 through 21.



PERFORMANCE HISTORY of a limestone reservoir with water drive. Fig. 1.

Fig. 1 shows pressure, cumulative water influx, and cumulative oil production plotted versus time. These graphs show that pressure declined at a lower rate in the early years compared with the later years of production. Rate of oil production was fairly constant throughout the entire history. Conversely, the rate of water influx was greater in the first 3 years of production than later.

It appears that the reverse should have been the case to be in accord with the pressure behavior. It is possible that the aquifer in contact with this reservoir is also in contact with other pools, the production from which has had an effect on pressure behavior. If such were the case, the pressures used are low and cumulative water-influx determinations are in error.

Irrespective of this, the performance data and water-influx calculations show a strong water-drive condition. Reservoir pressure dropped only 440 psi. in 9 years of production in which 3,400,000 st -tk. bbl of oil were produced.

References

1. Schilthuis, R. J., "Active Oil and Reservoir Energy": Trans. AIME, 118, 33, 1936.

2. "Standard Letter Symbols for Petroleum Reservoir Engineering and Electric Logging": Supplement to Jour. of Pet. Tech., October 1956.

3. Muskat, Morris: "Physical Principles of Oil Production," first edition, McGraw-Hill Book Co., Chapter 9, 1949.

vater-influx history of limestone reservoir

(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
$_t$+(11))+(11)	N_p×(12) (3)×(12) STB	W_p+(13), (5)+(13) bbl.	B_g/B_{gi} (6)/0.892	$(B_g/B_{gi})-1.000$ (15)−1.000	$(B_{oi})(m_i)$ (16) (1.464) (0.644) ×(16)	B_t-B_{oi} (8)−1.464	(18)+(17)	N×(19) 39.5×10[6] ×(19) bbl.	W_e, (14)−(20), bbl.
....	....	....	1.0000	....	....	....	....	....	....
7.347	483,800	483,800	1.0034	0.0034	0.0032	0.002	0.0052	205,400	278,400
6.368	2,322,410	2,323,400	1.0078	0.0078	0.0074	0.004	0.0114	450,300	1,873,100
5.369	4,251,228	4,334,338	1.0146	0.0146	0.0138	0.005	0.0188	742,600	3,591,73[illegible]
4.964	6,009,915	6,271,175	1.0291	0.0291	0.0274	0.012	0.0394	1,556,300	4,714,875
4.904	7,560,350	7,972,525	1.0493	0.0493	0.0465	0.018	0.0645	2,547,750	5,424,775
4.507	9,376,363	9,977,723	1.0729	0.0729	0.0687	0.027	0.0957	3,780,150	6,197,573
4.329	11,150,249	12,074,924	1.1009	0.1009	0.0951	0.037	0.1321	5,217,950	6,856,974
4.081	13,881,195	15,256,900	1.1368	0.1368	0.1290	0.055	0.1840	7,268,000	7,988,900

Part 46

How to predict time required for reservoir pressure to reach bubble point at a well

GIVEN: A well is located in an oil field producing by liquid and rock expansion. Well has been producing through 6-in. casing at a rate of 190 bbl. per day since its discovery. Initial reservoir pressure was 3,100 psi. and can be considered fixed at the outer limits of the radius of drainage. Other physical properties of the reservoir were found to be as follows:

p_w = initial sand face flowing pressure = 1,500 psi.

μ_o = viscosity of oil at average reservoir conditions above bubble-point pressure = 1.8 cp.

p_b = bubble-point pressure = 1,450 psia.

B_{ob} = oil-formation volume factor at p_b = 1.321.

ρ_o = specific gravity of oil = 0.85.

c_e = effective compressibility of oil, water and rock for average reservoir conditions above bubble-point pressure = 1.1×10^{-4} vol./vol./psi.

h = net sand thickness = 29 ft.

ϕ = porosity, fraction = 0.26.

k_o = permeability = 10.4 md.

r_e = estimated radius of drainage = 800 ft.

FIND:

1. Whether the system can be considered steady-state or unsteady-state.
2. Time from discovery when reservoir pressure will fall to the bubble point if present rate of production is maintained constant.

METHOD OF SOLUTION: The following equations provided by Muskat,[1] Chatas,[2], and Van Everdingen and Hurst[3] are used to solve this problem:

$$v_s/v_f = \frac{1.173 \times 10^8 \mu_o r_w \ln (r_e/r_w)}{k_o (p_e - p_w) \sqrt{\rho_o c_e}} \quad (1)$$

$$t_s = 158.1 \frac{\phi c_e r_e^2 \mu_o}{4 k_o} \quad (2)$$

$$p_i - p_b = \frac{887.4 \mu_o B_{ob} Q_o P_T}{2\pi k_o h} \quad (3)$$

$$t = \frac{T \phi \mu_o c_e r_w^2}{k_o 6.33 \times 10^{-3}} \quad (4)$$

Where: ρ_o, μ_o, p_b, B_{ob}, c_o, h, ϕ, k_o, p_w, and r_e were defined with the data and

v_s = speed of propagation of disturbance.

v_f = maximum speed of fluid movement.

r_w = radius of well, ft.

p_e = pressure at radius of drainage, psi.

t_s = time required for readjustment after pressure disturbance, days.

p_i = initial reservoir pressure, psi.

Q_o = rate of oil production, bbl. per day.

P_T = dimensionless pressure-change term.

t = time, days.

T = dimensionless time.

SOLUTIONS:

Solutions of Equations 1 and 2 (see box) are used to tell if the system is steady-state or unsteady-state.

Solutions of Equations 1 and 2

$$\frac{v_s}{v_f} = \frac{(1.173 \times 10^8)(1.8)(0.25) \ln (800/0.25)}{(10.4)(3{,}100 - 1{,}500)\sqrt{(1.1 \times 10^{-4})(0.85)}}$$

$$= \frac{4.256 \times 10^{10}}{16{,}091} = 2{,}645{,}000$$

$$t_s = \frac{(158.1)(0.26)(1.1 \times 10^{-4})(800)^2(1.8)}{(4)(10.4)} = \frac{5{,}207.4}{41.6} = 125 \text{ days}$$

For steady-state flow the ratio given by Equation 1 must be large (greater than 10,000) and the time for readjustment obtained with Equation 2 must be small (less than 10 days).

Although the ratio v_s/v_f is large, the time for readjustment is also large and consequently the system is unsteady-state.

Using Equation 3:

$$3{,}100 - 1{,}450 = \frac{(887.4)(1.8)(1.321)(190) P_T}{(2)(3.14)(10.4)(29)}$$

$$1{,}650 = \frac{400{,}900 P_T}{1{,}894}$$

$$P_T = 7.795$$

In this problem the pressure is constant at the external boundary and:

$$r_e/r_w = \frac{800}{0.25} = 3{,}200 \text{ or}$$

about 3,000

From Table 5 of Reference 2 or Table 4 of Reference 3:

$$T = 2{,}807{,}700$$

Solving Equation 4:

$$t = \frac{(2{,}807{,}700)\,(0.26)\,(1.8)\,(1.1 \times 10^{-4})\,(0.25)^2}{(10.4)\,(6.33 \times 10^{-3})}$$

$$= \frac{9{,}033.8}{65.83} = 137 \text{ days}$$

DISCUSSION: Flow of fluids through porous media is inherently of a time-varying nature. Frequently the degree of time-variance is insignificant and may be considered as independent of time. Systems in which the flow of fluids is independent of time are considered as steady-state while systems in which the fluid flow is dependent on time are nonsteady-state.

Ability to differentiate between these two extremes of fluid flow, where time variations are negligible and where time variations are predominant, is essential in reservoir engineering work.[1] Two principles can assist in determining whether a system is steady-state or nonsteady-state. Equations 1 and 2 are forms of these principles.

Equation 1 involves the ratio of the speed of propagation of disturbances in a porous media to the maximum speed of fluid movement. This ratio is a measure of time, relative to fluid movement, required for pressure variations at the boundaries of a system to be transmitted to internal points.[2] For steady-state conditions the ability for a disturbance to be transmitted through a porous media must be great compared to the ability of the porous media to transmit fluids. In steady-state flow the mass at any point in the porous media remains constant with time. Thus rapid propagation of a disturbance, relative to fluid movement, assists in maintaining the mass constant at all points. Such conditions are approached in the single-phase flow of slightly compressible fluids.

While the ratio given by Equation 1 assists in defining the type of flow, it is necessary rather than a sufficient condition for the existence of steady-state.[2] A better criterion is the ratio of the rate of change in the fluid-mass content of a porous media caused by pressure variations at its boundaries to the steady-state mass flowability of the same media.[1,2] A simplified version of this ratio is given by Equation 2 where t_s is obtained from Equation 4 in Reference 2 and represents change in mass divided by mass rate of flow.

Thus a measure is obtained of the time required for the readjustment of the internal-pressure distribution in a reservoir to a steady-state distribution when pressure variations occur at the boundaries. The ratio determines the extent that a system deviates from strictly steady-state conditions.

By considering the ratio simply as the change in fluid-mass content to the steady-state mass flow, the type of readjustment given by Equation 2 is obtained. Equation 2 shows the significant physical properties that determine the extent of nonsteady-state behavior in porous-media systems. When a pressure change occurs at a boundary of a porous-media system, the time required for the system to achieve a steady-state internal-pressure distribution is directly proportional to the volume of contained fluid and to the fluid compressibility, and inversely proportional to the mobility. A system is steady-state when the time of readjustment given by Equation 2 is small (less than 10 days). Thus steady-state conditions are approached in small reservoirs containing fluids of low compressibility and high mobility. Such conditions are approached in pattern water-flooding operations after fillup has occurred.

In this problem a large ratio of the speed of propagation of the disturbances to the speed of sound in the fluid was obtained with Equation 1 indicating possible steady-state conditions. However, the time necessary for readjustment to steady-state conditions computed with Equation 2 was also large, indicating nonsteady-state behavior. Since the latter is a more comprehensive and stringent condition, the system must be considered as unsteady-state.

Van Everdingen and Hurst[3] have solved the diffusivity equation for linear and radial systems of infinite and finite extent. Their finite boundary systems include solutions for a closed exterior boundary and solutions for fixed constant pressure at the exterior boundary. The mathematical developments and final solutions are given in References 2 and 3. These solutions can be used to predict pressure behavior from assumed or known rates of flow and cumulative fluid influx from assumed or known pressure histories. Assumptions involved in the development of these equations include: (1) the effects of gravity on the fluid flow is negligible; (2) flow through porous media is macroscopically laminar and governed by Darcy's law; (3) the systems are ideally radial or linear; and (4) a single-phase, slightly compressible fluid exists between the boundary surfaces of the system. An additional limitation of these equations is that values for the pressure change and fluid influx terms are available only for a few specified boundary conditions. In spite of the rigid assumptions and limitations of these solutions many reservoir systems have been encountered to which this method can be applied.

Equation 3 defines the drop in reservoir pressure resulting with time while production is kept constant. The dimensionless pressure term, P_T, is defined by Equations 19, 20, 21, or 22 in Reference 2. These complex equations have been solved for various ratios of exterior to interior radii.[2,3]

Results are reported with the pressure-change term, P_T, versus dimensionless time. Thus in this problem the pressure-change term is computed with Equation 3, which, in turn, gives the corresponding dimensionless time, T.

Equation 4 relates dimensionless time and actual time. Since the other factors are known, Equation 4 can be solved for the time necessary for the pressure to decrease from initial conditions to bubble-point conditions.

References

1. Muskat, M., "The Flow of Homogeneous Fluids Through Porous Media," J. W. Edwards, Inc., second printing, Chapters 3, 4, and 10, 1946.

2. Chatas, A. T., "A Practical Treatment of Nonsteady-State Flow Problems in Reservoir Systems," Part 1, Pet. Eng., May 1953, B-42; Part 2, June 1953, B-38; Part 3, August 1953, B-44.

3. Van Everdingen, A. F., and Hurst, W., "The Application of the LaPlace Transformation to Flow Problems in Reservoirs," Trans. AIME, Vol. 186, pp. 305-324, 1949.

Part 47

How to predict future pressure of a water-drive reservoir

. . . from known production-rate history

GIVEN: A reservoir system is comprised of an oil field located in an aquifer. This field has produced oil for the past 2 years under a complete water drive. The production-rate history is as follows:

Time, days	Oil-production rate, st.-tk. bbl.
0	9,800
90	19,900
181	30,100
273	39,700
365	50,050
455	40,200
546	59,500
638	69,800
730	80,400

Other data for this reservoir system are:

Initial reservoir pressure, p_i = 4,100 psia.

Current reservoir pressure, p = 3,475 psia.

Absolute permeability, k = 210 md.

Area of oil pool, A_o = 2,100 acres.

Area of aquifer, A_w = 4,900 sq. miles.

Net sand thickness, h = 86 ft.

Porosity, ϕ = 21.9%.

Compressibility of water at average reservoir conditions (assumed constant), $c_w = 5.0 \times 10^{-6}$ vol./vol./psi.

Oil formation-volume factor at average reservoir conditions (assumed constant), B_o = 1.16.

Viscosity of water at average reservoir conditions (assumed constant), μ_w = 0.32 cp.

FIND: Reservoir pressure 10 years hence if the oil-production rate is maintained constant at 80,400 st.-tk. bbl./day.

METHOD OF SOLUTION: Unsteady-state equations have been derived[1, 2] to define the pressure change at the interior boundary of linear or radial systems with infinite exterior boundary, finite closed exterior boundary, and finite exterior boundary with constant pressure. Thus it is necessary to determine the nature of a reservoir's boundary to select the proper solution. A reservoir system may be considered infinite if the ratio of the aquifer pore volume to the pool pore volume is on the order of 1,000 or greater.[1]

In this problem

$$\frac{\text{Aquifer pore vol.}}{\text{Pool pore vol.}} = \frac{A_w h \phi}{A_o h \phi}$$

$$= \frac{640 \times 4{,}900}{2{,}100} = 1{,}493$$

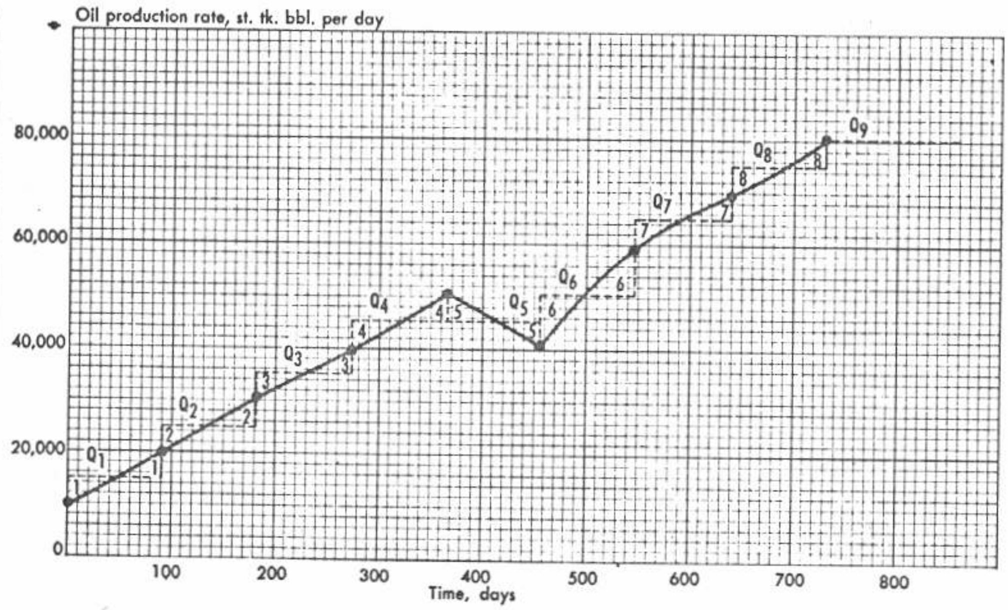
Hence the system is infinite, radial in nature, and is approximated by the equation[1]

Table 1—Computation of $\sum_{j=1}^{j=9} (Q_{oj} - Q_{o(j-1)}) (P_{Tn-T(j-1)})$

(1) Actual time, t, days	(2) T = 0.130 t = 0.130×(1)	(3) j	(4) $T_n - T_{(j-1)} = 569.40 - T_{(j-1)}$	(5) Q_o St.-tk. bbl./day	(6) Q_o avg. st.-tk. bbl./day $[(5)_{(j-1)} + (5)_j] \div 2$	(7) $Q_{oj} - Q_{o(j-1)}$ st.-tk. bbl./day $(6)_j - (6)_{(j-1)}$	(8) $P_{Tn-T(j-1)}$ Table 1 Ref. 1	(9) $(Q_{oj} - Q_{o(j-1)}) \times P_{Tn-T(j-1)}$ (7)×(8)
0	0	0	......	9,800	0	0	......	......
90	11.70	1	569.40	19,900	14,850	14,850	3.5811	53,179
181	23.53	2	557.70	30,100	25,000	10,150	3.5710	36,246
273	35.49	3	545.87	39,700	34,900	9,900	3.5603	35,247
365	47.45	4	533.91	50,050	44,875	9,975	3.5489	35,400
455	59.15	5	521.95	40,200	45,125	250	3.5374	884
546	70.98	6	510.25	59,500	*50,600	5,475	3.5262	19,306
638	82.94	7	498.42	69,800	64,650	14,050	3.5147	49,382
730	94.90	8	486.46	80,400	75,100	10,450	3.5022	36,598
4,380	569.40	9	474.50	80,400	80,400	5,300	3.4897	18,495

$$\sum_{j=1}^{j=9} (Q_{oj} - Q_{o(j-1)}) (P_{Tn-T(j-1)}) = 284{,}737$$

*Raised from 49,850 to 50,600 to make areas 6 in Fig. 1 equal.



Application of superposition theorem to oil-production history Fig. 1

$$p_i - p = \frac{887.4\,\mu_w B_o}{2\pi kh} \times$$

$$[Q_{o1}P_{Tn} + Q_{o2}P_{Tn-T1} - Q_{o1}P_{Tn-T1} + Q_{o3}P_{Tn-T2} - Q_{o2}P_{Tn-T2} + \ldots + Q_{o(n-1)}\,P_{Tn-T(n-2)} - Q_{o(n-2)}\,P_{Tn-T(n-2)}] \qquad (1)$$

$$p_i - p = \frac{887.4\,\mu_w B_o}{2\pi kh} \times$$

$$[Q_{o1}P_{Tn} + (Q_{o2} - Q_{o1})P_{Tn-T1} + (Q_{o3} - Q_{o2})P_{Tn-T2} + \ldots + (Q_{o(n-1)} - Q_{o(n-2)}) \times P_{Tn-T(n-2)}] \qquad (2)$$

$$p_i - p = \frac{887.4\,\mu_w B_o}{2\pi kh} \times \sum_{j=1}^{j=(n-1)} [(Q_{oj} - Q_{o(j-1)})P_{Tn-T(j-1)}] \qquad (3)$$

P_T is a dimensionless pressure change term that is a function of dimensionless time. Dimensionless time is related to actual time by the relation

$$T = \frac{6.33 \times 10^{-3}\,kt}{\phi\,\mu_w\,c_w\,r_w^2} \qquad (4)$$

Where:

ΔQ_o = increment of oil-production rate, st.-tk. bbl./day.

p = reservoir pressure of interest, psia.

P_T = dimensionless pressure change function defined by Equations 13, 14, or 15 of Reference 1.

T = dimensionless time.

t = actual time, days.

r_w = radius of interior boundary (represented by radius of oil pool in this application).

SOLUTION: To solve the problem it is necessary to convert actual time to dimensionless time, using Equation 4.

$$T = \frac{(6.33 \times 10^{-3})\,(210)\,t}{(0.219)\,(0.32)\,(5.0 \times 10^{-6})\,(5{,}400)^2}$$

$$= \frac{1{,}329{,}300\,t}{(0.3504)\,(29{,}160{,}000)}$$

$$= 0.130\,t$$

Where:

$$r_w = \sqrt{A/\pi}$$

$$= \sqrt{(2{,}100)\,(43{,}560)/(3.14)}$$

$$= \sqrt{29{,}132{,}500}$$

$$= 5{,}400 \text{ ft.}$$

Table 1 shows the computation procedure for the evaluation of

$$\sum_{j=1}^{j=n} (Q_{oj} - Q_{o(j-1)})\,(P_{Tn-T(j-1)})$$

in Equation 3.

Thus

$$4.100 - p = \frac{(887.4)\,(0.32)\,(1.16)}{(2)\,(3.14)\,(210)\,(86)} \times (284{,}737)$$

$$= \frac{(329.403)\,(284{,}737)}{(6.28)\,(18{,}060)} = 827$$

$$p = 4{,}100 - 827 = 3{,}273 \text{ psia.}$$

DISCUSSION: The most exact form of Equation 3 is given as Equation 23 in Reference 1. This equation gives the cumulative pressure drop at the interior boundary if, starting at zero time, the average rate of production over an interval of time has some constant value Q. The fluid and rock properties are assumed to remain constant throughout the time of flow. The equation applies to the unsteady-state flow of a single slightly compressible fluid. In actual practice the rate of production rarely remains constant but varies with time. If the production rates vary and their values at discrete time instants $T_o, T_1, T_2 \ldots, T_n$ are known, then the cumulative pressure drop at the interior boundary of this system over the total period of interest is given by Equation 3. Equation 3 represents a linear combination of solutions or step functions. It involves a constant term and a group of functions derived from the diffusivity equation.[1,2] Since the diffusivity equation is a linear homogeneous differential equation then the sum of any number of solutions each multiplied by a constant is also a solution.[3,4] This represents an application of the superposition theorem.[3,4]

It can be noted in Fig. 1 that the variation of oil production rates has been represented by nine constant rates, each applicable over a certain time interval. Theoretically, the shorter the time interval selected, the better should be the accuracy; however, consideration of other factors involved may not warrant smaller time intervals than those chosen in this problem.

In this problem oil produced in the vicinity of the well bore is replaced by oil flowing toward the well bore which in turn is replaced by edge-water encroachment across the oil-water contact. Since Equation 3 applies to the unsteady-state flow of a single slightly compressible fluid, it must be assumed that the compressibility of the oil is equivalent to the compressibility of the water. This, of course, is not true and some error is introduced. This is further complicated by the fact that the rock also expands as pressure declines. Addition of the rock expansibility factor to that of the slightly compressible fluid under study tends to take rock expansion into consideration. In spite of the assumptions involved and approximations made, useful results are obtained.

In solving Equation 3 it is necessary to convert actual time to dimensionless time which, in turn, is used to obtain the dimensionless pressure change functions from Reference 1. Equation 3 is a simplified form of Equation 2. Equation 2 essentially states that rate one is in effect from time zero to time one; rate two is in effect from time one to time two; rate three is in effect from time two to time three, etc. Another way of obtaining the same answer is to assume that rate one is in effect from time zero on; the difference between rate two and rate one is in effect from time one on; the difference between rate three and rate two is in effect from time two on, etc. This latter type of solution was used in this problem, as is shown in Table 1.

References

1. Chatas, A. T., "A Practical Treatment of Nonsteady-State Flow Problems in Reservoir Systems": Petroleum Engineer, Part 1, May 1953, p. B-42; Part 2, June 1953, p. B-38; Part 3, August 1953, p. B-44.
2. Van Everdingen, A. F., and Hurst, W., "The Application of the Laplace Transformation to Flow Problems in Reservoirs": Trans. AIME (1949), 186, p. 305.
3. Churchill, R. V., "Fourier Series and Boundary Value Problems": McGraw-Hill Book Co., Inc., first edition, 1941, pp. 3 and 99.
4. Sokolnikoff, I. S., and Redheffer, R. M., "Mathematics of Physics and Modern Engineering": McGraw-Hill Book Co., Inc., 1958, p. 51.

Part 48

How to determine cumulative water influx by the unsteady-state method

GIVEN: An oil pool is surrounded by an aquifer and has produced oil for 3 years under combined gas-cap expansion and water drives. The pressure history of this pool is as follows:

Date	Avg. reservoir pressure, psia.
July 1, 1957	6,000
Jan. 1, 1958	5,960
July 1, 1958	5,900
Jan. 1, 1959	5,805
July 1, 1959	5,620
Jan. 1, 1960	5,510
July 1, 1960	5,390

Other properties of this reservoir are:

Area, A = 7,000 acres

Area of aquifer, A′ = 1,000 sq. miles

Net sand thickness, h = 24 ft.

Oil formation-volume factor at avg. reservoir conditions (assumed constant), $B_o = 1.25$

Water compressibility factor at average reservoir conditions (assumed constant), $c_w = 4.1 \times 10^{-6}$ bbl./bbl./psi.

Viscosity of water at average reservoir conditions (assumed constant), $\mu_w = 0.25$ cp.

Absolute permeability, k = 255 md.

Porosity, $\theta = 23.0\%$

Rock compressibility factor, $c_r = 3.5 \times 10^{-6}$ pore volume per unit pore volume per psi.[1]

FIND: Cumulative water influx to January 1, 1960.

Method of solution: To select the appropriate equation for the solution of this problem it is necessary to establish whether the reservoir system is finite or infinite. A reservoir system may be considered infinite if the ratio of the aquifer pore volume to the oil-pool pore volume is on the order of 1,000 or greater.[2] For this problem

$$\frac{\text{aquifer pore vol.}}{\text{oil pool pore vol.}} = \frac{640\,A'\phi h}{A\phi h}$$

$$= \frac{(640)(1{,}100)}{7{,}000} = 101$$

where 640 represents acres per square mile. Thus the reservoir system is finite in extent.

Geologic evidence further indicates that the aquifer is surrounded by an impermeable shale. Hurst and Van Everdingen[3] developed an equation for the determination of cumulative water influx using the diffusivity and Darcy equations:

$$W_e = 2\pi\phi\, c_w r_w^2 h\, \Delta p Q_T \qquad (1)$$

Table 1—Computation of $\sum_{i=1}^{i=n} [(p_{(i-1)} - p_i) Q_{Tn-T(i-1)}]$ neglecting rock compressibility

(1) Date	(2) time, t, days	(3) pressure, p, psia.	(4) Dimensionless Time, T = 0.0705t	(5) $T_n - T_{(i-1)}$	(6) $Q_{Tn-T(i-1)}$ From Table 3 Ref. 2	(7) Δp psi. $(3)_n - (3)_{n-1}$	(8) Avg. Δp, psi. $[(7)_n + (7)_{n-1}] \div 2$	(9) $\Delta p Q_{Tn-T(i-1)}$ (8) × (6)
7/1/57	0	6,000	....	...	....	...	....	....
1/1/58	182	5,960	12.8	77.2	30.98	40	20.0	619.60
7/1/58	365	5,900	25.7	64.4	27.86	60	50.0	1,393.00
1/1/59	547	5,805	38.6	51.5	24.18	95	77.5	1,873.95
7/1/59	730	5,620	51.5	38.6	19.87	185	140.0	2,781.80
1/1/60	912	5,510	64.3	25.7	14.82	110	147.5	2,185.95
7/1/60	1,095	5,390	77.2	12.8	8.98	120	115.0	1,032.70

$\sum_{i=1}^{i=6} (p_{(i-1)} - p_i) Q_{Tn-T(i-1)}$ = 9,887.00

Table 2—Computation of $\sum_{i=1}^{i=n} [(p_{(n-1)} - p_i) Q_{Tn-T(i-1)}]$ considering rock compressibility

(1) Date	(2) time, t, days	(3) pressure, p. psia.	(4) Dimensionless Time, T = 0.0380 t	(5) $T_n - T_{(i-1)}$	(6) $Q_{Tn-T(i-1)}$ From Table 3 Ref. 2	(7) Δp psi. $(3)_n - (3)_{n-1}$	(8) Avg. Δp, psi. $[(7)_n + (7)_{n-1}] \div 2$	(9) $\Delta p Q_{Tn-T(i-1)}$ (8) × (6)
7/1/57	0	6,000	....	....	.....	...	....	....
1/1/58	182	5,960	6.9	41.6	20.93	40	20.0	418.60
7/1/58	365	5,900	13.9	34.7	18.42	60	50.0	921.00
1/1/59	547	5,805	20.8	27.7	15.66	95	77.5	1,213.65
7/1/59	730	5,620	27.7	20.8	12.68	185	140.0	1,775.20
1/1/60	912	5,510	34.7	13.9	9.91	110	147.5	1,461.73
7/1/60	1,095	5,390	41.6	6.9	6.28	120	115.0	722.20

$\sum_{i=1}^{i=6} (p_{(i-1)} - p_i) Q_{Tn-T(i-1)}$ = 6,512.38

This equation is applicable over any dimensionless time T for the constant pressure drop Δp. In practice the pressure drop varies with time. Since the diffusivity equation is linear[4] the superposition theorem can be applied to Equation 1 and the actual pressure history simulated with a series of constant pressure drops as shown in Fig. 1.

$$W_e = 2\pi\phi\, c_w r_w^2\, h\{(p_o - p_1) Q_{Tn} + (p_1 - p_2) Q_{Tn-T1} + (p_2 - p_3) Q_{Tn-T2} + \ldots + (p_{(n-1)} - p_n) Q_{Tn-T(n-1)} \quad (2)$$

$$W_e = 2\pi\phi\, c_w r_w^2 h \sum_{i=1}^{i=n} (p_{i-1} - p_i) Q_{Tn-T(i-1)} \quad (3)$$

or using units of barrel, psi., acres and feet.

$$W_e = 15{,}516\, \phi\, c_w A h \sum_{i=1}^{i=n} (p_{(i-1)} - p_i) Q_{Tn-T(i-1)} \quad (4)$$

Where $\pi r_w^2 = A$

W_e = cumulative water influx, bbl.

c_w = water compressibility at average reservoir conditions (assumed constant), bbl./bbl./psi.

p = pressure, psi.

Q_T = total volume of fluid passing a unit thickness of the interior boundary of a reservoir system over a period of time t and caused by a unit pressure drop at this boundary

n = time point at which cumulative water influx is desired

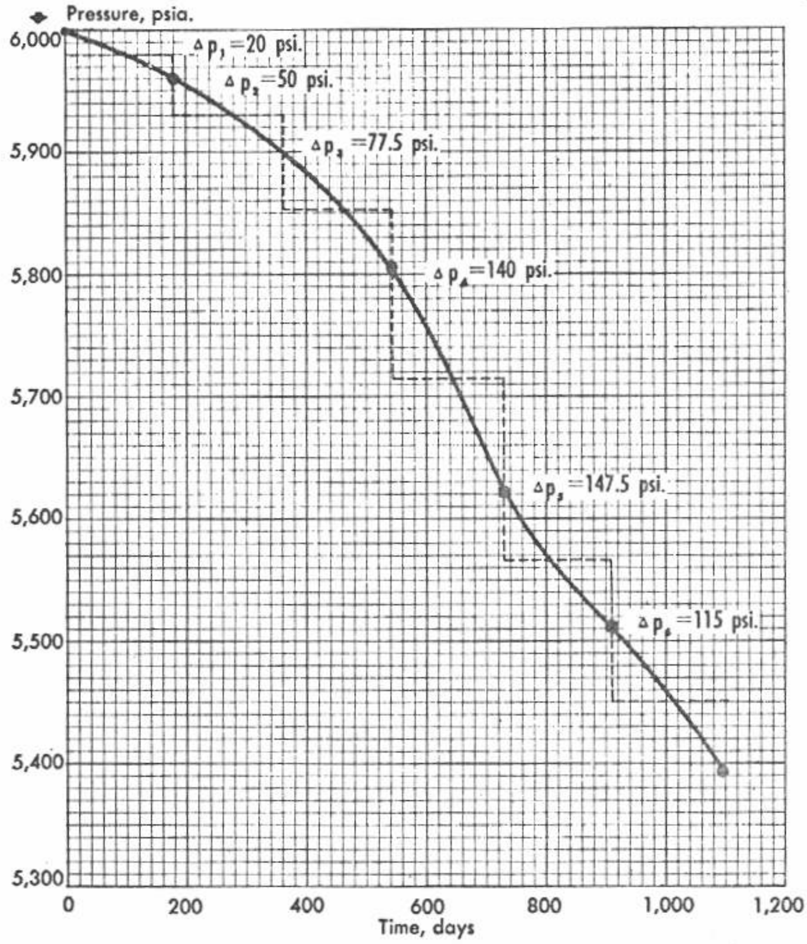
i = computation step points

The Q_T functions are defined by Equations 25, 26, 27, 28, and 29 of Reference 1 for infinite and finite reservoir systems. Values given by these equations have been computed[2,3] for essentially all ranges of dimensionless time and exterior radii of drainage encountered in practice. These values are available in graphical and tabular forms.[2,3]

SOLUTION: To obtain Q_T values from Reference 2 it is necessary to define the ratio of the exterior radius to the interior radius (r_e/r_w) and to convert actual time, t, to dimensionless time, T.

$$r_e/r_w = \sqrt{(A'/\pi)/(A/\pi)} = \sqrt{(A'/A)}$$
$$= \sqrt{(640)(1{,}100)/7{,}000}$$
$$= \sqrt{101} \approx 10$$

$$T = \frac{6.33 \times 10^{-3}\, kt}{\phi\, \mu_w\, c_w\, r_w^2} \quad (5)$$



APPLICATION OF SUPERPOSITION THEOREM to pressure history. Fig. 1.

Where:

T = dimensionless time

t = actual time, days

r_w = radius of interior boundary (represented by radius of oil pool in this application)

$$= \sqrt{A/\pi}$$
$$= \sqrt{(7{,}000)(43{,}560) \div 3.14}$$
$$= 9{,}855 \text{ ft.}$$

$$T = \frac{(6.33)(10^{-3})(255)\,t}{(0.23)(0.25)(4.1 \times 10^{-6})(9{,}855)^2} = 0.0705\,t$$

Table 1 shows the computation of

$$\sum_{i=1}^{i=n}(p_{(i-1)} - p_i)\,Q_{T_n - T(i-1)}$$

for Equation 4. Thus

$$W_e = (15{,}516)(0.23)(4.1 \times 10^{-6}) \times (7{,}000)(24)(9{,}887.00)$$
$$= (3{,}568.68)(0.0287)(237{,}288)$$

= 24,303,000 bbl. not taking rock expansion into consideration. Since water is slightly compressible (the compressibility of rock will be significant by comparison) it is good practice to consider rock compressibility in developing the effective system compressibility (or expansibility) factor, c_e. This is done by adding the water and rock compressibility factors.

$$c_e = c_w + c_r \qquad (6)$$

Where:

c_r = rock compressibility factor pore vol./unit pore vol./psi.

$$c_e = 4.1 \times 10^{-6} + 3.5 \times 10^{-6} = 7.6 \times 10^{-6}$$

Thus

$$T = \frac{(6.33\times10^{-3})(255)\,t}{(0.23)(0.25)(7.6\times10^{-6})(9{,}855)^2} = 0.0380\,t$$

Table 2 shows the computation of

$$\sum_{i=1}^{i=n}(p_{(i-1)} - p_i)\,Q_{T_n - T(i-1)}$$

for Equation 4 when rock compressibility is included. Thus

$$W_{e(corr.)} = (15{,}516)(0.23) \times 7.6\times10^{-6})(7{,}000) \times (24)(6{,}512.38) = 29{,}674{,}000 \text{ bbl.}$$

DISCUSSION: This problem illustrates the computation of cumulative water influx, W_e, from a known pressure history and known or estimated aquifer area, rock, and fluid properties. The unsteady-state flow equations of Van Everdingen and Hurst were used in the computations.[3]

Most of the assumptions underlying the development and application of these equations were given on Page 39. In addition it should be noted that Equation 4 assumes constant values for ϕ, c_w, A, and h. It gives the cumulative water influx at the interior boundary or initial oil-water contact. The method further assumes uniform thickness and

100% water saturation at the oil-water contact and all points in the aquifer.

The procedure was originally developed taking into consideration only the compressibility of the saturating fluid. Since Hall[1] has shown that rock expansion with reservoir-pressure decline can be significant compared with water expansion, the problem was solved by both neglecting and taking the changes in rock volume into consideration. It is seen that taking rock expansion into consideration results in 22% more water influx than when neglecting it. This is a result of the expansion of rock matrix into the pore volume displacing water.

The problem has been solved taking rock expansion into consideration by assuming that the effects of rock and water expansibilities can be taken into consideration with a hypothetical or effective compressibility factor having a value equal to the sum of the water and rock compressibility factors. This approximation represents a correction which should yield better estimations of water influx. It is perhaps as valid as the equations used since the latter are also approximations and involve many restricted assumptions, i.e., uniform rock properties, radial or linear reservoir geometry, etc.[5]

Although the effective compressibility factor is 85% larger than the water compressibility factor, the water influx obtained using the effective compressibility factor was only 22% larger than that obtained using the water-compressibility factor. As the compressibility increases, the actual time required to reach a certain reservoir pressure for a constant production rate increases. Or the dimensionless time obtained with Equation 5 for a certain actual time decreases.

This latter trend affects the summation term of Equation 4 as indicated by Tables 1 and 2. As fluid compressibility increases, the transmission to the aquifer of pressure disturbances occurring at the oil-water contact is retarded. This affects the amount of water influx that takes place and offsets to some degree the water displacement from the aquifer into the oil pool that takes place due to rock expansion.

References

1. Hall, H. N., "Compressibility of Reservoir Rocks": Trans. AIME, (1953), p. 309.
2. Chatas, A. T., "A Practical Treatment of Nonsteady-State Flow Problems in Reservoir Systems": Petroleum Engineer, Part 1, May 1953, p. B-42; Part 2, June 1953, p. B-38; Part 3, August 1953, p. B-44.
3. Van Everdingen, A. F., and Hurst, W., "The Application of the Laplace Transformation to Flow Problems in Reservoirs": Trans. AIME (1949), 186, p. 305.
4. Sokolnikoff, I. S., and Redheffer, R. M., "Mathematics of Physics and Modern Engineering": McGraw-Hill Book Co., Inc., 1958, p. 51.
5. Guerrero, E. T., Reservoir Engineering. Part 46, The Oil and Gas Journal, March 19, 1962, p. 182.

Part 49

How to predict future performance of a reservoir with partial edge-water drive

GIVEN: Production, rock, and fluid data for an initially undersaturated black-oil reservoir with a partial edge-water drive. These data were reported in Table 1. columns 1 through 10, Page 8.

FIND: Using the results from the problem starting on Page 7[1] compute average reservoir pressures for future successive time increments of 91, 182, 273, and 365 days if production rate is kept constant at 3,000 b/d of stock-tank oil. Estimate average producing GOR as 2,000 scf/st tk bbl and average producing WOR as 0.10 bbl of water/st tk bbl of oil.

SOLUTION: A recommended method[2 3 4] for predicting performance of an oil reservoir with edge-water drive is by the simultaneous trial-and-error solution of the material-balance and unsteady-state water-influx equations. The generalized material-balance equation when neglecting expansion of interstitial water, rock, and gas in solution with the water (these factors are negligible compared to expansion of evolved solution gas, water influx, etc.) is written as

$$W_e = N_p [B_t + B_g (R_p - R_{si})] + W_p - N \{B_t - B_{oi} + m_i B_{oi} [(B_g/B_{gi}) - 1]\} \quad (1)$$

Since an initially undersaturated reservoir is involved in this problem, $m_i = 0$. Thus, writing the material-balance equation for application at and below the bubble-point pressure (bubble-point pressure represents original conditions in this application) gives

$$W_e = N_p [B_t + B_g (R_p - R_{sb})] + W_p - N [B_t - B_{ob}] \quad (2)$$

Cumulative water influx can also be computed with the unsteady-state equation of Van Everdingen and Hurst[2] which is

$$W_e = B \sum_{j=1}^{j=n} [(p_{j-1} - p_j) \times Q_{T_n - T(j-1)}] \quad (3)$$

Where:

W_e = cumulative water influx, bbl

N_p = cumulative oil production, st tk bbl

B_t = two-phase formation-volume factor

B_g = gas formation-volume factor

R_p = cumulative gas-oil ratio, scf/st tk bbl

$\bar{R}_{sb}$ = initial or bubble-point solution gas-oil ratio, scf/st tk bbl

W_p = cumulative water production, bbl

N = initial oil in place at bubble-point conditions, st tk bbl

B_{ob} = initial or bubble-point oil formation-volume factor

$B = 15{,}516\ \phi\ c_e A h$ = proportionality factor to convert reduced units into bbl [2,4] (assumed constant)

ϕ = porosity, fraction (assumed constant)

c_e = effective fluid compressibility in aquifer vol/vol/psi

A = area of oil reservoir, acres

h = net sand thickness in aquifier, ft

p = average reservoir pressure, psia (in Equation 3, p represents the pressure at the oil-water contact)

j = computation step points

n = time point at which cumulative water influx is desired

Q_T = dimensionless total volume of slightly compressible fluid passing a unit thickness of the interior boundary (oil-water contact) of a reservoir system over a period of time t and caused by a unit pressure drop at this boundary.

SOLUTION: Cumulative production data are known for 1,460 days. (Table 1, Page 8[1]). In predicting the performance 91 days into the future, the total actual time is 1,551 days (1,460 + 91). The dimensionless time is 240 + 15 = 255. Let the estimated average reservoir pressure be 3,218 psia. From the estimated future oil-production rates, gas-oil ratios, and water-oil ratios and the cumulative oil, gas, and water-production figures given in Reference 1 for a time of 1,460 days, cumulative recoveries to 1,551 days, and future time points, can be determined. Thus at 1,551 days

$$N_p = 3.004 \times 10^6 + (3{,}000)(91) = 3.277 \times 10^6 \text{ st tk bbl}$$

$$G_p = 3{,}052.1 \times 10^6 + (2{,}000)(3{,}000)(91) = 3{,}598.1 \times 10^6 \text{ scf}$$

Where[1] $3{,}052.1 \times 10^6 = N_p R_p = (3{,}004{,}064)(1{,}016)$ at 1,460 days

$$W_p = 0.380 \times 10^6 + (3{,}000)(91)(0.10) = 0.407 \times 10^6 \text{ bbl}$$

$$R_p = \frac{G_p}{N_p} = \frac{3{,}598.1 \times 10^6}{3.277 \times 10^6}$$

$$= 1{,}098 \text{ scf/st tk bbl.}$$

The two-phase oil formation volume factors, B_t, and the gas formation volume factors, B_g, were obtained from Table 1 (Part 1) of Reference 4. The average reservoir pressure of 3,178 psia shown in this table is in error and was changed (by interpolation) to read 3,138 psia. Substitution into Equation 2 gives

$$W_e = 3.277 \times 10^6 \times [1.6113 + 0.0009623\,(1{,}089\text{-}900)] + 0.407 \times 10^6 - 24{,}567{,}500\,[1.6113 - 1.5385]$$

$$W_e = 4{,}523{,}000 \text{ bbl}$$

Procedure for solving Equation 3 was given in the problem starting on Page 7.[1] To obtain cumulative water influx from bubble-point pressure, this equation is solved for total time, and cumulative water influx to the bubble point is then subtracted. Data in colmuns 17 and 19, Table 1 Page 8[1] are used along with the data of columns 18 and 20, Table 1 of this problem. In the earlier problem, pressure drops, $(p_{j-1} - p_j)$, for each time increment were determined for the oil-water contact, while in the predictions of the present problem it was assumed that the pressure drops at the oil-water contact are equivalent to the average pressure drops of the reservoir. Modifying Equation 3 to conform to this actual condition of the problem gives

$$W_e = B \sum_{j=1}^{j=n} (p_{j-1} - p_j)\, Q_{Tn-T(j-1)} - B \sum_{j=1}^{j=4} (p_{j-1} - p_j)\, Q_{T4-T(j-1)} \quad (4)$$

In this equation $B \sum_{j=1}^{j=n} (p_{j-1} - p_j) \times Q_{Tn-T(j-1)}$ gives total water influx from original reservoir conditions while $B \sum_{j=1}^{j=4} (p_{j-1} - p_j)\, Q_{T4-T(j-1)}$ gives cumulative water influx from original reservoir condition to the bubble point. There are four time increments from original reservoir conditions to the bubble point and consequently the limit of $j = 4$.

For 1,551 days

$$W_e = 184\,[(2.5)(92.589) + (9.5)(88.062) + (20.0)(83.497) + (32.5)(78.886) + (34.0)(74.226) + (33.0)(69.512) + (42.5)(64.737) + (48.0)(59.895) + (38.5)(54.976) + (31.0)(49.968) + (40.5)(44.858) + (34.5)(39.626) + (29.0)(34.247) + (29.0)(28.691) + (20.5)(22.897) + (14.5)(16.742) + (37.5)(9.949)] - 184\,[(2.5)(28.691) + (9.5)(22.897) + (20.0)(16.742) + (32.5)(9.949)]$$

$$W_e = 184\,[25{,}505.526] - 184\,[947.432]$$

$$W_e = 4{,}519{,}000 \text{ bbl}$$

This value of cumulative water influx is nearly the same as that obtained with the material-balance equation and thus the estimated average reservoir pressure of 3,218 psia is correct.

DISCUSSION: The prediction of future performance for a water-drive reservoir is more difficult than that for solution-gas, or gas-cap drives. Instead of solving two equations for instantaneous gas-oil ratio or incremental oil recovery, two equations are solved for cumulative water influx. Also, time is now directly involved in the calculations.

In this problem cumulative water influx was computed for estimated constant values of oil-production rate, producing gas-oil ratio, and producing water-oil ratio. Similar calculations could be performed for other combinations of these estimated factors. The oil-production rate is normally set by practice or conservation regulations and can be assumed constant for all calculations. Of the remaining factors the water-oil producing ratios can best be predicted from past performance history. Thus, for fixed estimated values of oil-production rate and producing water-oil ratios, groups of cumulative water-influx calculations can be made for various constant values of producing gas-oil ratios. In this way limiting values for the reservoir pressure can be obtained.

Although more tedious than the procedure used, another method eliminates the necessity of estimating the producing gas-oil ratio. For the estimated pressure Equation 3 (or 4) is solved for W_e. Then the oil saturation is computed in the oil-producing area with the equation

$$S_o = \frac{\left[N - N_p - (S_{or}/B_o)\dfrac{(W_e - W_p)}{(1 - S_{wi} - S_{or} - S_{gr})}\right] B_o}{\dfrac{NB_{ob}}{1 - S_{wi}} - \dfrac{W_e - W_p}{1 - S_{wi} - S_{or} - S_{gr}}} \quad (5)$$

Where:

S_o = oil saturation, fraction
S_{or}, S_{gr} = residual oil and gas saturations, fractions
B_o = oil formation-volume factor
S_{wi} = interstitial-water saturation in uninvaded portion of oil column

With this value of S_o and a representative k_g/k_o plot the instantaneous gas-oil-ratio equation can be solved for the producing (instantaneous) gas-oil ratio. This gas-oil ratio is averaged with the gas-oil ratio at the beginning of the pressure decrement and the result used to determine R_p. The material-balance equation can then be solved for W_e. If the cumulative water influx obtained with the material-balance equation does not equal the water-influx value calculated by the unsteady-state-flow equation, a new pressure is selected and the procedure is repeated.

Table 1 shows the data and calculation of performance for the problem. Three to four estimates of reservoir pressure normally need to be made at each time increment to obtain the proper values. A study of Equations 2 and 3 shows that an increase in reservoir pressure decreases the cumulative water-influx calculations by the unsteady-state-flow equation and increases those given by the material-balance equation. This behavior can be used to obtain a reasonable value for the estimated pressure from a plot of pressure versus cumulative water influx computed by the two equations. The correct pressure is that where the two plots intersect. Such trial-and-error calculations are readily adaptable for solution by digital computers.

Columns 3-17 of Table 1 are a solution of Equation 2, while columns 18-23 are a solution of Equation 4. The correct estimated reservoir pressure gives equivalent values of cumulative water influx in columns 17 and 23. In the problem solved no initial free gas cap

Table 1—Calculation of performance for water-drive reservoir

Oil production rate = 3,000 st tk b/d
Producing GOR = 2,000 scf/st tk bbl, Producing WOR = 0.10 bbl water/st tk bbl oil

(1) Actual time, days	(2) $T_n = T_{n-1} + \Delta T = T_{n-1} + 15$	(3) Estimated avg. reservoir pressure, psia	(4) N_p 10^6 bbl	(5) G_p MM scf	(6) R_p scf/st tk bbl	(7) W_p 10^6 bbl	(8) B_t
1,460	240	3,277	3.004	3,052.1	1,016	0.380	1.6005
1,551	255	3,218	3.277	3,598.1	1,098	0.407	1.6113
1,642	270	3,168	3.550	4,144.1	1,167	0.435	1.6209
1,733	285	3,126	3.823	4,690.1	1,227	0.462	1.6292
1,825	300	3,090	4.099	5,242.1	1,279	0.490	1.6366

(9) $B_g \times 10^3$ Res. bbl/scf	(10) $R_p - R_{sb}$ = (6)−900	(11) $(R_p - R_{sb}) B_g$ = (10)×(9)	(12) $B_t + B_g(R_p - R_{sb})$ = (8)+(11)	(13) $N_p \times (12)$ = (4)×(12) 10^6 bbl	(14) $(13) + W_p$ = (13)+(7)	(15) $B_t - B_{ob}$ = (8)−1.5385	(16) $N(B_t - B_{ob})$ 10^6 bbl
0.9469	116	.....	.....	....	....	.....	....
0.9624	198	0.1906	1.8019	5.905	6.312	0.0728	1.789
0.9760	267	0.2606	1.8815	6.679	7.114	0.0824	2.024
0.9879	327	0.3230	1.9522	7.463	7.925	0.0907	2.228
0.9983	379	0.3784	2.0150	8.259	8.749	0.0981	2.410

(17) W_e = (14)−(16) 10^6 bbl	(18) Q_{Tn} Table 2 Ref. 3	(19) $\Delta p_n = (p_{n-1} - p_n) = (3)_{n-1} - (3)_n$ psi	(20) Avg. $\Delta p = [\Delta p_{n-1} + \Delta p_n] \div 2$ $[(19)_{n-1} + (19)_n] \div 2$	(21) $\sum_{j=1}^{j=n} \Delta p_j Q_{T n - T(j-1)}$	(22) $W_e = B \times (21) = 184 \times (21)$ 10^6 bbl	(23) W_e from bubble point[2] (22)−0.174 10^6 bbl
...	88.062	16	....	....	....	...
4.523	92.589	59	37.5	25,505.526	4.693	4.519
5.090	97.081	50	54.5	28,604.462	5.263	5.089
5.697	101.540	42	46.0	31,919.625	5.873	5.699
6.339	105.968	36	39.0	35,391.614	6.512	6.338

From Page 9, N = 24,567,500 bbl, ΔT = 15.0, B = 184.

$0.174 \times 10^6 = 184 \times 947.432$

was involved. The same procedure can be used for a reservoir initially containing a free gas cap. The size of the gas cap must be determined independently.

The application of Equation 4 to define water-drive reservoir performance is restricted to reservoirs that approximate ideal linear or radial geometry. The Q_T terms of Equation 3 have been computed for infinite-boundary conditions and for finite-boundary conditions with constant pressure or with no flow across the interior boundary.[2,3] Geometry and boundary-condition restrictions can be eliminated by following calculation procedures suggested by Hutchinson and Sikora.[5] They developed an equation which relates pressure at the water-oil contact to the water-influx rate as a function of time by a factor called the resistance function. This resistance function covers the composite effect of the aquifer geometry and flow-resistance distribution.[5]

In solving the problem by the method of Van Everdingen and Hurst,[1,2,4] it was necessary to use the Superposition Theorem. Such calculations are tedious when the work is done without using a digital computer. Carter and Tracy[6] have developed a method for computing water influx based on the work of Hurst[10] that eliminates the superposition calculations.

Other analytical methods of predicting water-flood performance have been suggested by Schilthuis[7] and Hurst.[7,8] Stewart et al.[9] made a comparison of results obtained with these and other methods and the method used in this problem. These authors found that the Van Everdingen-Hurst method used in this problem gave the best results along with an electric-analyzer approach.

References

1. Guerrero, E. T., "How to Find Original Oil in Place by Material Balance for Reservoir with Partial Water Drive": The Oil and Gas Journal, May 15, 1961, pp. 164-166, 168.

2. Van Everdingen, A. F., and Hurst, W., "The Application of the LaPlace Transformation to Flow Problems in Reservoirs": Trans. AIME (1949), 186, pp. 305-324 B.

3. Chatas, A. T., "A Practical Treatment of Nonsteady-State Flow Problems in Reservoir Systems": Petroleum Engineer, Pt. 1, May 1953, p. B-42; Pt. 2, June 1953, p. B-38; Pt. 3, August 1953, p. B-44.

4. Van Everdingen, A. F., Timmerman, E. H., and McMahon, J. J., "Application of the Material-Balance Equation to a Partial-Water-Drive Reservoir": Trans. AIME (1953), 198, 51.

5. Hutchinson, T. S., and Sikora, V. J., "A Generalized Water-Drive Analysis": Trans. AIME (1959), 216, 169.

6. Carter, R. D., and Tracy, G. W., "An Improved Method for Calculating Water Influx": Journal of Petroleum Technology (1960), 12, 12, 58-60.

7. Pirson, S. J., "Oil Reservoir Engineering": McGraw-Hill Book Co., Second Edition, 1958.

8. Hurst, W., "Water Influx Into a Reservoir and Its Application to the Equation of Volumetric Balance": Trans. AIME (1943), 151, 57.

9. Stewart, F. M., Callaway, F. H., and Gladfelter, R. E., "Comparison of Methods for Analyzing a Water Drive Field, Torchlight Tensleep Reservoir, Wyoming": Trans. AIME (1954) 201, 197.

10. Hurst, W., "The Simplification of the Material Balance Formulae by the LaPlace Transformation": SPE of AIME Petr. Conf. on Production and Reservoir Eng., Tulsa, Mar. 20-21, 1958, Paper 1030-G, 16 pp.

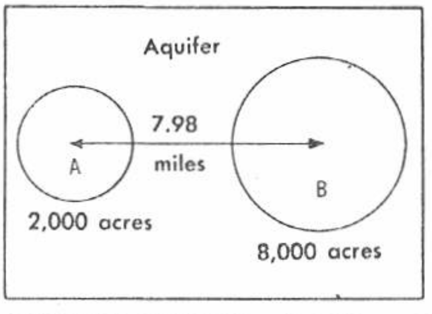
Part 50

How to predict future pressure performance of two reservoirs located in a common aquifer

GIVEN: A reservoir system comprised of two fields, A and B, located in an extensive (assumed infinite) aquifer. The distance between the centers of the fields is 7.98 miles. Other data (average) for the pools and the aquifer are as follows:

	Field A	Field B	Aquifer
A = area, acres	2,000	8,000	....
p_i = initial reservoir pressure, psi	4,000	4,000	....
p = current reservoir pressure, psi	3,220	3,220	....
k_w = permeability, md	....	....	212
ϕ = porosity, fraction	....	....	0.22
h = net sand thickness, ft	....	....	84
c_e = effective compressibility, vol/pore vol/psi	....	....	5×10^{-6}
μ_w = viscosity of water at average aquifer conditions, cp	....	....	0.30
B_o = average oil-formation volume factor	1.15	1.15	....
t = production time to date, years	4	2	....

Oil-production-rate histories for the two reservoirs are shown in columns 1 and 2 of Tables 1 and 2 and in Figs. 2 and 3. Field A has been producing 4 years while Field B has been producing 2 years. Production rate of Field A has remained constant at 75,000 b/d since the beginning of the third year.



SCHEMATIC DIAGRAM of fields and aquifer system. Fig. 1.

FIND: Pressure in both fields 4 years hence if Field A is produced at 75,000 st tk bbl/d and Field B is produced at 120,000 st tk bbl/d.

Method of solution. This problem can be solved using the unsteady-state equations developed by Van Everdingen and Hurst.[1,2,3] Fig. 1 shows a schematic diagram of the fields and aquifer system. For such conditions:

$$(p_i - p)_A = (p_i - p)_{AA} + (p_i - p)_{AB} \quad (1)$$

$$(p_i - p)_{AA} = \frac{887.4\,\mu_w B_o}{2\pi k_w h} \sum_{j=1}^{j=n} [Q_{oj} - Q_{o(j-1)}]_A \, [P_{Tn-T(j-1)}]_{AA} \quad (2)$$

$$(p_i - p)_{AB} = \frac{887.4\,\mu_w B_o}{2\pi k_w h} \sum_{j=1}^{j=n} [Q_{oj} - Q_{o(j-1)}]_B \, [P_{Tn-T(j-1)}]_{AB} \quad (3)$$

Therefore:

$$(p_i - p)_A = \frac{887.4\,\mu_w B_o}{2\pi k_w h} \{ \sum_{j=1}^{j=n} [Q_{oj} - Q_{o(j-1)}]_A \, [P_{Tn-T(j-1)}]_{AA} + \sum_{j=1}^{j=n} [Q_{oj} - Q_{o(j-1)}]_B \, [P_{Tn-T(j-1)}]_{AB} \} \quad (4)$$

Similarly

$$(p_i - p)_B = \frac{887.4\,\mu_w B_o}{2\pi k_w h} \{ \sum_{j=1}^{j=n} [Q_{oj} - Q_{o(j-1)}]_B \, [P_{Tn-T(j-1)}]_{BB} + \sum_{j=1}^{j=n} [Q_{oj} - Q_{o(j-1)}]_A \, [P_{Tn-T(j-1)}]_{BA} \} \quad (5)$$

Where:

$(p_i - p)_A$ = total pressure drop at A from initial conditions due to production at A and B, psi

$(p_i - p)_{AA}$ = pressure drop at A from initial conditions due to production at A, psi

$(p_i - p)_{AB}$ = pressure drop at A from initial conditions due to production at B, psi

$(p_i - p)_B$ = total pressure drop at B from initial conditions due to production at B and A, psi

p_i = initial reservoir pressure, psi

p = pressure, psi

Q_o = oil-production rate, b/d

P_T = dimensionless pressure change at dimensionless time T and radius r

j = computation step points

n = time point at which cumulative pressure drop $(p_i - p)$ is desired

Symbols A, p_i, p, k_w, ϕ, h, c_e, μ_w, B_o, and t are defined with the data.

SOLUTION: The solution to this problem is shown in Tables 1 and 2. To solve Equations 4 and 5 actual time must be converted to dimensionless time and actual radii to dimensionless radii. Dimensionless time is related to actual time by the relationship of

$$T = \frac{6.33 \times 10^{-3} k_w t}{\phi \mu_w c_e r_w^2} \quad (6)$$

For Field A

$$r_{wA} = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{(2{,}000 \text{ acres})(43{,}560 \text{ sq ft/acre})}{3.14}} = 5{,}265 \text{ ft}$$

$$T = \frac{(6.33 \times 10^{-3})(212)t}{(0.22)(0.30)(5 \times 10^{-6})(5{,}265)^2} = \frac{1{,}342\,t}{(0.33)(27{,}720)} = 0.1467\,t$$

For Field B

$$r_{wB} = \sqrt{\frac{(8{,}000 \text{ acres})(43{,}560 \text{ sq ft/acre})}{3.14}} = 10{,}535 \text{ ft}$$

$$T = \frac{(6.33 \times 10^{-3})(212)t}{(0.22)(0.30)(5 \times 10^{-6})(10{,}535)^2} = \frac{1{,}342\,t}{(0.33)(110{,}986)} = 0.0366\,t$$

$$\frac{887.4\,\mu_w B_o}{2\pi k_w h} = \frac{(887.4)(0.30)(1.15)}{(2)(3.14)(212)(84)} = \frac{306.15}{111{,}834.24} = 0.002738$$

The dimensionless radii are

$$R_A = \frac{r_{wA}}{r_{wA}} = 1.0$$

$$R_{AB} = \frac{r_{AB}}{r_{wA}} = \frac{(7.98)(5{,}280)}{5{,}265} = \frac{42{,}134}{5{,}265} = 8.0$$

$$R_B = \frac{r_{wB}}{r_{wB}} = 1.0$$

$$R_{BA} = \frac{r_{AB}}{r_{wB}} = \frac{(7.98)(5{,}280)}{10{,}535} = \frac{42{,}134}{10{,}535} = 4.0$$

Where:

r_{wA}, r_{wB} = radii of pools A and B respectively

r_{AB} = distance between centers of pools.

R_A, R_B = dimensionless radii of pools A and B.

R_{AB} = dimensionless distance between centers of oil pools in multiples of r_{wA}.

R_{BA} = dimensionless distance between centers of oil pools in multiples of r_{wB}.

Values for $P_{Tn-T(j-1)}$ in Equations 4 and 5 can be obtained[1,2,4] using dimensionless time and radius.

The solution of this problem involves the use of the superposition theorem.[3,5,6] Column 8 of Table 1 is obtained as follows:

$$\sum_{j=1}^{j=9} [Q_{oj} - Q_{o\,(j-1)}]_A \, [P_{T_n - T\,(j-1)}]_{AA} = (14{,}000)\,(3.44) + (10{,}000)\,(3.42)$$

$$+ (5{,}500)\,(3.41) + (10{,}500)\,(3.39) + (12{,}000)\,(3.37) + (-500)\,(3.36)$$

$$+ (15{,}500)\,(3.34) + (4{,}000)\,(3.32) + (4{,}000)\,(3.30) = 253{,}720.$$

Thus, substituting in Equations 4 and 5 gives

$$(p_i - p)_A = 4{,}000 - p_A = (0.002738)\,[253{,}720 + 141{,}370]$$

$$4{,}000 - p_A = (0.002738)\,(395{,}090) = 1{,}082$$

$$p_A = 2{,}918 \text{ psi}$$

$$(p_i - p)_B = 4{,}000 - p_B = (0.002738)\,[305{,}510 + 98{,}905]$$

$$4{,}000 - p_B = (0.002738)\,(404{,}415) = 1{,}107$$

$$p_B = 2{,}893 \text{ psi}$$

DISCUSSION: This problem shows a method for computing the interference of two oil fields located in a common aquifer. The computation procedure was described in the problem starting on Page 41.

Water-drive oil fields sharing a common aquifer are in hydrodynamic communication. Fluid production from one of the oil fields results in a pressure loss transmitted through the aquifer to the other pool or pools and is manifested as pressure interference. This behavior can result in pressure reduction many miles away from a producing pool.

Production from several Woodbine-sand pools caused the initial reservoir pressure in Hawkins pool to be 280 psi below the initial value in the aquifer.[4] This indicates that the pressure-interference effect may be significant and should be considered in predicting the pressure-production behavior of pools in a common aquifer.

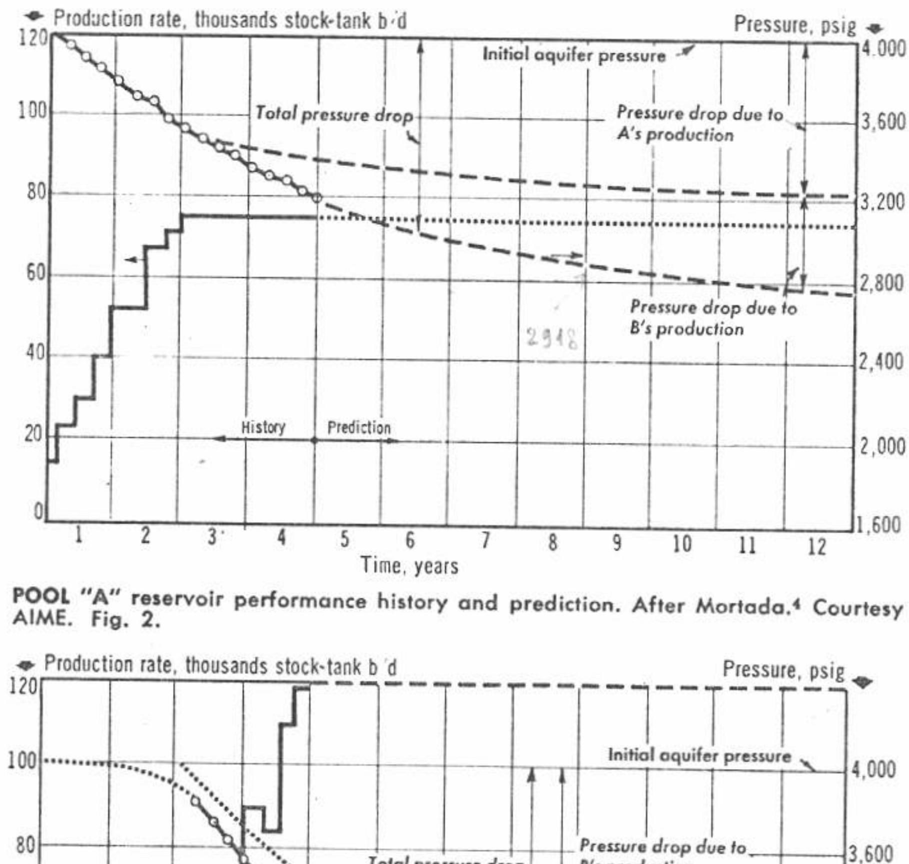
Analytical procedure used in this problem assumes (1) circular geometry of the pools, (2) uniform reservoir rock and fluid properties, (3) effective compressibilities of oil pools and aquifer are equivalent, and (4) infinite aquifer conditions. In attempts to account for the non-uniform behavior of aquifer properties, Hurst[7] and Mortada[8] have presented different equations for treating pressure interference in an infinite aquifer consisting of two permeable zones in series. Neither of their equations was used in this problem. The complications which result from use of the more extensive equations are not considered warranted for most reservoir-engineering calculations. Use and further development of such methods may prove them more valid than the method used in this problem.

The solution of Equation 6 and the factor $\dfrac{887.4\,\mu_w B_o}{2\pi k_w h} = C$ (assumed constant) involve uncertainties, particularly with regard to k_w, c_e, and r_w. If reliable past pressure and production-rate histories are available, it is possible to determine values for C and $(n\,\Delta T = T)$

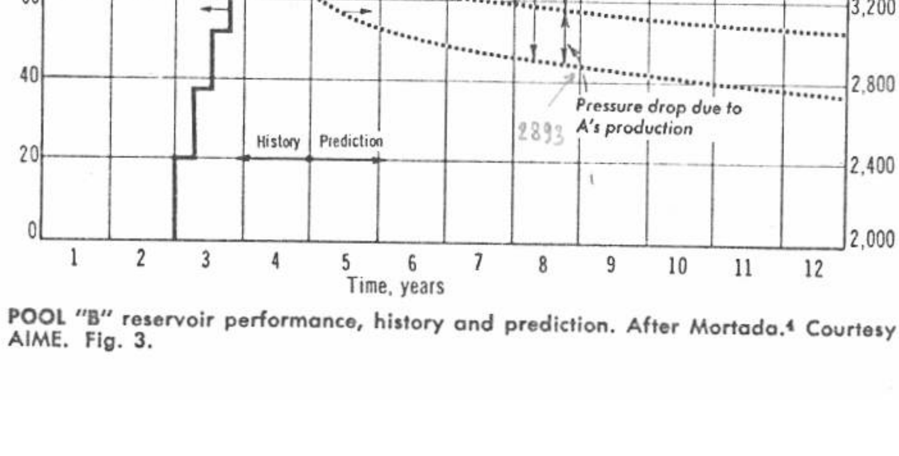
$$\Delta T = \frac{6.33 \times 10^{-3}\, k_w\, \Delta t}{\phi\, \mu_w\, c_e\, r_w^2}$$

using a procedure similar to that used in the problem starting on Page 7. Either of Equations 4 and 5 can be used. These equations become

$$(p_i - p)_A = C\,X \qquad (7)$$



POOL "A" reservoir performance history and prediction. After Mortada.[4] Courtesy AIME. Fig. 2.



POOL "B" reservoir performance, history and prediction. After Mortada.[4] Courtesy AIME. Fig. 3.

Table 1—Calculation of dimensionless pressure changes in Fields A and B due to production from Field A

(1) Time, t days	(2) Q_{oj} b/d	(3) $Q_{oj}-Q_{o(j-1)}$ $(2)_n-(2)_{n-1}$ b/d	(4) Dimensionless time $T_j=$ 0.1467 t	(5) T_n-T_{j-1}	(6) $P_{Tn-T(j-1)}$ $R_A=1.0$ (*)	(7) $P_{Tn-T(j-1)}$ $R_{AB}=8.0$ (†)	(8) $[Q_{oj}-Q_{o(j-1)}]_A \times[P_{Tn-T(j-1)}]_{AA}$ (3)×(6)	(9) $[Q_{oj}-Q_{o(j-1)}]_A \times[P_{Tn-T(j-1)}]_{BA}$ (3)×(7)
0	14,000	14,000	....	428.7	3.44	1.36	48,160	19,040
90	24,000	10,000	13.2	415.5	3.42	1.35	34,200	13,500
181	29,500	5,500	26.6	402.1	3.41	1.34	18,755	7,370
273	40,000	10,500	40.0	388.7	3.39	1.33	35,595	13,965
365	52,000	12,000	53.5	375.2	3.37	1.32	40,440	15,840
455	51,500	−500	66.7	362.0	3.36	1.30	−1,680	−650
546	67,000	15,500	80.1	348.6	3.34	1.28	51,770	19,840
638	71,000	4,000	93.6	335.1	3.32	1.26	13,280	5,040
730	75,000	4,000	107.1	321.6	3.30	1.24	13,200	4,960
2,922			428.7					
							Σ=253,720	Σ=98,905

*From Table 1, Ref. 2. †From Fig. 3, Ref. 4.

Table 2—Calculation of dimensionless pressure changes in Fields B and A due to production from Field B

(1) Time, t days	(2) Q_{oj} b/d	(3) $Q_{oj}-Q_{o(j-1)}$ $(2)_n-(2)_{n-1}$ b/d	(4) Dimensionless time $T_j=$ 0.0366 t	(5) T_n-T_{j-1}	(6) $P_{Tn-T(j-1)}$ $R_B=1.0$ (*)	(7) $P_{Tn-T(j-1)}$ $R_{BA}=4$ (†)	(8) $[Q_{oj}-Q_{o(j-1)}]_B \times[P_{Tn-T(j-1)}]_{BB}$	(9) $[Q_{oj}-Q_{o(j-1)}]_B \times[P_{Tn-T(j-1)}]_{AB}$
0	20,000	20,000	....	80.2	2.62	1.24	52,400	24,800
90	38,000	18,000	3.3	76.9	2.59	1.22	46,620	21,960
181	52,000	14,000	6.6	73.6	2.57	1.21	35,980	16,940
273	71,500	19,500	10.0	70.2	2.55	1.19	49,725	23,205
365	90,000	18,500	13.4	66.8	2.53	1.17	46,805	21,645
456	84,000	−6,000	16.7	63.5	2.50	1.14	−15,000	−6,840
547	111,000	27,000	20.0	60.2	2.48	1.11	66,960	29,970
639	119,000	8,000	23.4	56.8	2.45	1.08	19,600	8,640
731	120,000	1,000	26.8	53.4	2.42	1.05	2,420	1,050
2,192			80.2					
							Σ=305,510	Σ=141,370

*From Table 1, Ref. 1. †From Fig. 3, Ref. 4.

$$(p_i - p)_B = C\,Y \qquad (8)$$

where X and Y represent the portions in parenthesis of Equations 4 and 5. These relations represent equations of straight lines having the same slope C. Thus trial-and-error calculations would be involved to select the ΔT that would result in straight-line correlations. It should be noticed that the ΔT's for the two pools are related by a constant

$$\Delta T_A = \Delta T_B \frac{(r_{wB})^2}{(r_{wA})^2} \qquad (9)$$

Even if such a procedure is not followed, usable results can be obtained if reasonable estimates can be made of k_w, h, c_e, and r_w since μ_w, B_o and ϕ are normally available.

In applying the procedures described for this problem, it must be remembered that infinite aquifer conditions are assumed. The presence of nearby faults would cause erroneous results. Also, erroneous results would be obtained if one or more other pools were located (in the same aquifer) nearby and were not taken into consideration. The effects of interference can often be significant enough to warrant taking them into consideration in spite of the limitations of the methods discussed. Figs. 2 and 3 show the performances of Pools A and B as computed by Mortada.[4] These graphs show the pressure drop in each of the pools caused by interference. Over a 10-year period the production of Pool B will cause a pressure drop of more than 400 psi in Pool A while the production of Pool A will cause a pressure drop of about 300 psi in Pool B.

References

1. Van Everdingen, A. F., and Hurst, W., "The Application of the LaPlace Transformation to Flow Problems in Reservoirs:" Trans. AIME Vol. 186, 1949, pp. 305-324.

2. Chatas, A. T., "A Practical Treatment of Nonsteady-State Flow Problems in Reservoir Systems:" Petroleum Engineer, Pt. 1, May 1953, p. B-42; Pt. 2, June 1953, p. B-38; Pt. 3, August 1953, p. B-44.

3. Guerrero, E. T.: Reservoir Engineering, Pt. 47, The Oil and Gas Journal, Apr. 23, 1962, p. 112.

4. Mortada, M., "A Practical Method for Treating Oil-Field Interference in Water-Drive Reservoirs": Trans. AIME Vol. 204, 1955, pp. 217-226.

5. Churchill, R. V.,, "Fourier Series and Boundary Value Problems:" McGraw-Hill Book Co., Inc., First Edition, 1941, pp. 3 and 99.

6. Sokolnikoff, I. S. and Redheffer, R. M., "Mathematics of Physics and Modern Engineering:" McGraw-Hill Book Co., Inc., 1958, p. 51.

7. Hurst, W., "Interference Between Oil Fields:" Trans. AIME Vol. 219, 1960, pp. 175-192.

8. Mortada, M., "Oil-field Interference in Aquifers of Nonuniform Properties": Trans. AIME Vol. 219, 1960, pp. 412-413.

9. Guerrero, E. T., "How to Find Original Oil in Place by Material Balance for Reservoir with Partial Water Drive:" The Oil and Gas Journal, Vol. 59, No. 20, May 15, 1961, pp. 164-166, 168.

Part 51

How to estimate oil recovery from a gravity-drainage pool

When the pressure is maintained constant at gas-displacement front

GIVEN: Data on the Elk Basin field gas pressure-maintenance project in the Embar-Tensleep reservoir as reported by Stewart et al., as follows:[1]

Average porosity, ϕ = 10.7%.

Average absolute permeability, k, = 91 md.

Interstitial-water saturation, S_w = 8.0%.

Initial reservoir pressure (−400 ft), p_i, = 2,234 psia.

Bubble-point pressure, p_b, = 1,250 psi at crest = 500 psi at lowest elevation.

Reservoir pressure at start of pressure maintenance = 1,200 psi.

Average reservoir fluid properties at start of pressure maintenance:

Viscosity of oil, μ_o, = 2.24 cp.

Oil gradient, $g\rho_o$, = 0.351 psi/ft.

Gas gradient, $g\rho_g$ = 0.026 psi/ft.

Oil formation-volume factor, B_o = 1.161.

Gas viscosity, μ_g = 0.0177 cp.

Average formation dip, α = 30° positive for downdip flow of gas and negative for updip flow of gas.

Average effective oil permeability, k_o, and average permeability ratios, k_g/k_o, are shown in columns 2 and 3 of Table 1.

Maximum oil-productive closure = 2,330 ft.

Average flow-path area, A = 5.78×10^6 sq ft.

FIND: Estimated ultimate oil recovery if oil-production rate is kept constant at 19,000 st tk b/d and reservoir pressure is maintained constant at the gas-oil contact and in the secondary gas cap.

METHOD OF SOLUTION: This problem can be solved using an equation originally presented by Leverett and Buckley.[2,3]

$$f_g = \frac{1.000 - 0.001128 \dfrac{k_o A}{Q_t \mu_o}\left(\dfrac{\partial P_c}{\partial L} - g\Delta\rho \sin\alpha\right)}{1.000 + \dfrac{k_o}{k_g}\dfrac{\mu_g}{\mu_o}} \qquad (1)$$

Where:

f_g = fraction of gas flowing at any point, i.e., (gas flow rate) ÷ gas flow rate + oil flow rate.

K_o, A, μ_o, g, α, k_g, and μ_g are defined with the data.

Q_t = total fluid production rate in reservoir b/d. In this problem no free gas or water is produced so that total fluid production rate is oil-production rate.

P_c = capillary pressure, psi.

L = length in the direction of flow, ft.

$g\Delta\rho = (\rho_o - \rho_g)\,g$, psi/ft.

g = gravitational constant.

ρ_o = density of oil at reservoir conditions.

ρ_g = density of gas at reservoir conditions.

Welge[4] has developed a simplified method for determining oil recovery that includes the use of Equation 1 with the capillary pressure term deleted. Thus

$$f_g = \frac{1.000 - 0.001128 \dfrac{k_o A}{Q_t \mu_o} g\Delta\rho \sin\alpha}{1.000 + \dfrac{k_o}{k_g}\dfrac{\mu_g}{\mu_o}} \qquad (2)$$

Table 1—Calculation of fractional flow equation

(1) Gas saturation, S_g %	(2) Effective oil permeability, k_o md	(3) Permeability ratio, k_g/k_o	(4) $0.001128 \frac{k_o A}{Q_t \mu_o} \times g\Delta\rho \sin\alpha = 0.02144\,k_o$	(5) 1.000−(4)	(6) $\frac{k_o}{k_g}\frac{\mu_g}{\mu_o}$ = 0.0079÷(3)	(7) $1.000 + \frac{k_o}{k_g}\frac{\mu_g}{\mu_o}$ = 1.000+(6)	(8) f_g (5)÷(7)
30.0	29.8	0.14	0.639	0.361	0.056	1.056	0.342
40.0	15.5	0.42	0.332	0.668	0.019	1.019	0.656
50.0	6.7	1.70	0.144	0.856	0.005	1.005	0.852
60.0	1.9	12.99	0.041	0.959	0.001	1.001	0.958
70.0	0.1	33.33	0.002	0.998	0.000	1.000	0.998
80.0	0.0	∞	0.000	1.000	0.000	1.000	1.000

SOLUTION: The solutions of Equation 2 for various gas saturations are given in Table 1. For a gas saturation of 40.0%.

$$Q_t = (19{,}000)\,(1.161) = 22{,}059 \text{ b/d}$$

$$g\Delta\rho = (\rho_o - \rho_g)\,g = 0.351 - 0.026 = 0.325 \text{ psi/ft}$$

and

$$f_g = \frac{1.000 - \dfrac{(0.001128)\,(15.5)\,(5.78 \times 10^6)\,(0.325)\,(\sin 30°)}{(19{,}000)\,(1.161)\,(2.24)}}{1.000 + \left(\dfrac{1}{0.42}\right)\left(\dfrac{0.0177}{2.24}\right)} = \frac{1.000 - \dfrac{(0.01748)\,(939{,}250)}{49{,}412}}{1.000 + (2.381)\,(0.0079)} = 0.656$$

Fig. 1 shows a plot of fractional flow of gas (column 8, Table 1), versus gas saturation (column 1, Table 1).

It has been proven[4,5] that this plot can be used to determine oil recovery by gravity drainage if a tangent is drawn to the curve from the point represented by the average displacing fluid saturation (zero for this problem) and the fractional flow of the displacing fluid (zero for this problem) existing at the start of injection and pressure-maintenance operations. The gas saturation at the displacement front or gas-oil contact is given at the point of tangency while the average gas saturation behind the displacement front or gas-oil contact is given at fg = 1.0. The latter gas saturation closely represents the average residual gas saturation at depletion.[1] Thus, from Fig. 1

$$S_{g\,av.} = 58.0\%$$

Since no gas saturation existed at start of pressure-maintenance operations

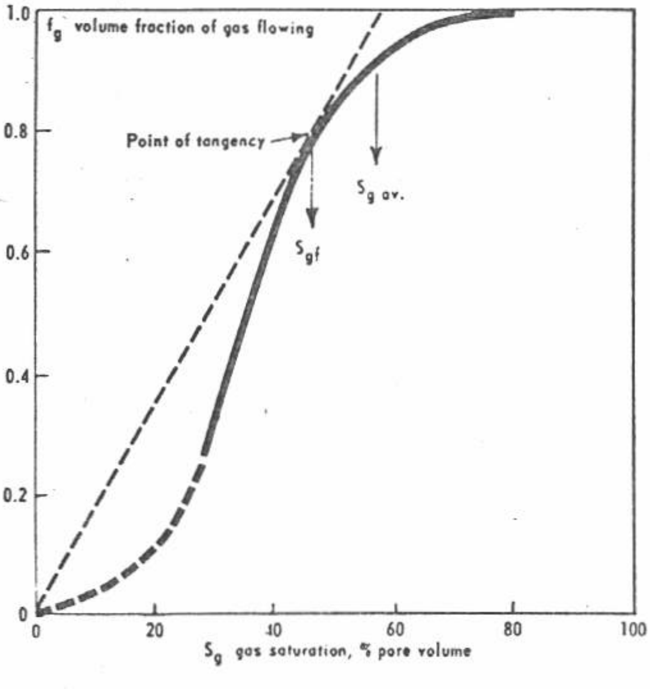
Oil recovery, (per cent initial oil in place)

$$= \left[\frac{\dfrac{1.00 - S_w}{B_{oi}} - \dfrac{1.00 - S_{g\,av.} - S_w}{B_o}}{\dfrac{1.00 - S_w}{B_{oi}}}\right] 100$$

Since $B_{oi} \approx B_o$

oil recovery, (per cent initial oil in place)

$$= 100\left[\frac{S_{g\,av.}}{1.00 - S_w}\right] = \frac{58}{0.92} = 63.0\%$$



GRAPHICAL method for finding average gas saturation in a depleted area. Fig. 1.

The product of 0.63 and the initial stock-tank oil in place gives the recovery in barrels.

Discussion. Because of the high oil-gas viscosity ratios and the gas-oil relative-permeability ratios at low gas saturations, oil recovery by gas displacement is generally much lower than oil recovery by water displacement.[6] This is not true if the gas displacement is accompanied by substantial gravitational segregation. Gravity drainage generally occurs under conditions of high permeability (20 md or more), low oil viscosity, and high structural dip (10° or more) or great closure. The gravitational segregation effects are normally less significant for water-drive reservoirs than for gas-drive reservoirs because of the much lower oil-water density differences.

In solving Equation 1, $\partial P_c/\partial L$ can be written as $(\partial P_c/\partial S_g) \times (\partial S_g/\partial L)$. For high-relief reservoirs and above the front (gas - oil contact), these terms are not significant and can be neglected. This is true because capillarity has an appreciable effect on recovery over only the stabilized zone which represents a very limited portion of the gas cap.[1,7] The gas-cap portion affected by capillarity is that immediately adjacent to the gas-oil contact. This explains the use of Equation 2 instead of Equation 1 in the solution.

In this problem oil recovery was obtained by determining the change in gas saturation that occurred in the oil column at gas breakthrough in the structurally lowest row of wells. Since pressure maintenance was assumed, the change in gas saturation represents change in oil saturation and oil recovery. Actually, slightly more recovery may occur after gas breakthrough. Determination of this additional recovery and the performance of a gravity-drainage reservoir will be considered in a subsequent problem.

The fractional flow equation used in the solution of this problem was originally derived by Leverett[2] and later used by Buckley and Leverett[3] to develop theory on the displacement of oil by water or gas. It is derived using the generalized Darcy equations for oil and gas and the equations defining capillary pressure and fractional flow.[5] Its application in the form of Equation 2 assumes that reservoir pressure changes so little that the gas can be considered incompressible, that all of the flow is parallel to the bedding planes, and that the influence of capillary forces is negligibly small.

A study of Fig. 1 shows that any behavior that causes the fractional flow curve to move to the right will increase oil recovery and vice versa. A change that reduces f_g for the same S_g causes the fractional flow curve to move to the right. In Equation 2 the gravity term, $g\Delta\rho \sin \alpha$, is positive for down dip flow of gas and thus subtracts from 1.000 to reduce f_g. This effect can be maximized by making Q_t (oil production rate) small (if economics permit).

Thus as Q_t is made smaller, oil recovery would increase. Also in this case high structural dip acts favorably to increase recovery. The gravity forces tend to maintain the gas above the oil. For gas flow updip the gravity term is negative and would add to 1.000 to increase f_g and decrease oil recovery. Under such condition Q_t (oil production rate) should be maximized to reduce the gravity effects. In this case the gravity forces are tending to cause the gas to bypass the oil and thus decrease the efficiency of displacement.

References

1. Stewart, F. M., Garthwaite, D. L., and Krebill, F. K., "Pressure Maintenance by Inert-Gas Injection in the High-Relief Elk Basin Field": Trans. AIME, Vol. 204, 1955, pp. 49-57.
2. Leverett, M. C., "Capillary Behavior in Porous Solids": Trans. AIME, Vol. 142, 1941, p. 152.
3. Buckley, S. E., and Leverett, M. C., "Mechanism of Fluid Displacement in Sands": Trans. AIME, Vol. 142, 1942, p. 107.
4. Welge, H. J., "A Simplified Method for Computing Oil Recovery by Gas or Water Drive": Trans. AIME, Vol. 195, 1952, p. 91.
5. Pirson, S. J., "Oil Reservoir Engineering": McGraw-Hill Book Co., second edition, 1958, Chap. 11.
6. Craft, B. C., and Hawkins, M. F., "Applied Petroleum Reservoir Engineering": Prentice-Hall, Inc., 1959, Chap. 7.
7. Terwilliger, P. L., Wilsey, L. E., Hall, Howard N., Bridges, P. M., and Morse, R. A., "An Experimental and Theoretical Investigation of Gravity Drainage Performance": Trans. AIME, Vol. 192, 1951, p. 285.

Part 52

How to find the performance of a gravity-drainage reservoir—

assuming average rock properties and pressure-maintenance operations at gas displacement front

GIVEN: Data reported by Stewart, et al.[1] on the Elk Basin field gas pressure-maintenance project in the Embar-Tensleep reservoir as follows:

f_g vs. S'_g relationship (Fig. 1)[2].

Average porosity, ϕ = 10.7%.

Average flow path area, A = 5.78 × 10⁶ sq ft

Interstitial water saturation, S_w = 8.0%.

Maximum oil-productive closure = 2,330 ft

Reservoir pressures, psia: Initial (−400 ft), p_i = 2,234; Bubble point, p_b = 1,250 at crest, = 500 at lowest elevation. At start of pressure maintenance = 1,200 at −400 ft.

Oil formation-volume factor, B_o = 1.161.

Oil-production rate (constant), Q_o = 19,000 stock tank b/d.

Average formation dip, α = 30°.

Maximum length in the direction of flow, L maximum = 4,400 ft.

Cumulative oil production, N_p = 32,000,000 stock tank barrel.

Total injection rate, Q_t = 22,060 reservoir b/d.

FIND: Performance of the pool and saturation distributions as a function of position of the gas front.

METHOD OF SOLUTION: For conditions at the displacement front or gas-oil contact, the Buckley-Leverett[3] frontal-advance equation can be written as

$$Q_t t = \frac{LA\phi(1 - S_w)}{5.615\,(df_g/dS'_g)_f} \quad (1)$$

The advancement of saturations behind the gas-oil contact are defined by

$$\times = \frac{5.615\,Q_t t}{A\phi(1 - S_w)}\left(\frac{df_g}{dS'_g}\right) \quad (2)$$

$$\text{Where } \times = h \div \sin\alpha \quad (3)$$

When the position of the front (gas-oil contact) is specified, the cumulative reservoir voidage term $Q_t t$ in Equation 3 can be replaced by Equation 1 to give[4]

$$h = \frac{L \sin\alpha\,(df_g/dS'_g)}{(df_g/dS'_g)_f} \quad (4)$$

In these equations

t = time, days

× = distance traveled along bedding plane by a plane of saturation, ft

df_g/dS'_g = derivative of the fractional flow equation evaluated for the saturation corresponding to ×

$(df_g/dS'_g)_f$ = derivative of the fractional flow equation for the gas saturation which exists at the displacement front or gas-oil contact

h = vertical distance corresponding to distance along bedding plane ×, or L ft

S'_g = gas saturation, fraction of hydrocarbon pore volume

L = distance traveled along bedding plane by the gas-oil contact

Q_t, A, ϕ, S_w, and α were defined with the data.

SOLUTION: The solution of this problem using Equations 1, 2, and

4, and Figs. 1 and 2 is given in Tables 1 and 2.

For L = 1,000 ft and S'_g = 60.0%

$$Q_t t = \frac{(1{,}000)(5.78 \times 10^6)(0.107)(1.00 - 0.08)}{(5.615)(1.587)}$$

$$= \frac{568{,}983{,}200}{8.911} = 63.852 \times 10^6 \text{ bbl}$$

$$t = \frac{63.852 \times 10^6}{(19{,}000)(1.161)} = \frac{63.852 \times 10^6}{22{,}060} = 2{,}894 \text{ days}$$

$$= 7.93 \text{ years}$$

$$h = \frac{(1{,}000)(\sin 30°)(0.92)}{1.59} = \frac{(500)(0.92)}{1.59} = 290 \text{ ft}$$

DISCUSSION: All of the expansive forces prevalent in reservoir systems that cause oil displacement can essentially be described by the fractional flow and frontal advance equations.[5] The fractional flow equation was discussed in the previous problem. The frontal-advance equation is derived assuming that the quantity of displacing fluid flowing during a small increment of time into a small volume of homogeneous, uniformly saturated sand minus the quantity of the same fluid flowing out is equal to the increase in displacing fluid content of the sand If the displacing fluid is a gas, then the total pressure is assumed to be large compared with pressure drop between the gas-oil contact and the first row of producing wells so that the gas can be considered incompressible.

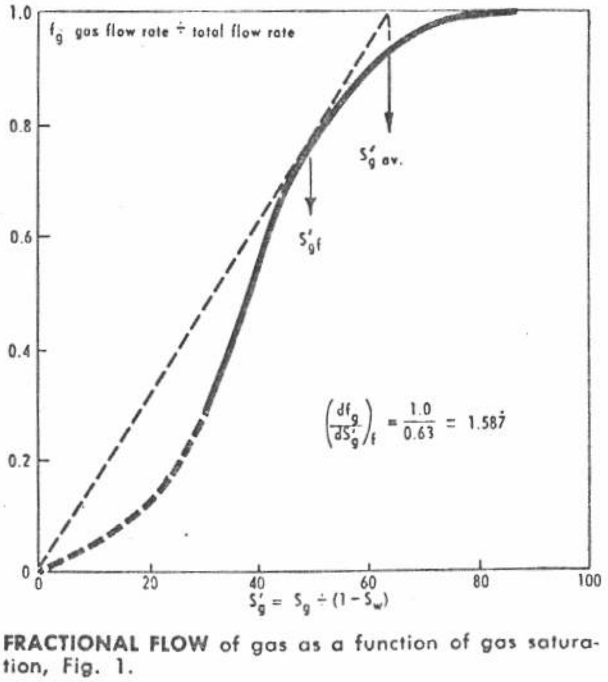
Also steady-state flow in a linear system is assumed.[6] In its derived form, Equation 2 expresses the linear advance of a surface of constant saturation during an increment of time, t, under the application of a constant displacing fluid injection rate, Q_t. Displacement of fluids by frontal advance creates stabilized saturation distribution conditions at the front and nonstabilized saturation distribution conditions behind the front.[5]

In applying the frontal-advance flow theory to Elk Basin field in this problem, it was necessary to estimate the maximum length, L, in the direction of flow; cumulative oil production; total hydrocarbon pore volume; and actual hydrocarbon pore-volume distribution (assumed). Therefore, in some respects the problem is hpothetical.

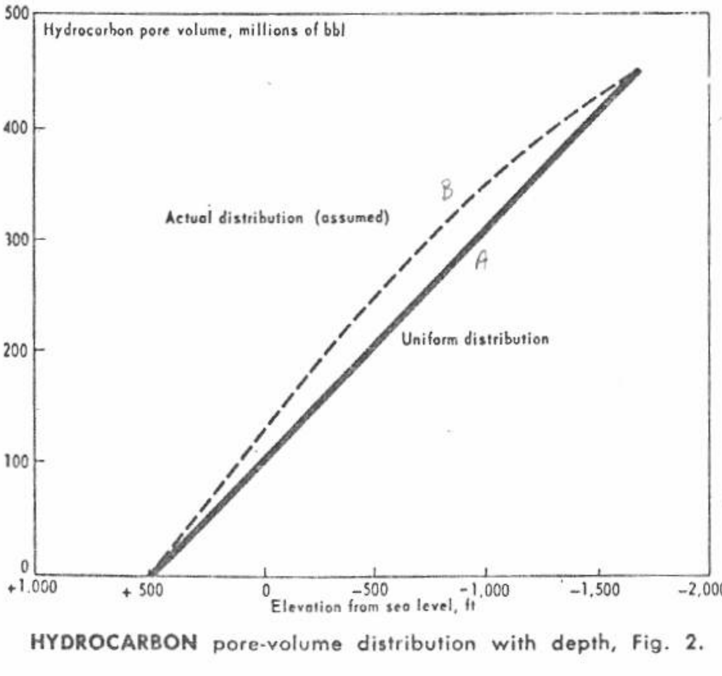
Although the total injection rate in reservoir barrels is nearly equal to the reservoir-oil production rate and the two rates are assumed equal, the method can also be applied where free gas and water are also produced. In this case it is necessary to know or be able to predict the quantities of free gas and water production. Such conditions result in less oil production over a period of time since the injection fluid is also displacing free gas and water.

Table 1 shows solutions of the problem assuming uniform and actual hydrocarbon pore volume distributions as given in Fig. 2. The two cases are solved to show the differences that exist when using actual hydrocarbon pore volume distribution as compared to the ideal case of uniform hydrocarbon pore volume distribution. Fig. 3 shows that although the ultimate recovery and variation of recovery with time are the same for the two cases, significant variations occur in oil recovery with position of the gas-oil contact.

Table 2 shows the computation and Fig. 4 the graphical results of saturation distribution as a function of gas-front position. Here can be



FRACTIONAL FLOW of gas as a function of gas saturation, Fig. 1.



HYDROCARBON pore-volume distribution with depth, Fig. 2.

Table 1—Computation of gas-cap posit

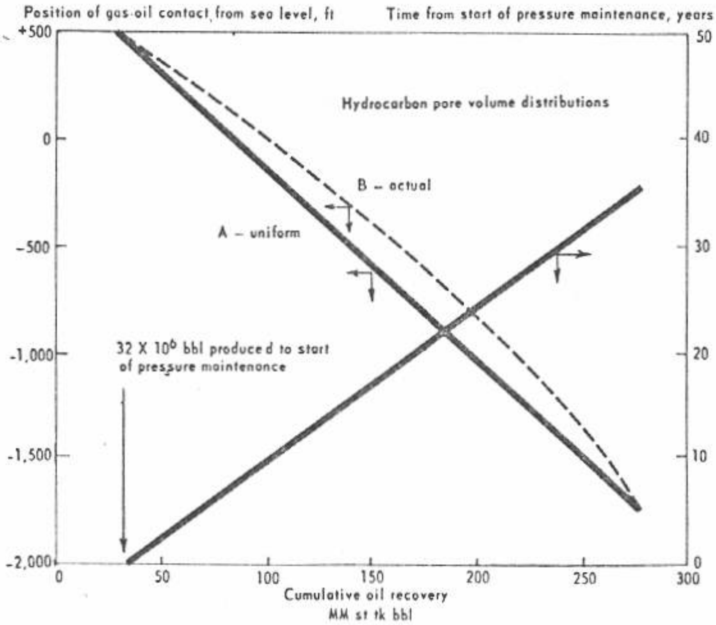
(1) L, ft	(2) h, ft	(3) Elevation from sea level, ft + 500 − (2)	(4) Hydrocarbon pore vol. = $LA\phi(1-S_w) \div 5.615$ MM bbl	(5) $Q_t t = (4) \div (df_g/dS'_g)_f = (4) \div 1.587$ MM bbl	(6) t days (5) ÷ 22,060	(7) $N_p =$ 0.019 × (6) MM bbl
		Uniform hydrocarbon pore volume distribution—curve A, Fig. 2				
...	...	+500	.....	......	.....	32.000
100	50	+450	10.13	6.383	285	37.491
300	150	+350	30.40	19.156	868	48.492
600	300	+200	60.80	38.311	1,737	65.003
1,000	500	0	101.33	63.852	2,894	86.986
1,500	750	−250	152.00	95.778	4,342	114.498
2,000	1,000	−500	202.67	127.706	5,789	141.991
2,500	1,250	−750	253.33	159.628	7,236	169.484
3,000	1,500	−1,000	304.00	191.556	8,683	196.977
3,500	1,750	−1,250	354.66	223.478	10,130	224.470
4,000	2,000	−1,500	405.33	255.406	11,578	251.982
4,440	2,220	−1,720	449.92	283.503	12,851	276.169

seen the stabilized saturation portion between S'_g of 0 to 50% and the nonstabilized portion between S_g of 50 to 87%. The shape of the nonstabilized portion changes with time and advance of the front.

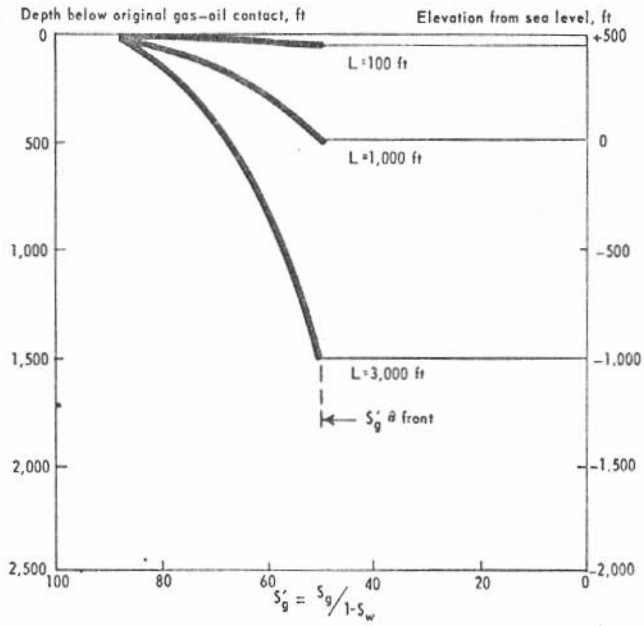
The theory and derivation of the frontal advance equation assume that flow occurs only in one direction. This implies that this method cannot be applied to gravity-drainage conditions where segregation of fluids (gas flowing up and oil down) is occurring. Such a condition is often prevalent for reservoirs having high permeabilities and high structural dips and at pressures below the

Table 2—Computation of saturation distribution as a function of gas-front position

(1) $S'_g = S_g \div (1-S_w)$	(2) f_g Fig. 1	(3) (df_g/dS'_g) Fig. 1	(4) $(df_g/dS'_g) \div (df_g/dS'_g)_f$ (3) ÷ 1.59	(5) Δh for L=100 ft	(6) Δh for L=1,000 ft	(7) Δh for L=3,000 ft
50.0	0.787	1.59	1.00	50.0	500	1,500
60.0	0.913	0.92	0.58	29.0	290	870
70.0	0.982	0.42	0.26	13.0	130	390
80.0	0.998	0.07	0.04	2.0	20	60
87.0	1.000	0	0	0	0	0



VARIATION of oil recovery with position of gas-oil contact and time, Fig. 3.



SATURATION DISTRIBUTION as a function of gas front position, Fig. 4.

umulative oil production

sumed actual hydrocarbon pore volume distribution, curve B, Fig. 2			
(8)	(9)	(10)	(11)
-bon HYDROC. ol., POR. VOL. 2 Fig 2 B Curve B ol	$Q_t t$ (8) ÷ $(df_g/dS'_g)_t$ (8) ÷ 1.587 MM bbl	t, days (9) ÷ 22,060	$N_p =$ (0.019) × (10) + 32 MM bbl
	......	.....	32.000
	6.931	314	37.966
	22.684	1,028	51.532
	45.369	2,057	71.083
	78.765	3,570	99.830
	117.832	5,341	133.479
	155.009	7,027	165.513
	189.036	8,569	194.811
	219.282	9,940	220.860
	246.377	11,168	244.192
	269.061	12,197	263.743
	283.554	12,854	276.226

bubble point. Such a case, using an alternate method of solution, will be treated at a later date.

References

1. Stewart, F. M., Garthwaite, D. L., and Krebill, F. K., "Pressure Maintenance by Inert-Gas Injection in the High-Relief Elk Basin Field": Trans. AIME, Vol. 204, 1955, pp. 49-57.
2. Guerrero, E. T., Part 51, Sept. 24, 1962, p. 179.
3. Buckley, S. E., and Leverett, M. C., "Mechanism of Fluid Distribution in Sands": Trans. AIME, Vol. 142, 1942, p. 107.
4. Dardaganian, S. G., "The Application of the Buckley-Leverett Frontal Advance Theory to Petroleum Recovery": Trans. AIME, Vol. 213, 1958, pp. 365-368.
5. Pirson, S. J., "Oil Reservoir Engineering": second edition, McGraw-Hill Book Co., 1958.
6. Hocott, C. R., "Mechanics of Fluid Injection—Gas": Chapter IX—Improving Oil Recovery, University of Texas.

Part 53

How to determine rate of gravity drainage

and time needed for oil column to become saturated with oil in a reservoir in which segregation of oil and gas occurs

GIVEN: Most of the following data on a steeply dipping southern Oklahoma reservoir were reported by Essley and coauthors.[1] Other data were assumed and/or estimated from Reference 2.

The reservoir under study initially contained a 4,200-ft oil column. For 12 years, production occurred by pressure depletion and gravity drainage, and the average reservoir pressure (at middle of oil column) fell from 1,850 to 600 psi. At this point gas-cap pressure-maintenance operations were initiated. Other average data used on this reservoir are as follows: (Top of oil column was assumed to be 2,000 ft below the surface, or middle of oil column is at a depth of 4,100 ft. All average pressures and fluid properties apply at this depth.)

Equilibrium gas saturation in oil column = 4.0% pore volume

Dip, $\alpha = 45°$

Interstitial-water saturation, $S_w = 14.0\%$

Pore volume, $V_p = 672{,}000{,}000$ bbl

Pore volume of top 250 ft of oil column = 22,200,000 bbl

Field-test data show the gas-oil contact 250 ft below top of oil column at start of pressure-maintenance operations.

Initial oil formation volume factor, $B_{oi} = 1.16$

Average residual-oil saturation in top 250 ft of oil column at start of gas cap pressure maintenance operations, $S_{or} = 29.0\%$

Area pressure gradient, $\rho_w = 0.45$ psi/ft

Absolute permeability, k = 175 md

Average cross-sectional area perpendicular to bedding plane, A = 3,550,000 sq ft

Cumulative oil recovery at start of pressure-maintenance operations, $N_p = 27{,}000{,}000$ st tk bbl

Average reservoir fluid properties from discovery to initiation of pressure maintenance operations:

Oil viscosity, $\mu_o = 5.5$ cp

Gas gradient, $g\rho_g = 0.03$ psi/ft

Oil gradient, $g\rho_o = 0.35$ psi/ft

Fig. 1 shows the variation of porosity, absolute permeability, and interstitial-water saturation with depth.[1]

Fig. 2 shows the average relative-permeability properties of the pool.[1]

Average reservoir fluid properties at 600 psi:

Oil viscosity, $\mu_o = 6.5$ cp

Gas gradient, $g\rho_g = 0.03$ psi/ft

Oil gradient, $g\rho_o = 0.38$ psi/ft

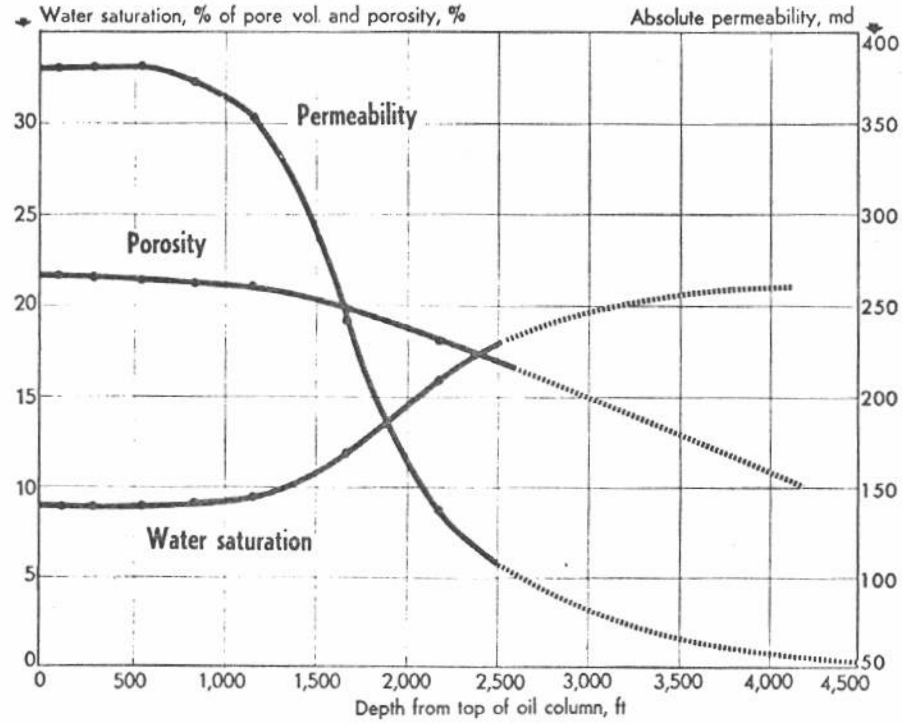
Change in pressure with depth in oil column, $dp/dD = 0.10$ psi/ft (this actually represents a combined oil and gas gradient)

Oil formation-volume factor, $B_o = 1.07$

Constant oil-production rate during gas pressure-maintenance operations, $Q_o = 7{,}000$ st tk b/d. This rate of oil production can be maintained until the gas-oil contact falls below 2,000 ft from top of producing formation. Assume that below this point each of the remaining producing wells can produce the same amount of oil and the producing wells go to gas uniformly as the gas-oil contact moves down with time.

FIND: 1. Rate of gravity drainage during first years of pressure-maintenance operations.

2. If resaturation of oil column occurs, time for this resaturation to take place down to $S_g = 4.0\%$ of pore volume.



VARIATION of permeability, porosity, and water saturation with depth for a southern Oklahoma reservoir. Fig. 1.

METHOD OF SOLUTION: Equations used are those recommended by Essley et al.[1] These equations are the generalized Darcy equation

$$q_o = 1.127\,(k_oA/\mu_o) \times (dp/dl - \rho_o g \sin\alpha) \quad (1)$$

A material-balance equation

$$\sum_{GOCi}^{GOC} (\Delta S_o \times \Delta V_p)_i = Q_oB_o\,\Delta t + \Delta S_{goc}\,V_{poc} \quad (2)$$

and a displacement equation

$$(dD/dt)_{So} = (6.33\,k \sin^2\alpha/\mu_o\,\phi)\,g\Delta\rho \times (dk_{ro}/dS_L) \quad (3)$$

Where:

A, μ_o, $g\rho_o$, $g\rho_g$, α, V_p, Q_o, B_o, and k were defined with the data and

q_o = oil-production rate, reservoir b/d.

k_o = effective oil permeability, darcies.

dp/dl = change in pressure with distance along bedding plane in oil column.

S_o = reservoir-oil saturation, fraction of pore volume.

t = time, days.

S_{goc} = reservoir gas saturation in oil column, fraction of pore volume.

oc = subscript for oil column.

GOC = gas-oil contact position or depth.

i = subscript for initial.

$(dD/dt)_{So}$ = change in depth with time for a given oil saturation above gas-oil contact.

ϕ = porosity, fraction.

$g\Delta\rho = g\,(\rho_o - \rho_g)$, psi/ft.

k_{ro} = relative oil permeability.

D = depth, ft.

Since $k_o = k \times k_{ro}$ and $dp/dl = (dp/dD)\sin\alpha$

Equation 1 can be written as

$$q_o = 1.127\,k \times (k_{ro}\,A/\mu_o) \times [(dp/dD) - g\rho_o]\sin\alpha \quad (4)$$

SOLUTION: The average oil saturation in the oil column (gas-oil contact has advanced 250 ft into oil column) at the start of pressure-maintenance operations is given by

$$S_o\,V_{poc} = [(\frac{1.00 - S_w}{B_{oi}})\,V_{poc} + (\frac{1.00 - S_w}{B_{oi}})\,V_{pGC} - \frac{S_{or}}{B_o}\,V_{pGC} - N_p]\,B_o$$

Where:

V_{poc} = pore volume in oil column below gas-oil contact, barrels.

V_{pGC} = pore volume in portion

Table 1—Calculations to determine distribution of reservoir fluids at start of press

(1) Time interval, Δt: Years	Days	(3) S_o, %	(4) Depth* from previous calculation, ft	(5) ΔD Est. ft	(6) Avg. depth est. ft (4)+½ (5)	(7) From Fig. 1: k, darcies	(8) From Fig. 1: φ fraction	(9) From Fig. 1: S_w Fraction of pore vol.	(10) S_L (3)+(9)	(11 dK$_{ro}$/ fror Fig.
12	4,380	25	0	70	35	0.380	0.215	0.088	0.338	0.0
		30	0	142	71	0.380	0.215	0.088	0.388	0.1
		35	0	228	114	0.380	0.215	0.088	0.438	0.1
		40	0	328	164	0.380	0.215	0.088	0.488	0.2
6.5	2,373	25	71	36	89	0.380	0.215	0.088	0.338	0.0
		30	142	71	178	0.380	0.215	0.088	0.388	0.1
		35	228	114	285	0.380	0.215	0.088	0.438	0.1
		40	250	164	332	0.380	0.215	0.088	0.488	0.2
		45	250	272	386	0.380	0.214	0.088	0.538	0.3
		50	250	446	473	0.380	0.213	0.088	0.588	0.6
		55	250	640	570	0.380	0.213	0.088	0.638	0.8
		60	250	900	700	0.377	0.212	0.089	0.689	1.2

*Expressed as feet from top of oil column.

of original oil column now above gas-oil contact, barrels.

Thus

$$S_o = \frac{\left[\left(\frac{1.00 - S_w}{B_{oi}}\right) 22.2 \times 10^6 - \left(\frac{0.29}{1.07}\right)(22.2 \times 10^6) - 27 \times 10^6\right] 1.07}{649.8 \times 10^6}$$

$$S_o = 0.767$$

Therefore $S_g = 1.000 - 0.767 - 0.140 = 0.093$.

Rate of gravity drainage is computed with Equation 4. To obtain k_{ro} for this equation requires knowledge of the average total liquid saturation ($S_o + S_w$), S_L. For this computation let it be assumed that oil resaturation of the oil column will take place (gas saturation will decrease to 4.0%) during the first few years of pressure-maintenance operations. Thus S_o will be (0.767 + 0.093 − 0.040) or 0.82.

$$S_{o(avg)} = (0.767 + 0.820) \div 2 = 0.794$$
$$S_L = 0.794 + 0.140 = 0.934$$

and from Fig. 2, $k_{ro} = 0.85$.

Solving Equation 4 gives

$$q_o = (1.127)(0.175)(0.85)(3.55 \times 10^6)(0.10 - 0.38) \sin 45° \div 6.5$$

= 18,120 reservoir b/d rate of gravity drainage.

The current production rate is 7,000 × 1.07 = 7,490 reservoir b/d.

Thus the oil column resaturates with oil at the rate of 18,120 − 7,490 = 10,630 reservoir b/d.

Resaturation will take place below the gas-oil contact and as resaturation takes place the gas-oil contact will move downstructure. To determine the time necessary for resaturation, trial-and-error computations are performed to satisfy Equation 2, where Q_o = 7,000 st tk bbl/d and $\Delta S_g = 0.093 - 0.040 = 0.053$. The calculation procedure is as follows:

1. Compute the saturation-depth profile using Equation 3 at the end of a 12-year pool life (period from discovery to start of pressure-maintenance operations). This is shown in Table 1.
2. Estimate the pool-life interval, Δt, (pool life = 12 years + Δt) required for resaturation of oil column to occur.
3. Compute the saturation-depth profile using Equation 3 for this Δt. This is shown for Δt = 6.5 years in Table 1.
4. Solve Equation 2. Table 2 shows the computation of the left portion of Equation 2.
5. Plot the sum of each side of Equation 2 versus depth from top of oil column. This is shown on Fig. 3.
6. Determine the position of the gas-oil contact from the point of intersection (1,170 ft) of the two curves in Fig. 3 and compute the pore volume, V_{poc}, in the oil column below this point. By summing column 5 in Table 2 and interpolating between the depths of 1,100 and 1,200 ft ($V_{p\,1,170} = 169.8 \times 10^6$ bbl).

$V_{poc} = 672.0 \times 10^6 - 169.8 \times 10^6 = 502.2 \times 10^6$ bbl.

7. Compute the time required for resaturation to occur in V_{poc} at the existing resaturation rate of 10,630 reservoir b/d.

$$\Delta t = V_{poc}\, \Delta S_{goc}/10{,}630$$
$$= (502.2 \times 10^6)(0.053)/10{,}630$$
$$= 2{,}504 \text{ days or } 6.86 \text{ years.}$$

8. The time interval computed in step 7 should agree with the time interval estimated in step 2. Trial-and-error calculations are continued until agreement within the desired accuracy is obtained. In this case 6.86 years is considered sufficiently close to the estimated time interval of 6.5 years.

Steps 1 and 3 require solution of Equation 3 for time intervals of 12 and 6.5 years. The procedure for solving this equation is as follows: (See Table 1 for illustration).

1. Estimate change in depth, ΔD, to occur for a particular saturation over the time interval.
2. Compute the average depth of the saturation over the time interval as the previous depth plus ½ ΔD.
3. Read absolute permeability (k), porosity (ϕ), and water saturation (S_w) for the average depth from Fig. 1.
4. Compute the total liquid saturation using the oil and water saturations.

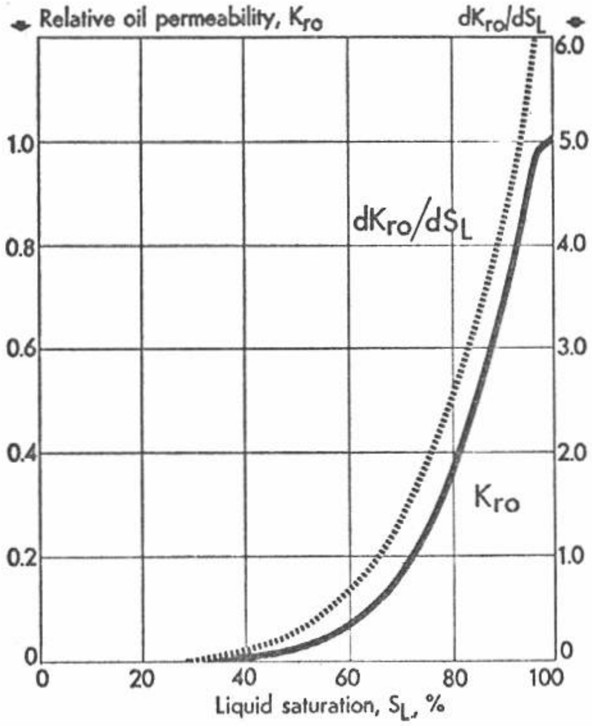
$S_L = S_o + S_w$

5. Read (dK_{ro}/dS_L) from Fig. 2.
6. Solve Equation 3 for $(dD/dt)_{So}$ noting that for pressure-maintenance operations ($6.33\ k \times \sin^2 \alpha$) $g\Delta\rho/\mu_o$ is a constant and equal to

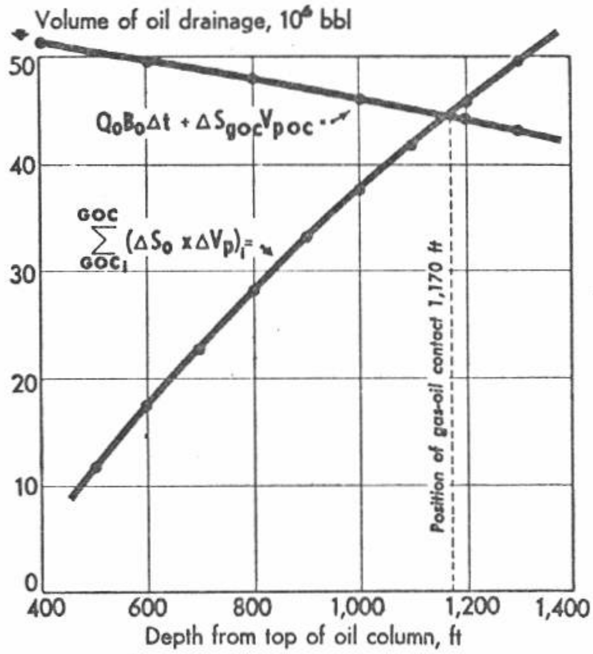
$$(6.33) \times (\sin 45°)^2 \times (0.38 - 0.03) \div 6.5 = 0.170$$

tenance and at a pool life of 18.5 years

(12) $\Delta\rho \sin^2 \alpha$ / μ_o	(13) $(dD/dt)_{So}$ (7)×(11)×(12)÷(8)	(14) ΔD Computed (2)×(13)	(15) Calculated depth, ft (4)+(14)	(16) Life of pool, years
184	0.0163	71	71	12
184	0.0325	142	142	. . .
184	0.0520	228	228	. . .
184	0.0748	328	328	. . .
170	0.0150	36	107	18.5
170	0.0300	71	213	. . .
170	0.0481	114	342	. . .
170	0.0691	164	414	. . .
170	0.1147	272	522	. . .
170	0.1880	446	696	. . .
170	0.2699	640	890	. . .
170	0.3779	897	1,147	. . .



RELATIVE PERMEABILITY PROPERTIES for a southern Oklahoma reservoir. Fig. 2.



GRAPHICAL SOLUTION of Equation 2. Fig. 3.

7. Multiply the $(dD/dt)_{S_o}$ computed by Δt to obtain ΔD. This value of ΔD should check with the value estimated in step 1. If it does not check within about 5% then another ΔD is estimated and the computations are repeated until the check is satisfied. As an example let the calculations be performed for a time increment of 6.5 years and a saturation of 45% where:

Depth from previous calculation = 250 ft (cannot be greater than depth to gas-oil contact).

ΔD estimated = 272 ft.

Average depth = 250 + (½) (272) = 386 ft.

For this depth (from Fig. 1)

$k = 0.380$

$\phi = 0.214$

$S_w = 0.088$

Thus $S_L = 0.450 + 0.088 = 0.538$ and $(dK_{ro}/dS_L) = 0.38$ from Fig. 2.

Thus, substituting into Equation 3 gives

$$dD/dt = (0.170)\,(0.380)\,(0.38)/0.213 = 0.1147$$

$$\Delta D = (0.1147) \times (6.5 \text{ years}) \times (365 \text{ days/year}) = 272 \text{ ft}$$

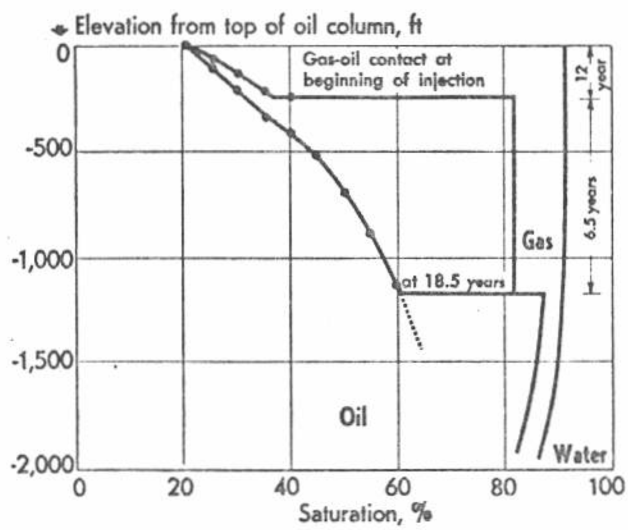
The computed value of ΔD agrees with the value estimated. The calculated depth of the 45% oil saturation at the end of 18.5 years. (12 + 6.5) is

$$250 + 272 = 522 \text{ ft}$$

from the top of the initial oil column. In Tables 1 and 2 the calculated depth values of column 15 are plotted versus the oil-saturation values of column 3 to give the saturation-time-depth profiles for pool lives of 12 and 18.5 years. These plots are shown in Fig. 4. For the data of Table 1 the gas-oil contact was placed at 250 ft from the top of the oil column based on field tests. To locate the gas-oil contact at the end of 18.5 years requires the solution of Equation 2.

Since both sides of this equation are functions of pore volume, each side of the equation is solved for various values of oil-column pore volume (assuming different positions of the gas-oil contact) and each set of results (volumes of oil drainage) plotted versus depth from the top of the oil column. The computations for the left portion of Equation 2 are shown in Table 2; those for the right portion are not shown but can readily be obtained using cumulative pore volume increments from column 5 of Table 2. For example, at a depth of 600 ft from the top of the oil column,

$$V_{p600} = (7.8 + 9.1 + 10.5 + 11.9 + 13.3 + 14.8)\,10^6 = 67.4 \times 10^6 \text{ bbl}$$



SATURATION-time-depth profiles for a southern Oklahoma reservoir. Fig. 4.

Table 2—Computation of gravity drainage and displacement of oil by movement of gas-oil contact for time interval of 12 to 18.5 years (Δt = 6.5 years)

(1) Depth from top of oil column, ft	(2) Avg. oil saturation, fraction of pore vol., from Fig. 4 At 12 years	(3) At 18.5 years	(4) ΔS_o (2) — (3)	(5) Pore vol. ΔV_p from Fig. 5 Ref. 1, MM bbl	(6) Pore vol. voided $\Delta S_o \times \Delta V_p$ (4) × (5) 10^6 bbl	(7) $\Sigma \Delta S_o \ \Delta V_p$ = Σ (6), 10^6 bbl
0-100	0.239	0.222	0.017	7.8	0.13	0.13
100-200	0.307	0.268	0.039	9.1	0.35	0.48
200-300	0.363	0.314	0.049	10.5	0.51	0.99
300-400	0.819	0.360	0.459	11.9	5.46	6.45
400-500	0.819	0.417	0.402	13.3	5.35	11.80
500-600	0.819	0.459	0.360	14.8	5.33	17.13
600-700	0.819	0.487	0.332	16.4	5.44	22.57
700-800	0.818	0.516	0.302	18.3	5.53	28.10
800-900	0.817	0.542	0.275	18.3	5.03	33.13
900-1,000	0.816	0.566	0.250	18.3	4.58	37.71
1,000-1,100	0.816	0.583	0.233	18.3	4.26	41.97
1,100-1,200	0.815	0.602	0.213	18.3	3.90	45.87
1,200-1,300	0.813	0.619	0.194	18.3	3.55	49.42

$$V_{poc} = V_p - V_{p600} = 672 \times 10^6 - 67.4 \times 10^6 = 604.6 \times 10^6 \text{ bbl}$$

and since Q_o = 7,000 st tk bbl/day, B_o = 1.07, Δt = (365) (6.5 years) = 2,373 days, ΔS_{goc} = 0.053

$$Q_o B_o \Delta t + \Delta S_{goc} V_{poc} = (7{,}000)(1.07)(2{,}373) + (0.053)(604.6 \times 10^6) = 49.8 \times 10^6 \text{ bbl}$$

DISCUSSION: A solution-gas-drive reservoir derives most of its energy from the expansion of gas that evolves from solution as the reservoir pressure falls below the bubble point. Initially at low gas saturations the expanding gas efficiently drives the oil to the producing wells. However, as the reservoir pressure continues to decline and the gas saturation increases, permeability to gas rapidly develops and the gas energy is dissipated without displacing most of the oil. Gravity has the effect of tending to cause segregation of the gas and oil. It is most effective under conditions of high permeability (20 md or more), low oil viscosity, and high structural dip (10° or more). If the forces, created by production and acting on unit volumes of oil and gas, are small compared with the gravitational forces also acting on the oil and gas, segregation will occur with oil moving down and gas up to a gas cap or to form a secondary gas cap. When gravity drainage is effective in a steeply dipping pool, gas usually flows to and accumulates at the crest.[1]

Gravity drainage results in a situation where oil saturations are high at the lower part of a reservoir and gas saturations are high at the crest of a structure. The additional recovery resulting from gravity drainage is oil that is replaced by gas in structurally high positions. The effect of gravity drainage on recovery can be computed by determining the rate at which oil is migrating downdip from a secondary (or expanding primary) gas cap or by determining the rate at which gas is accumulating and expanding in the cap.

In the previous problem gravity-drainage performance was computed for a reservoir where segregation does not occur; oil moves down but gas does not flow to a gas cap. For the problem under discussion a more general solution procedure is required since segregation does occur. This solution can be achieved with Equations 1, 2, 3, and 4. The solution is facilitated by pressure maintenance at the gas-oil contact and in the gas cap. Below the gas-oil contact some variation in pressure occurs due to gravity alone. Thus steady-state theory can be applied.

Equations 1 and 4 are forms of the generalized Darcy equation that is used here to compute the rate of gravity drainage. Average values for the various factors in the equation were used to compute the rate of drainage. This rate of drainage is more than twice the oil-production rate and results in resaturation of the oil column to an equilibrium gas saturation of 4% in about 6½ years after initiation of pressure-maintenance operations, Fig. 4. After resaturation of the oil column (to Sg = 4%), segregation of gas ceases and oil moves down at a rate comparable to oil-production rate.

Equation 2 states that the amount of drainage from within the gas cap plus the volume of oil displaced at the gas-oil contact are equal to the volume of oil produced from below the gas-oil contact plus the decrease in pore volume occupied by gas in the oil column (zero after resaturation to Sg = 4%). This is a material-balance equation used to locate the position of the gas-oil contact with time. Its solution requires knowledge concerning the variation of oil saturation with time at all points within the expanding gas cap. This information is obtained with Equation 3 which defines the distance traveled by an oil saturation during time increment, Δt. Its solution requires the use of average rock properties over the distance ΔD. Since ΔD is sought, the solution is trial and error as shown.

References

1. Essley, P. L., Jr., Hancock, G. L., Jr., and Jones, K. E., "Gravity Drainage Concepts in a Steeply Dipping Reservoir": Paper, SPE of AIME, Petroleum Conference on Production and Reservoir Engineering, Tulsa, March 20-21, 1958, Paper 1029-G, 11 pp.

2. Standing, M. B., "A Pressure-Volume-Temperature Correction for Mixtures of California Oils and Gases": API Drilling and Production Practice, 1947, p. 275.

3. Guerrero, E. T., "Reservoir Engineering—Part 52: How to Find the Performance of a Gravity Drainage Reservoir"; The Oil and Gas Journal Vol. 60, No. 43, Oct. 22, 1962, pp 102-103, 106.

Part 54

How to determine performance of a gravity-drainage reservoir in which segregation of oil and gas occurs

GIVEN: Data on a steeply dipping southern Oklahoma reservoir as reported by Essley and coauthors,[1] and in previous problem.

FIND: 1. Compute the saturation-time-depth profiles for 27, 50, and 70 years of history.

2. Determine the variation of oil-recovery rate and cumulative oil recovery with time.

METHOD OF SOLUTION: This problem is a continuation of the previous one where performance solutions were presented for pool lives of 12 and 18.5 years. Since resaturation of the oil column has occured, the $\Delta S_{goc} V_{poc}$ term, shown in Equation 2 of the previous part, is not included in Equation 1 of this problem. This indicates that no further gas segregation is occurring and that the total of oil displacement at the gas-oil contact and gravity drainage behind the gas-oil contact cannot exceed the total amount of oil produced under pressure maintenance operations. The procedure[1] used includes a material-balance equation.

$$\sum_{GOCI}^{GOC} (\Delta S_o \times \Delta V_p)_i = Q_o B_o \Delta t \quad (1)$$

and a displacement equation

$$(dD/dt)_{So} = (6.33\ k\ \mathrm{Sin}^2\alpha/\mu_o\ \phi) \times g\Delta\rho\ (dk_{ro}/dS_L) \quad (2)$$

Where:

GOC = gas-oil contact position or depth.

i = subscript for initial.

S_o = reservoir oil saturation in oil column, fraction of pore volume.

V_p = pore volume.

Q_o = oil production rate, st tk bbl/day.

B_o = average oil formation volume factor.

t = time, days.

Δ = symbol for "increment."

D = depth, ft.

k = absolute permeability, darcies.

α = dip, degrees.

μ_o = average reservoir oil viscosity, cp.

ϕ = porosity, fraction.

$g\Delta\rho = g\ (\rho_o - \rho_g)$.

Table 1—Calculations to determine distribution of reservoir flu

(1) Time interval, Δt: Years	(2) Time interval, Δt: Days	(3) So %	(4) Depth* from previous calculation, ft	(5) ΔD est., ft	(6) Avg. depth est., (4)+½ (5)	(7) From Fig. 1, Ref. 2: k darcies	(8) From Fig. 1, Ref. 2: φ fraction	(9) From Fig. 1, Ref. 2: S_w Fraction of pore volume	(1 S (3)+
8.5	3,103	25	107	47	131	0.380	0.215	0.088	0.3
		30	213	93	260	0.380	0.215	0.088	0.3
		35	342	150	417	0.380	0.214	0.088	0.4
		40	414	217	523	0.380	0.213	0.088	0.4
		45	522	362	703	0.377	0.212	0.089	0.5
		50	696	572	982	0.367	0.210	0.090	0.5
		55	890	802	1,291	0.337	0.205	0.096	0.6
		60	1,147	990	1,642	0.253	0.196	0.114	0.7
23	8,395	25	154	126	217	0.380	0.215	0.088	0.33
		30	306	252	432	0.380	0.214	0.088	0.38
		35	492	432	708	0.377	0.212	0.089	0.43
		40	631	600	931	0.368	0.210	0.090	0.49
		45	888	996	1,386	0.321	0.203	0.099	0.54
		50	1,268	1,220	1,878	0.190	0.189	0.132	0.63
		55	†1,312	1,550	2,087	0.149	0.182	0.148	0.69
20	7,300	25	280	110	335	0.380	0.215	0.088	0.33
		30	560	220	670	0.378	0.212	0.089	0.38
		35	924	360	1,104	0.358	0.209	0.092	0.44
		40	1,231	518	1,490	0.298	0.201	0.104	0.50
		45	1,881	670	2,216	0.132	0.178	0.158	0.60
		50	†1,930	915	2,388	0.114	0.172	0.169	0.66

*Expressed as feet from top of oil column. †Depth of gas-oil contact.

$g\rho_o$ = reservoir oil gradient or density, psi/ft.

$g\rho_g$ = reservoir gas gradient or density, psi/ft.

k_{ro} = relative oil permeability.

S_L = total liquid saturation, fraction of pore volume.

SOLUTION: Mechanics of solution to this problem are shown in Tables 1 and 2. Table 1 gives the solution of Equation 2 for time increments of 8.5, 23, and 20 years, which correspond to pool lives of 27, 50, and 70 years. Similar calculations were given for pool lives of 12 and 18.5 years in the previous problem which also showed the trial-and-error procedure for solving Equation 2. For example, this equation is solved as follows for a time increment of 8.5 years and at an oil saturation of 45%:

$$\Delta t = (8.5 \text{ years})\ (365 \text{ days/year}) = 3{,}103 \text{ days}$$

$$6.33 \operatorname{Sin}^2 \alpha\ g\Delta\rho/\mu_o = (6.33)\ (0.500)\ (0.35)/6.5 = 0.170$$

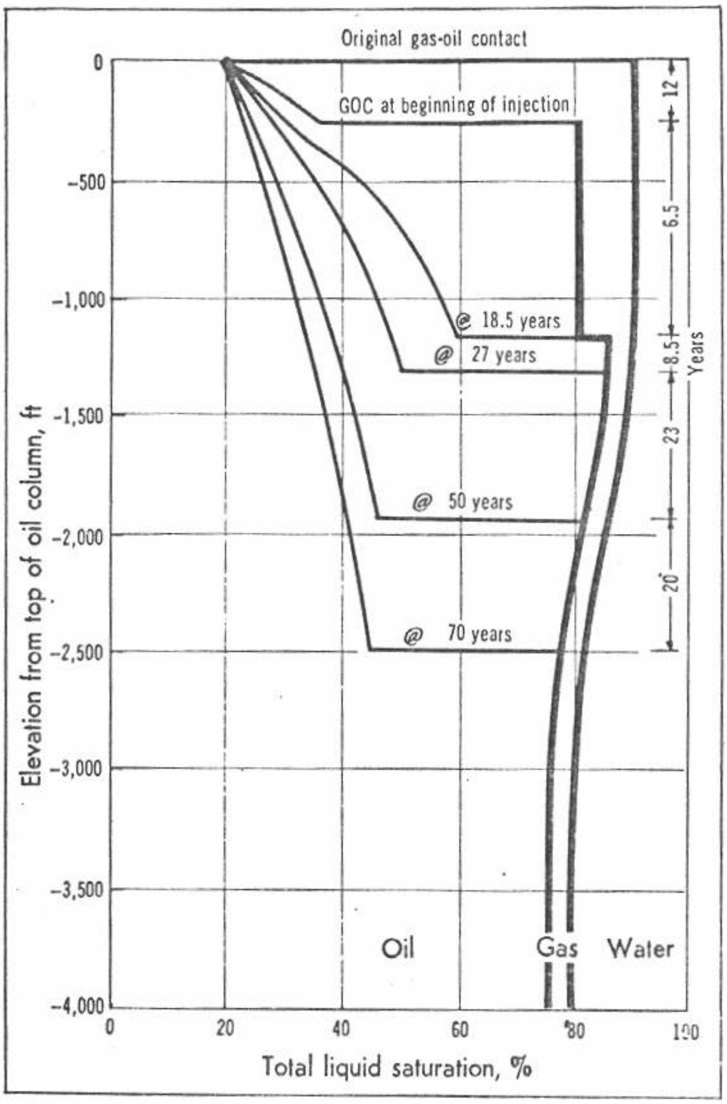
Where (from Part 53):

$\alpha = 45°$

$g\Delta\rho = 0.35$ psi/ft

$\mu_o = 6.5$ cp

In this illustration the position is



SATURATION - time - depth profiles southern Oklahoma reservoir. Fig. 1.

522 ft. It is estimated the 45% oil saturation will move 362 ft in 8½ years., The average depth of this saturation will be

522 + (362) (½) = 703 ft.

At 703 ft, using Fig. 1, Part 53

k = 0.377 darcies

ϕ = 0.212

S_w = 0.089 or S_L = 0.450 + 0.089 = 0.539

From Fig. 2, Part 53 (dk_{ro}/dS_L) = 0.39.

Substituting into Equation 2 gives

$$(dD/dt)_{So} = (0.170)\ (0.377)\ (0.39)/0.212 = 0.1179 \text{ ft/day}$$

$$\Delta D = (0.1179 \text{ ft/day}) \times (3{,}103 \text{ days}) = 366 \text{ ft}$$

The value of 366 ft checks sufficiently well with the estimated value of 362 ft. The position of the 45% oil saturation at the end of the 8½

ool lives of 27, 50, and 70 years

) S_L	(12) $6.33\ \Delta\rho \operatorname{Sin}^2 \alpha / \mu_o$	(13) $(dD/dt)_{So}$ (7)×(11)×(12) ÷(8)	(14) ΔD computed (2)×(13)	(15) Calculated depth, ft (4)+(14)	(16) Life of pool, years
5	0.170	0.0150	47	154	27
0	0.170	0.0300	93	306	..
5	0.170	0.0483	150	492	..
3	0.170	0.0698	217	631	..
9	0.170	0.1179	366	888	..
2	0.170	0.1842	572	1,268	..
3	0.170	0.2599	806	1,696	..
5	0.170	0.3204	994	2,141	..
5	0.170	0.0150	126	280	50
0	0.170	0.0302	254	560	..
7	0.170	0.0514	432	924	..
4	0.170	0.0715	600	1,231	..
4	0.170	0.1183	993	1,881	..
5	0.170	0.1453	1,220	2,488	..
3	0.170	0.1851	1,554	2,866	..
5	0.170	0.0150	110	390	70
0	0.170	0.0303	221	781	..
7	0.170	0.0495	361	1,285	..
3	0.170	0.0706	515	1,746	..
3	0.170	0.0920	672	2,553	..
1	0.170	0.1251	913	2,843	..

Table 2—Computation of gravity drainage and displacement of oil by movement of gas-oil contact for time interval of 27 to 50 years ($\Delta t = 23$ years)

(1) Depth from top of oil column, ft	(2) Avg. oil saturation, fraction of pore vol. from Fig. 1 — At 27 years	(3) At 50 years	(4) ΔS_o (2) − (3)	(5) Pore vol. ΔV_p from Fig. 5 Ref. 1, MM bbl	(6) Pore vol. voided, $\Delta S_o \times \Delta V_p$ (4) × (5), 10^6 bbl	(7) $\Sigma \Delta S_o \Delta V_p = \Sigma$ (6) 10^6 bbl
0- 100	0.211	0.209	0.002	7.8	0.02	0.02
100- 200	0.247	0.227	0.020	9.1	0.18	0.20
200- 300	0.281	0.245	0.036	10.5	0.38	0.58
300- 400	0.313	0.263	0.050	11.9	0.60	1.18
400- 500	0.346	0.279	0.067	13.3	0.89	2.07
500- 600	0.374	0.294	0.080	14.8	1.18	3.25
600- 700	0.402	0.312	0.090	16.4	1.48	4.73
700- 800	0.425	0.328	0.097	18.3	1.78	6.51
800- 900	0.443	0.343	0.100	18.3	1.83	8.34
900-1,000	0.460	0.358	0.102	18.3	1.87	10.21
1,000-1,100	0.472	0.372	0.100	18.3	1.83	12.04
1,100-1,200	0.486	0.386	0.100	18.3	1.83	13.87
1,200-1,300	0.498	0.399	0.099	18.3	1.81	15.68
1,300-1,312	0.504	0.404	0.100	2.2	0.22	15.90
1,312-1,400	0.863	0.409	0.454	16.1	7.31	23.21
1,400-1,500	0.859	0.419	0.440	18.3	8.05	31.26
1,500-1,600	0.853	0.427	0.426	18.3	7.80	39.06
1,600-1,700	0.846	0.434	0.412	18.3	7.54	46.60
1,700-1,800	0.840	0.442	0.398	18.3	7.28	53.88
1,800-1,900	0.832	0.449	0.383	18.3	7.01	60.89
1,900-2,000	0.825	0.458	0.367	18.3	6.72	67.61
2,000-2,100	0.816	0.466	0.350	18.3	6.41	74.02
2,100-2,200	0.806	0.475	0.331	18.3	6.06	80.08

Using data in this table, and Equation 1, gas-oil contact is located at 1,930 ft from top of reservoir.

years (life of 27 years) will be

$$522 + 366 = 888 \text{ ft}$$

The position of each saturation at the start of a pressure increment (depth from previous calculation) is obtained by referring to column 15, Table 1, of the previous time-increment calculations. This is true for all saturations above the gas-oil contact. However, as pointed out earlier in the computation for saturations within the gas-oil contact, a two-step method is used. First it is assumed that the saturation-depth relationships as computed for conditions above the gas-oil contact extend below the gas-oil contact. These saturations are then adjusted through use of a material-balance equation. This is illustrated for an oil saturation of 55% in the calculations for the time increment of 23 years. The previous calculations showed a position of 1,696 ft; however, since an oil saturation of 55% falls in the gas-oil contact zone in Fig. 1 (at 27 years), the position of the gas-oil contact (1,312 ft) is used.

The data of column 3, Table 1, are plotted versus the data of column 15 for each time increment on Fig. 1. This gives the position of each oil saturation that lies above the gas-oil contact at each time increment but not the position of the gas-oil contact. Data corresponding to the beginning of pressure-maintenance operations and 18.5 years are shown in Fig. 1 as obtained from the previous problem. To obtain the position of the gas-oil contact for time increments of 8½, 23, and 20 years, Equation 1 is solved for each corresponding set of data. This type of calculation is shown in Table 2. The data for column 2 are obtained from the complete curve for a pool life of 27 years. Data for column 3 are obtained from a plot of column 3 vs. column 15 of Table 1 for a time increment of 23 years. Most of these data are shown plotted on Fig. 1 for a time of 50 years. The last data point shown is for S_o = 45%. The depth of 2,488 ft for S_o = 50% was plotted and used in establishing the position of the gas-oil contact but is not shown on Fig. 1. The data for column 5 of Table 2, were obtained from Fig. 5 of Reference 1. Pore volume voided shown in column 6 represents the volume of gravity-drainage oil and oil displaced at the gas-oil contact for each increment of pore volume over the time increment under consideration. In Table 2 gravity drainage occurs above a depth of 1,930 ft and oil displacement occurs below this depth.

In the previous problem it was stated that the constant oil-production rate would be maintained until the gas-oil contact fell below a depth of 2,000 ft from the top of the producing formation. Below this depth it was assumed that the remaining producing wells would go to gas uniformly as the gas-oil contact moved down with time.

For Δt = 23 years

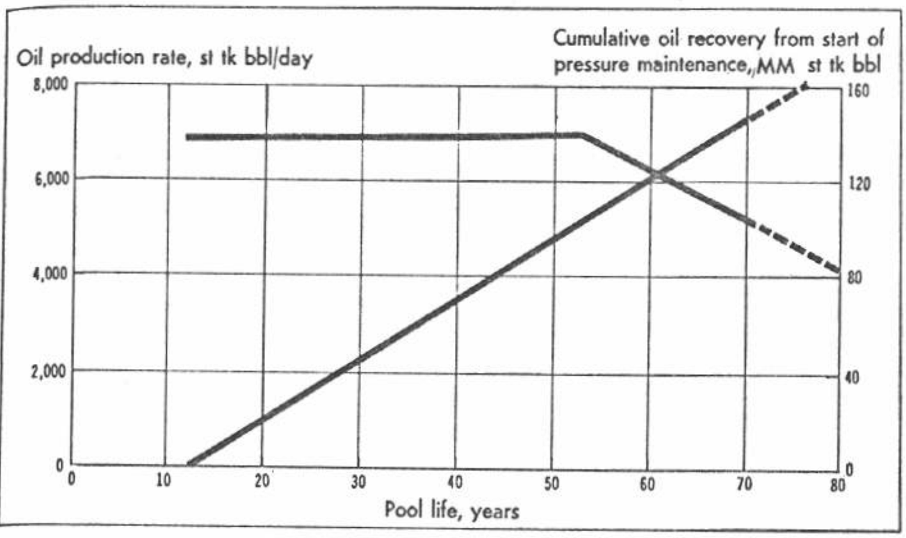
$$Q_o\, B_o\, \Delta t = (7{,}000 \text{ b/d} \times (1.07)\,(23 \text{ years})\,(365 \text{ days/year}) = 62.88 \times 10^6 \text{ reservoir bbl}$$

This sum occurs in column 7, Table 2, at a depth between 1,900 and 2,000 ft or

$$\text{Position of gas-oil contact} = 1{,}900 + \frac{(62.88 - 60.89)\,10^6}{(67.61 - 60.89)\,10^6} \times 100 \text{ ft} = 1{,}930 \text{ ft}$$

This information completes the curve for 50 years in Fig. 1.

To locate the gas-oil contact for a pool life of 70 years a trial and error procedure was used. Between pool lives of 27 and 50 years, the



GRAVITY DRAINAGE performance of a southern Oklahoma reservoir. Fig. 2.

gas-oil contact moved (1,930 − 1,312 = 618 ft) 618 ft. By extrapolation of this trend it will take 2.6 years more.

Time for gas-oil contact to reach depth of 2,000 ft =

$$\frac{(2{,}000 - 1{,}930)}{618 \text{ ft}/23 \text{ years}} + 50.00 = 52.6 \text{ years}$$

for the gas-oil contact to reach a depth of 2,000 ft below the top of the initial oil column. Below this point the oil producing capacity is assumed equal to that fraction of 7,000 st tk bbl/day given by the ratio of the pore volume of the remaining oil column to the pore volume of the oil column below a depth of 2,000 ft. The position of the gas-oil contact is assumed and an average oil producing rate is computed between this position and 2,000 ft. Oil recovery for the 20-year time increment is then computed, and the position of the gas-oil contact is found as shown in Table 2. Assumed and computed gas-oil-contact positions should check, otherwise another position is assumed and the process repeated.

DISCUSSION: In this problem the maximum rate at which oil can flow downdip limits the rate of oil displacement at the gas-oil contact and the rate of movement of the gas-oil contact. Since the residual-oil saturation in the gas cap is a direct function of time, conditions causing rapid movement of the gas-oil contact down dip result in high residual oil saturation and low oil recovery at gas breakthrough down structure.[3] Conversely, when the rate of oil displacement at the gas-oil contact is relatively small compared to the gravity drainage from the gas cap, a well-defined gas-oil contact or stabilized zone will exist.

Procedures in this and the preceding problem are applicable only when reservoir pressure is essentially maintained through gas-injection. Pressure differences in the oil column are due to gravity.

Calculations for gravity drainage where reservoir pressure declines are more complex and have not yet been solved. A method developed independently and similar to that used in this problem has been presented by Shreve and Welch.[4]

These authors suggest dividing the reservoir into a series of assumed isotropic segments of different volumes. Such a procedure eliminates trial-and-error solution but introduces saturation discontinuities across the segment boundaries.

The procedure used in this problem for computing the average oil-producing rate when the gas-oil contact dropped below a depth of 2,000 ft from the top of the initial oil column did not take into account variations in oil saturation.

Other methods could be used to compute this oil-production rate. A pool map showing structure and well locations could be used most effectively since for any assumed position of the gas-oil contact, the average oil-producing rate could readily be determined by summing the rates of the producing wells.

The predicted performance of the pool under study is shown in Figs. 1 and 2. Fig. 1 gives the saturation-time-depth profiles for five time increments. These profiles show the advancement of oil-saturation planes with time, and the changes of oil saturation with time and position. Note that the water saturation increases with depth, as was indicated by Fig. 1 of the preceding problem.

The gas saturation remains constant after 18.5 years since resaturation of the oil column to an equilibrium gas saturation of 4% occurred about this time. Fig. 2 was plotted with data obtained from Table 2, and calculations for other time increments. Oil-production rate and performance curves shown are not complete. Similar calculations are needed, to abandonment conditions, to obtain the entire performance curves.

References

1. Essley, P. L., Jr., Hancock, G. L. Jr., and Jones, K. E., "Gravity-Drainage Concepts in a Steeply Dipping Reservoir": Paper, SPE of AIME, petroleum conference on production and reservoir engineering, Tulsa, Mar. 20-21, 1958, Paper 1029-G, 11 pp.

2. Guerrero, E. T.: Part 53 Reservoir Engineering: The Oil and Gas Journal, Dec. 17, 1962 p. p. 103.

3. Terwilliger, P. L., Wilsey, L. E., Hall, H. N., Bridges, P. M., and Morse, R. A., "An Experimental and Theoretical Investigation of Gravity Drainage Performance": Trans. AIME v. 192, p. 285, 1951.

4. Shreve, D. R., and Welch, L. W., Jr., "Gas Drive and Gravity-Drainage Analysis for Pressure-Maintenance Operations": Trans. AIME v. 207, p. 136, 1956.

Acknowledgment

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Part 55

How to predict performance of a gravity-drainage reservoir in the stripper stage

GIVEN: Ten years of past performance for a lease (columns 1 and 2 of Table 1) that has been producing during this time in the stripper stage by gravity drainage.

FIND: Performance (remaining life and reserves) of the lease if abandonment occurs at an oil-production rate of 200 bbl/month.

METHOD OF SOLUTION: Equations recommended by Lefkovits and Matthews[1] will be used in the solution of this problem. These investigators found that if lease life to abandonment is 25 years or less and n is less than 1.0 or if lease life is greater than 25 years and n is greater than 1.0, suitable predictions can be made with

$$\frac{Q_o}{Q_{oi}} = \frac{1}{(1 + \gamma t)^n} \qquad (1)$$

$$N_p = \frac{12\,Q_{oi}}{\gamma(1 - n)} \times [(1 + \gamma t)^{1-n} - 1] \qquad (2)$$

If lease life to abandonment is greater than 25 years and n is less than 1.0, better predictions are obtained with

Table 1—Past performance and computations

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
X = t = time, years	Avg. rate of oil production, Q_o bbl/month	Q_{oi}/Q_o 1,000 ÷ (2)	$Y_{observed} = (Q_{oi}/Q_o)^{1/n} = (3)^{1/n}$				
			n = 0.50	n = 0.75	n = 1.00	n = 1.25	n = 0.9
0	1,000	1.000	1.000	1.000	1.000	1.000	1.000
1	900	1.111	1.234	1.151	1.111	1.088	1.124
2	805	1.242	1.543	1.336	1.242	1.189	1.272
3	735	1.361	1.852	1.509	1.361	1.280	1.408
4	680	1.471	2.164	1.674	1.471	1.362	1.536
5	638	1.567	2.455	1.840	1.567	1.433	1.647
6	598	1.672	2.796	1.986	1.672	1.509	1.771
7	560	1.786	3.190	2.168	1.786	1.591	1.905
8	528	1.894	3.587	2.344	1.894	1.667	2.034
9	498	2.008	4.032	2.534	2.008	1.745	2.168
10	470	2.128	4.528	2.738	2.128	1.830	2.313
55			28.381	20.280	17.240	15.694	18.178

(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
Y Computed				$[Y_{observed} - Y_{computed}] \times 10^3$			
n = 0.75 Eq. 8	n = 0.90 Eq. 9	n = 1.00 Eq. 10	n = 1.25 Eq. 11	n = 0.75 (5) − (15)	n = 0.90 (8) − (16)	n = 1.00 (6) − (17)	n = 1.25 (7) − (18)
0.986	1.005	1.011	1.019	14	−5	−11	−19
1.157	1.134	1.122	1.100	−6	−10	−11	−12
1.329	1.264	1.234	1.182	7	8	8	7
1.500	1.393	1.345	1.263	9	15	16	17
1.672	1.523	1.456	1.345	2	13	15	17
1.844	1.653	1.567	1.427	−4	−6	0	6
2.015	1.782	1.678	1.508	−29	−11	−6	1
2.187	1.912	1.790	1.590	−19	−7	−4	1
2.358	2.041	1.901	1.671	−14	−7	−7	−4
2.530	2.171	2.012	1.753	4	−3	−4	−8
2.702	2.301	2.123	1.835	36	12	5	−5

$$\frac{Q_o}{Q_{oi}} = \frac{1}{1 + bc} \times \left[\frac{1}{\left(1 + \frac{\gamma t}{d}\right)^2} + \frac{bc}{\left(1 + \frac{c\gamma t}{d}\right)^2}\right] \quad (3)$$

$$\frac{N_p}{Q_{oi}t} = \frac{12}{1 + bc} \times \left(\frac{1}{1 + \frac{\gamma t}{d}} + \frac{bc}{1 + \frac{c\gamma t}{d}}\right) \quad (4)$$

Where:

Q_o = oil-production rate at time t, bbl/month

Q_{oi} = initial or oil production rate at zero time, bbl/month

γ = a constant in years^{-1}

n = exponent

t = time, years

b, c, d = constants in Equations 3 and 4 (obtained from Table 1 of Reference 1)

N_p = oil recovery at time t, stock-tank barrels

METHOD OF SOLUTION: To solve this problem, first compute values for γ and n based on past history, using Equation 1 which can be written as

$$(Q_{oi}/Q_o)^{1/n} = 1 + \gamma t \quad (5)$$

This form of the equation indicates that the correct values of n and γ give a linear plot when $(Q_{oi}/Q_o)^{1/n}$ is plotted versus t. The slope of such a plot is γ. Table 1 gives most of the computation steps involved in the determination of n and γ. These steps are: .

1. Compute $(Q_{oi}/Q_o)^{1/n}$ with time for various values of n; 0.50, 0.75, 0.90, 1.00, and 1.25 in this case as shown in columns 4 through 8 of Table 1.

2. Plot $(Q_{oi}/Q_o)^{1/n}$ versus time for each value of n (Fig. 1) and select the cases that appear to be linear.

3. For the cases selected develop the equation of each linear plot using the least-squares method. This gives the values of γ for each case. In this problem approximately linear plots were obtained for n = 0.75, 0.90, 1.00, and 1.25. Columns 9 through 14 in Table 1 show calculations needed for the least-squares method.

4. Compute the standard deviation for each case selected using the observed values of $(Q_{oi}/Q)^{1/n}$ and corresponding values of $(Q_{oi}/Q_o)^{1/n}$ computed with the equations developed. These calculations are shown in columns 15 through 26 of Table 1.

5. Plot standard deviation and γ versus n, Fig. 2, to obtain the best values of n and γ at the lowest standard deviation.

6. The values of n and γ obtained in step 5 and the initial and abandonment oil-production rates are used in Equation 1 to compute total lease life.

7. If total lease life to abandonment as computed with Equation 1 is 25 years or less and n is less than 1.0 or if lease life is greater than 25 years and n is greater than 1.0, Equation 2 is used to compute the lease reserves.

8. If total lease life to abandonment as computed with Equation 1 is greater than 25 years and n is less than 1.0, Equations 3 and 4 are used to obtain better predictions. The constants b, c, and d are obtained by interpolation, if necessary, from Table 1 of Reference 1 for the n determined.

In step 3, γ (slope) and the ordinate intercept of the linear plots are computed with[2]

$$\gamma = \frac{z\Sigma XY - \Sigma X \Sigma Y}{z\Sigma X^2 - (\Sigma X)^2} \quad (6)$$

$$f = \frac{\Sigma X^2 \Sigma Y - \Sigma X \Sigma XY}{z\Sigma X^2 - (\Sigma X)^2} \quad (7)$$

Where:

γ = slope

z = number of data points

X = t = time

Y = $(Q_{oi}/Q_o)^{1/n}$

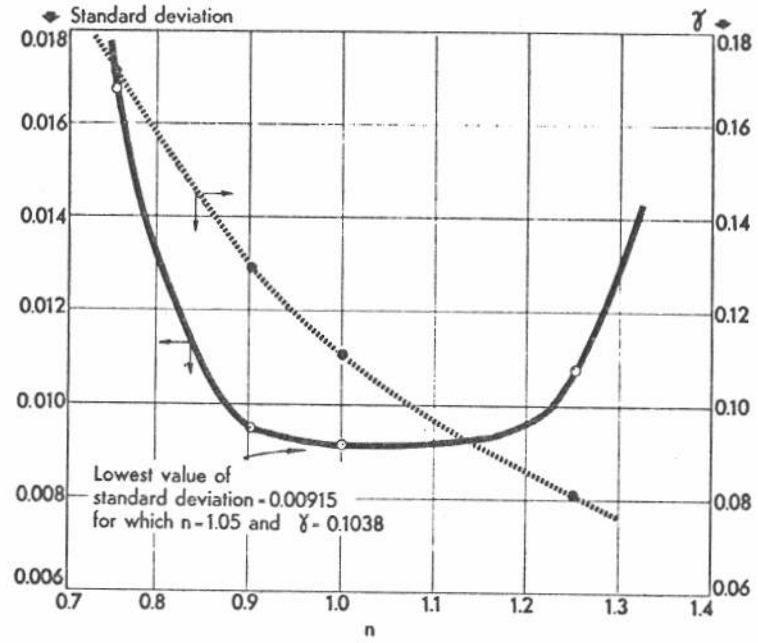
for factors n and γ

(9)	(10)	(11)	(12)	(13)	(14)
	XY				
X² (1)²	n = 0.50 (1) × (4)	n = 0.75 (1) × (5)	n = 1.00 (1) × (6)	n = 1.25 (1) × (7)	n = 0.90 (1) × (8)
..	......	......	.....	.....	......
1	1.234	1.151	1.111	1.088	1.124
4	3.086	2.672	2.484	2.378	2.544
9	5.556	4.527	4.083	3.840	4.224
16	8.656	6.696	5.884	5.448	6.144
25	12.275	9.200	7.835	7.165	8.235
36	16.776	11.916	10.032	9.054	10.626
49	22.330	15.176	12.502	11.137	13.335
64	28.696	18.752	15.152	13.336	16.272
81	36.288	22.806	18.072	15.705	19.512
00	45.280	27.380	21.280	18.300	23.130
85	180.177	120.276	98.435	87.451	105.146

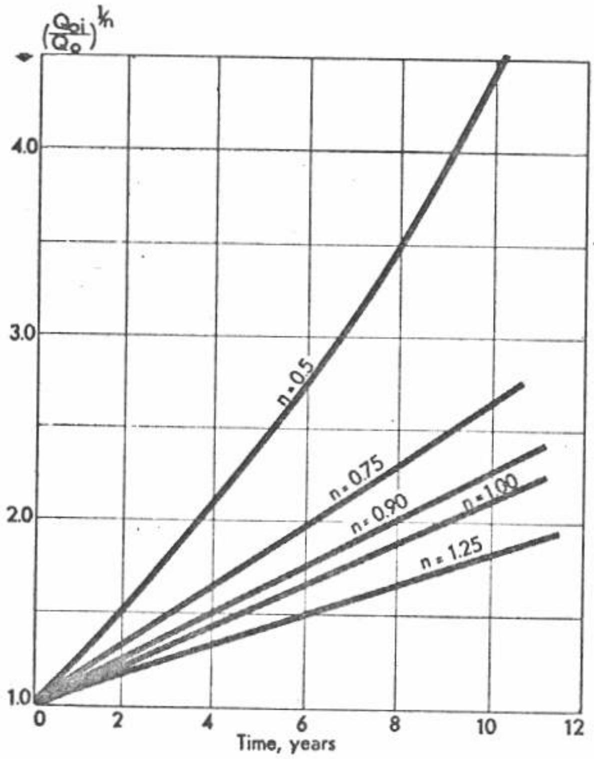
(23)	(24)	(25)	(26)	(27)	(28)
$(Y_{observed} - Y_{computed})^2 \times 10^6$					
n = 0.75 (19)²	n = 0.90 (20)²	n = 1.00 (21)²	n = 1.25 (22)²	ΔN_p	N_p
196	25	121	361	.....	......
36	100	121	144	11,400	11,400
49	64	64	49	10,230	21,630
81	225	256	289	9,240	30,870
4	169	225	289	8,490	39,360
16	36	0	36	7,308	46,668
841	121	36	1	7,416	54,084
361	49	16	1	6,948	61,032
196	49	49	16	6,528	67,560
16	9	16	64	6,156	73,716
1,296	144	25	25	5,808	79,524
3,092	991	929	1,275	79,524	

f = ordinate intercept which actually is 1.0000 but must be computed in the least-squares method.

For n = 0.90 and using the data of columns 1, 8, 9, and 14 of Table 1



VARIATION of standard deviation and factor γ with n. Fig. 2.



PLOT to determine behavior of $(Q_{oi}/Q_o)^{1/n}$ with time in the evaluation of n and γ. Fig. 1.

$$\gamma = \frac{(11)(105.146) - (55)(18.178)}{(11)(385) - (55)^2}$$

$$= 0.1296$$

$$f = \frac{(385)(18.178) - (55)(105.146)}{(11)(385) - (55)^2}$$

$$= 1.0045$$

The equations obtained were:

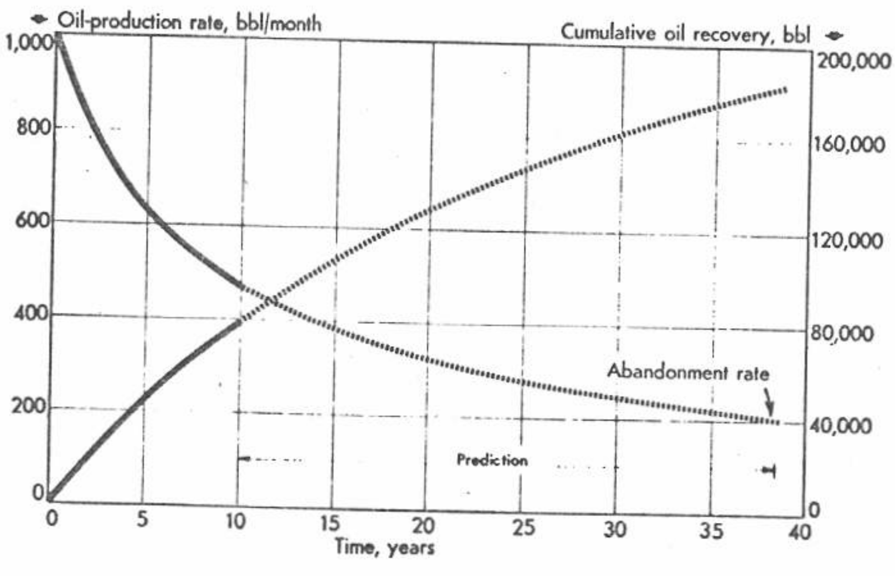
$$(Q_{oi}/Q_o)^{1/0.75} = 0.9856 + 0.1716\ t \quad (8)$$

$$(Q_{oi}/Q_o)^{1/0.90} = 1.0045 + 0.1296\ t \quad (9)$$

$$(Q_{oi}/Q_o) = 1.0111 + 0.1112\ t \quad (10)$$

$$(Q_{oi}/Q_o)^{1/1.25} = 1.0185 + 0.0816\ t \quad (11)$$

These equations were used in the computations shown in columns 15, 16, 17, and 18 of Table 1. Standard deviations (actually standard errors of estimate[2]) σ_γ are computed with



LEASE PERFORMANCE predicted on the basis of Equations 3 and 4. Fig. 3.

$$\sigma_y = \sqrt{\frac{\Sigma\,[(Q_{oi}/Q_o)^{1/n}{}_{observed} - (Q_{oi}/Q_o)^{1/n}{}_{computed}]^2}{z}} \qquad (12)$$

for n = 0.9

$$\sigma_y = \sqrt{\frac{0.000991}{11}} = 0.00949$$

Similarly

$\sigma_y = 0.0168$ for $n = 0.75$

$= 0.00919$ for $n = 1.00$

$= 0.0108$ for $n = 1.25$

These values and the slopes (γ) of Equations 8, 9, 10, and 11 are plotted versus n in Fig. 2. This plot shows the lowest standard deviation to be 0.00915 for which $n = 1.05$ and $\gamma = 0.1038$ year^{-1}. Thus solving Equation 1 for the abandonment and initial oil-production rates of 300 bbl/month and 1,000 bbl/month gives

$$\frac{200}{1{,}000} = \frac{1}{(1.000 + 0.1038\,t)^{1.05}}$$

$$t = \frac{(1{,}000/200)^{1/1.05} - 1.000}{0.1038}$$

$$= 35.0 \text{ years}$$

or the remaining lease life = 35.0 − 10.0 = 25.0 years.

Solving Equation 2 gives

$$N_p = \frac{(12)(1{,}000)}{(0.1038)(1.00 - 1.05)}\{[1.0000 + (0.1038)(35.0)]^{1.00-1.05} - 1.0000\}$$

$$= -\frac{12{,}000}{0.00519}[(4.633)^{-0.05} - 1.0000] = 170{,}174 \text{ bbl}$$

or the remaining lease reserves are

$$= 170{,}174 - 79{,}524 = 90{,}650 \text{ bbl}$$

The value of 79,524 bbl (column 27 of Table 1) represents cumulative oil recovery at the end of 10 years. This value was obtained by using average oil-production rates for each year of history (i.e., for the first year recovery = [(1,000 + 900) ÷ 2] 12 = 11,400 bbl).

Since the lease life to abandonment as computed with Equation 1 is greater than 25 years and n is about 1.0, Equations 3 and 4 are also used in this problem to compare predictions. From Table 1 of Reference 1 for n = 1.05 and by interpolation b = 0.325, c = 6.65, and d = 9.25. Thus

$$\frac{200}{1{,}000} = \frac{1}{1.000 + (0.325)(6.65)} \times \left\{\frac{1}{[1.000 + \frac{(0.1038)\,t}{9.25}]^2} + \frac{(0.325)(6.65)}{[1.000 + \frac{(6.65)(0.1038)\,t}{9.25}]^2}\right\}$$

$$0.200 = \frac{1}{3.161}\left[\frac{1}{(1.000 + 0.0112\,t)^2} + \frac{2.161}{(1.000 + 0.0746\,t)^2}\right]$$

$$0.6322 = \frac{1}{(1.000 + 0.0112\,t)^2} + \frac{2.161}{(1.000 + 0.0746\,t)^2}$$

Trial-and-error solution of this question shows that for t = 38.5 years

$$0.6322 = \frac{1}{(1.4312)^2} + \frac{2.161}{(3.8721)^2} = 0.6324$$

The remaining life is 38.5 − 10.0 = 28.5 years.

With Equation 4

$$N_p = \frac{(1{,}000)(12)(38.5)}{1.000 + (0.325)(6.65)}\left[\frac{1}{1.000 + (0.1038)(38.5)/9.25} + \frac{(0.325)(6.65)}{1.000 + (6.65)(0.1038)(38.5)/9.25}\right]$$

$$= 183{,}616 \text{ bbl}$$

The remaining reserves are

$$= 183{,}616 - 79{,}524 = 104{,}092 \text{ bbl or } 104{,}000 \text{ bbl}$$

In this particular case the performance of the lease is obtained by substituting selected values of t (i.e.. 15, 20, 25, 30, 35 years) into either Equations 1 and 2 or Equations 3 and 4. The results obtained using Equations 3 and 4 are shown in Fig. 3.

At t = 25 years.

$$Q_o = \frac{1{,}000}{1.000 + (0.325)(6.65)}\left[\frac{1}{(1.000 + (0.1038)(25)/9.25)^2} + \frac{(0.325)(6.65)}{(1.000 + (6.65)(0.1038)(25)/9.25)^2}\right] = 276 \text{ bbl/month}$$

and

$$N_p = \frac{(12)(1{,}000)(25)}{1.000 + (0.325)(6.65)}\left[\frac{1}{1.000 + (0.1038)(25)/9.25} + \frac{(0.325)(6.25)}{1.000 + (6.65)(0.1038)(25)/9.25}\right] = 145{,}740 \text{ bbl}$$

DISCUSSION: Experimental work with models and theoretical considerations have shown that wells having a free surface (contain gas-oil contact) in a homogeneous reservoir producing by pure gravity drainage (reservoir pressure is depleted) have a performance that follows a hyperbolic decline with the exponential factor n = 2.[3] Lefkovits and Matthews performed this initial work and later extended the ideal homogeneous reservoir behavior to actual field cases.[1]

Both the initial and extended procedures assume that the reservoir pressure has been depleted and that oil moves to the well bores by gravity drainage. In the study of actual field cases, it was found that the decline was hyperbolic in many cases, but that the exponent n was normally not equal to 2 and often was less than 1. For leases and reservoirs the exponent is affected by the number of wells containing and not containing free-surface layers and the uniformity of the pay sand.

Performance of leases can be predicted for time periods up to 25 years using a hyperbolic decline (Equations 1 and 2) where the exponent n is based on past performance of about 7 years or more. This procedure can be satisfactory for time periods greater than 25 years when the exponent n given by past performance is greater than 1.

When the exponent n is less than 1, the time to the economic limit may be much greater than 50 years and better predictions are obtained by using the best-fitting two-layer curve (Equations 3 and 4).[1] In this problem, n = 1.05, and nearly similar results were obtained by both the hyperbolic and best-fitting two-layer procedures. For both the hyperbolic and best-fitting two-layer methods, the procedure for computing γ and n is the same.

The performance and ultimate recovery of a reservoir declining hyperbolically have been determined by using the loss ratio and graphical methods.[4] Although more tedious, the methods used in this problem (Equations 1, 2, 3, and 4) are considered more rigorous and reliable. The disadvantage of many trial-and-error calculations can be eliminated when a computer is used.

References

1. Lefkovits, H. C., and Matthews, C. S., "Application of Decline Curves to Gravity-Drainage Reservoirs in the Stripper Stage" Trans. AIME Vol. 213, 1958, pp. 275-280.
2. Richardson, C. H., "An Introduction to Statistical Analysis": revised edition, Harcourt, Brace & Co. 1944, p. 218.
3. Matthews, C. S., and Lefkovits, H. C., "Gravity-Drainage Performance of Depletion-type Reservoirs": Trans, AIME Vol. 207, 1956, p. 265.
4. Guerrero, E. T., Reservoir Engineering Part 38, "Performance and Ultimate Recovery of a Reservoir Declining Hyperbolically": The Oil and Gas Journal, Vol. 59, No. 29, July 17, 1961, pp. 94-96.

Table 1—Calculations for

(1)	(2)	(3)	(4)	(5)	(6)
Pressure, p psia	B_o	R_s scf/st-tk bbl	B_g Res. bbl/scf	μ_o cp	ϕ_o bbl/bbl
900	1.161	334	0.00262	1.35	0.657
700	1.147	278	0.00344	1.50	0.242
500	1.117	220	0.00495	1.80	0 021
300	1.093	160	0.00860	2.28	−0.092
100	1.058	84	0.02720	3.22	−0.100
(13)	(14)	(15)	(16)	(17)	(18)
ϕ_o + (12) (6) + (12)	$N_{p(n-1)}\ \phi_o$ $(18)_{n-1} \times (6)$	$G_{p(n-1)}\ \phi_g$ $(33)_{n-1} \times (7)$	1 − (14) − (15)	ΔN_p (16) ÷ (13)	$N_p = N_{p(n-1)} + (17)$ Fraction of N
......	......	......	......	......	0.1347
4.7843	0.0326	0.6638	0.3036	0.0635	0.1982
5.5065	0.0042	0.7439	0.2519	0.0457	0.2439
5.6348	−0.0224	0.8032	0.2192	0.0389	0.2828
4.2708	−0.0283	0.8245	0.2038	0.0477	0.3305
(25)	(26)	(27)	(28)	(29)	(30)
$S_{Le} = S_w + (24) = 0.25 + (24)$	$(k_g/k_o)_e$ from Fig. 1 of Ref. 1	$\mu_o/\mu_g =$ (5) ÷ 0.02	$(k_g/k_o)_e\ (\mu_o/\mu_g) \times (B_o/B_g) =$ (26) × (27) × (8)	$R_{Ie} = R_s + (28) =$ (3) + (28)	Computed $R_{avg.}$ $[(29)_{n-1}+(29)_n] \div 2$
......	......	67.5	......	1,900	......
0.775	0.118	75.0	2,947	3,225	2,563
0.722	0.225	90.0	4,577	4,797	4,011
0.631	0.375	114.0	5,429	5,589	5,193
0.634	0.670	161.0	4,207	4,291	4,940

Part 56

How to find dispersed gas-drive performance taking conformance into account

GIVEN: Production history, rock and fluid properties, and depletion-drive performance as shown and computed earlier[1] (OGJ, Nov. 27, 1961, p. 106, and Practical Reservoir Engineering, Vol. 2, p. 26). Data from this problem are shown in columns 1-7 of table 1. Other data are:

1. Gas viscosity (μ_g) at reservoir conditions remains about 0.02 cp for pressures between the bubble point and 100 psia.
2. Initial oil formation - volume factor, B_{oi} = 1.265.
3. Interstitial water saturation, S_w = 25.0% of pore vol.

FIND: The dispersed gas-drive performance for the reservoir if 60% of the produced gas is reinjected after reservoir pressure has declined to 900 psia. Assume a conformance factor of 0.5.

SOLUTION: The Tracy form of the material-balance equation for conditions of no water drive and dispersed gas injection is

$$N = [N_{p(n-1)} + \Delta N_p^* (1-e)]\,\phi_o + G_p\,\phi_g \qquad (1)$$

Where:

N = initial oil in place, st-tk bbl

$N_{p(n-1)}$ = cumulative oil production to beginning of pressure decrement, st-tk bbl

ΔN_p^* = depletion oil recovery for pressure decrement, Δp, st-tk bbl

e = conformance factor

ϕ_o = oil pressure factor (see Reference 1)

G_p = cumulative gas production, scf

ϕ_g = gas-pressure factor, bbl/scf

[d]ispersed gas-drive performance

(7)	(8)	(9)	(10)	(11)	(12)
ϕ_g bbl/scf	B_o/B_g (2) ÷ (4)	$R_{I\,est.}$	$\frac{R_{I(n-1)}+R_{Ie}}{2}$ $[(9)_{n-1}+(9)_n]\div 2$	$\left(\frac{R_{I(n-1)}+R_{Ie}}{2}\right)\phi_g$ (10) × (7)	(1 − I) (11) (0.4) (11)
0.00602	443	1,900	.....	.....	......
0.00441	333	3,250	2,575	11.3558	4.5423
0.00345	226	4,700	3,975	13.7138	5.4855
0.00278	127	5,600	5,150	14.3170	5.7268
0.00223	39	4,200	4,900	10.9270	4.3708

(19)	(20)	(21)	(22)	(23)	(24)
N_p^* fraction of N from Ref. 1	$N_p^*(1-e)$ (19) × (0.5)	$N_p - N_p^*(1-e)$ (18) − (20)	$B_o/e\,B_{oi}$ (2) ÷ (0.5) (1.265)	e − (21) 0.5 − (21)	$(1-S_w)$ (23) (22) (0.75) (23) (22)
0.1347	......	......	1.836	.....	......
0.1690	0.0845	0.1137	1.813	0.3863	0.525
0.2005	0.1003	0.1436	1.766	0.3564	0.472
0.2307	0.1154	0.1674	1.728	0.3326	0.431
0.2726	0.1363	1.1942	1.673	0.3058	0.384

(31)	(32)	(33)	(34)	(35)	(36)
ΔG_p (17) × (30)	$\Delta G_p (1-I) =$ (31) × (0.4)	$G_p = G_{p(n-1)} + (32)$ multiples of N	$G_p\,\phi_g =$ (33) × (7)	$N_p\,\phi_o$ (18) × (6)	N = 1.000 (34) + (35)
......	......	.150.516	......	......	......
162.751	65.100	215.616	0.9509	0.0480	0.9989
183.303	73.321	288.937	0.9968	0.0051	1.0019
202.008	80.803	369.740	1.0279	−0.0260	1.0019
235.638	94.255	463.995	1.0347	−0.0331	1.0016

In this problem the conformable portion of the reservoir is that fraction of the net effective pore space, e, contacted or flushed by the injected gas. Here performance is determined by a combination of depletion and gas-drive energies. The nonconformable portion of the reservoir is that fraction of the net effective pore space, $1 - e$, that is not contacted or flushed by the injected gas. Here a straight depletion-drive performance occurs. It is assumed in the solution that oil from the nonconformable portion flows into the conformable portion and thence to the producing wells. Oil from the nonconformable portion of the reservoir does not flow directly to the producing wells.

In Equation 1

$$G_p = G_{p(n-1)} + \Delta G_p (1 - I) \quad (2)$$

and when production occurs into the boreholes only from the conformable portion of the reservoir

$$\Delta G_p = \frac{R_{I(n-1)} + R_{Ie}}{2} \times [\Delta N_p^* (1 - e) + \Delta N_{pe}] \quad (3)$$

Thus

$$\Delta N_p = \Delta N_{pe} + \Delta N_p^* (1 - e) = \frac{1 - N_{p(n-1)} \phi_o - G_{p(n-1)} \phi_g}{\phi_o + (1 - I)\left(\frac{R_{I(n-1)} + R_{Ie}}{2}\right) \phi_g} \quad (4)$$

In Equation 4, $N = 1.0$ and ΔN_p, ΔN_{pe}, ΔN_p^*, $N_{p(n-1)}$ and G_p are expressed as fractions or multiples of N. Also

$p_{(n-1)} - p_n$ = reservoir pressure decrement, psi

$p_{(n-1)}$ = beginning of pressure decrement, psi

p_n = end of pressure decrement, psi

ΔN_p = oil recovery in pressure decrement, $p_{(n-1)} - p_n$

ΔN_{pe} = oil recovery from conformable portion of rock in pressure decrement, $p_{(n-1)} - p_n$

I = fraction of produced gas reinjected

$R_{I(n-1)}$ = instantaneous gas-oil ratio at $p_{(n-1)}$, scf/st-tk bbl

R_{Ie} = instantaneous gas-oil ratio at p_n, scf/st-tk bbl

Equations 1 and 4 are used in the solution of this problem along with the total liquid saturation and instantaneous gas-oil ratio equations.

$$S_{Le} = S_w + \frac{(1 - S_w)\{e - [N_p - N_p^* (1-e)]\} B_o}{e B_{oi}} \quad (5)$$

$$R_{Ie} = R_s + (k_g/k_o)_e(\mu_o/\mu_g)(B_o/B_g) \quad (6)$$

Where:

S_{Le} = total liquid saturation in conformable portion of reservoir at p_n

S_w = interstitial water saturation, a constant

N_p = cumulative oil production from conformable and nonconformable portions of reservoir at p_n, fraction of N

N_p^* = cumulative oil production from reservoir by straight depletion (primary) drive at p_n, fraction of N

B_{oi}, B_o = initial and current (at pressure p_n) oil formation-volume factors

R_s = solution gas-oil ratio at p_n, scf/st-tk bbl

$(k_g/k_o)_e$ = permeability ratio for conformable portion of reservoir

μ_o, μ_g = oil and gas viscosities at p_n, centipoises

B_g = gas formation volume factor at p_n, bbl/scf

Finally Equation 1 can be written for conditions at the end of the pressure decrement as

$$1.0 = N_p \phi_o + G_p \phi_g \quad (7)$$

Where:

$N_p = N_{pe} + N_p^* (1 - e)$, and

N_{pe} = cumulative oil recovery at p_n from conformable portion of reservoir, fraction of N

SOLUTION: This problem is solved by trial-and-error solution of Equations 4, 5, 6, and 7. In these equations I, S_w, e, N, and B_{oi} are constants. For calculations performed for a pressure decrement, $(p_{(n-1)} - p_n)$, $N_{p(n-1)}$, $G_{p(n-1)}$ and $R_{I(n-1)}$ refer to the pressure at the beginning of the decrement, $p_{(n-1)}$, while the remaining terms that are functions of pressure refer to p_n.

A recommended solution procedure is as follows:

1. Equation 4 contains two unknowns, ΔN_p and R_{Ie}; estimate R_{Ie} and solve for ΔN_p.
2. Solve for $N_p = N_{p(n-1)} + \Delta N_p$.
3. Solve Equation 5 for S_{Le} and with this S_{Le} read $(k_g/k_o)_e$ from Fig. 1 of Reference 1.
4. Solve Equation 6 for R_{Ie}. This instantaneous gas-oil ratio value should check with the value estimated in step 1 (within 50 to 100 scf/st-tk bbl depending on the magnitude of R_{Ie}). If it does not, repeat steps 1, 2, and 3, using the computed R_{Ie} for the next estimated gas-oil ratio.
5. The trial-and-error calculations are continued until R_{Ie}, computed, is about equal to R_{Ie}, estimated.
6. As a final check at each pressure, solve Equation 7, the right-hand side of which must give a value between 0.991 and 1.009. This will be obtained at each pressure if R_{Ie}, computed, and R_{Ie}, estimated, are about equal.

For a pressure of 500 psia (pressure decrement from 700 to 500 psia) assume R_{Ie} = 4,700 scf/st-tk bbl and

$$\Delta N_p = \left[\frac{1.000 - (0.1982)(0.021) - (215.616)(0.00345)}{0.021 + (1.00 - 0.60)\left(\frac{3,250 + 4,700}{2}\right)(0.00345)}\right]$$

$$= \frac{1.000 - 0.0042 - 0.7439}{0.021 + (0.40)(3,975)(0.00345)}$$

$$= \frac{0.2519}{0.021 + 5.4855}$$

$$= \frac{0.2519}{5.5065} = 0.0457$$

$$N_p = 0.1982 + 0.0457 = 0.2439$$

$$S_{Le} = 0.250 + (1.00 - 0.25)\,\{0.5000 - [0.2439 - 0.2005\,(1.0000 - 0.5000)]\}\,(1.117)$$

$$= 0.250 + \frac{(0.75)\,[0.5000 - 0.1436]\,1.117\ \text{(0.5000) (1.265)}}{0.6325}$$

$$= 0.250 + \frac{(0.75)\,(0.3981)}{0.6325} = 0.250 + 0.472 = 0.722$$

From Fig. 1 of Reference 1, k_g/k_o for $S_{Le} = 0.722$ is 0.225

$$R_{Ie} = 220 + (0.225)\,(1.80/0.02) \times (1.117/0.00495)$$
$$= 220 + (0.225)\,(90)\,(226)$$
$$= 220 + 4{,}577 = 4{,}797$$

Since the computed R_{Ie} is close to the estimated value (4,797 compared to 4,700), the final check is made

$$N = 1.000 = (0.2439)\,(0.021) + (288.937)\,(0.00345)$$
$$= 0.0051 + 0.9968$$
$$= 1.0019$$

This checks within the accuracy desired of 0.991 to 1.009 for the right-hand side of Equation 7.

DISCUSSION: Procedure used to solve this problem assumes that the injected gas disperses evenly throughout the conformable portion of the oil column and that no frontal displacement effect is present. This is equivalent to assuming that the portion of the produced gas, equivalent to the injected gas, (I), was never produced.

When the entire oil column is believed contacted (e = 1.0), then Equation 4 reduces to the more familiar form[2] of dispersed gas-drive equations. The writer believes that in most reservoirs injected gas will not contact all the rock because of location of injected wells, heterogeneous rock properties, and structural characteristics of the formation.

Two major difficulties in applying the method used in this problem are (1) determination of the value of e and (2) determination of whether the nonconformable portion of the formation is producing directly into the well bores or into the conformable portion. In this problem the latter conditions were assumed. If reservoir conditions were such that both the conformable and nonconformable portions of the formation predominantly produced into the well bores, then Equation 4 would need to be modified to

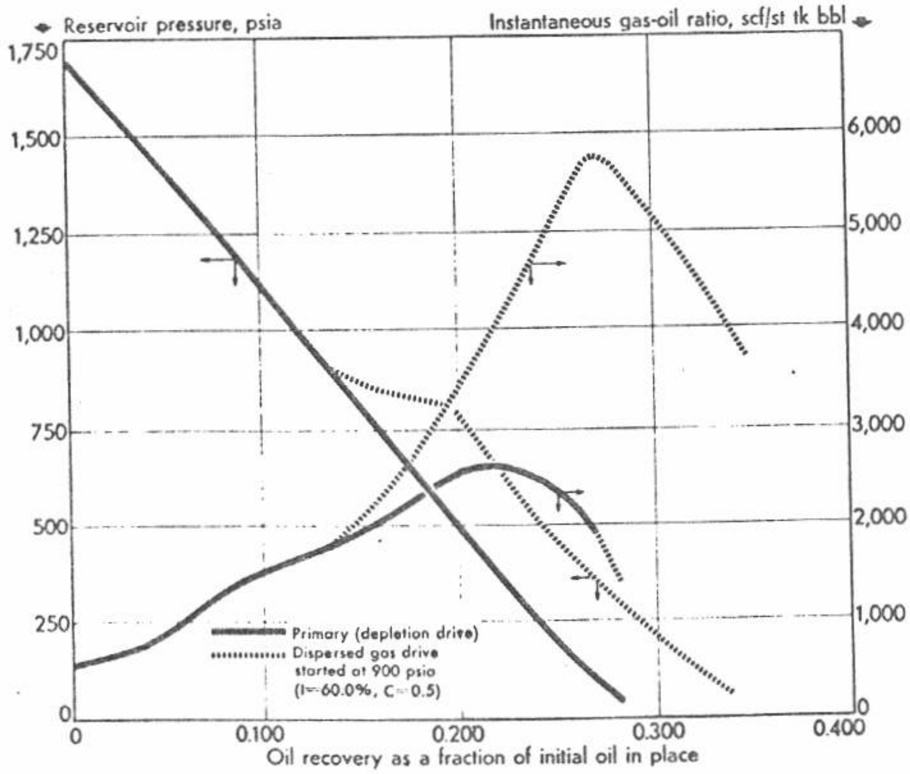
$$\Delta N_{pe} = \frac{1 - \phi_o N_{p(n-1)} - \phi_g G_{p(n-1)}}{\phi_o + (1 - I)\left(\frac{R_{I(n-1)} + R_{In}}{2}\right)_e \phi_g} - \frac{\Delta N_p^{*}\,(1 - e)\left[\phi_o + \phi_g\,(1 - I)\left(\frac{R_{I(n-1)} + R_{In}}{2}\right)_d\right]}{\phi_o + (1 - I)\left(\frac{R_{I(n-1)} + R_{In}}{2}\right)_e \phi_g} \qquad (8)$$

where the subscripts d and e refer to the nonconformable (depletion drive) and conformable portions respectively.

It is difficult to state that any particular reservoir will entirely conform to the conditions of either Equation 4 or Equation 8. The writer believes that most reservoirs would more closely follow the conditions of Equation 4; the nonconformable portion produces into the conformable and all production into the well bores comes through the latter.

This method is believed to be particularly applicable in reservoirs containing 10% or more gas saturation. Where the free gas saturation is below 5%, frontal advance may take place and this method would predict a conservative recovery. Under such conditions the conformance factor increases with time and approaches a stabilized value.

There is no reliable method available for finding the conformance factor, e. It must be estimated from core analysis, logs, and other sources or by solving Equation 6 for $(k_g/k_o)_e$, using field test data to obtain R_{Ie}. Having a value for $(k_g/k_o)_e$,



COMPARATIVE PERFORMANCES of a depletion-type pool under depletion drive and under dispersed-gas drive. I = 60.0%, e = 0.5. Fig. 1.

S_{Le} can be read from the k_g/k_o curve and Equation 3 can be solved for e.

In Table 1, columns 9 to 17 solve Equation 4, 18 to 25 solve Equation 5, 26 to 29 solve Equation 6, and 30 to 36 solve Equation 7. Calculations are required at each pressure. The final results are plotted on Fig. 1 and compared with the depletion-drive performance obtained from Reference 1.

References

1. Guerrero, E. T., "Reservoir Engineering, Part 42—How to Find Performance and Ultimate Oil Recovery of a Depletion-Type Pool Using the Tracy Material Balance Approach": The Oil and Gas Journal, Vol. 59, No. 48, pp. 106-110, Nov. 27, 1961.

2. Pirson, S. J., "Oil Reservoir Engineering": second edition, Chap. 10, McGraw-Hill Publishing Co., 1958.

Part 57

How to determine pressure distribution for steady-state and semisteady-state conditions

GIVEN: A well is producing undersaturated oil from a formation with a net sand thickness of 31 ft. Other data pertaining to the well are:

Diameter of well = 8 in.

Estimated radius of drainage, r_e = 660 ft

Flowing sand face pressure, p_w = 2,240 psia

Bubble point pressure, p_b = 1,250 psia

Oil viscosity at average reservoir conditions, μ_o = 1.35 cp

Oil formation volume factor at average reservoir conditions, B_o = 1.44

Effective permeability to oil, k_o = 10 md

Oil production rate, Q_o = 150 st tk b/d.

FIND: Steady-state pressure distributions for radial flow of an incompressible liquid (constant pressure at r_e) and a compressible liquid (no flow across r_e).

SOLUTION: Equations for the solution of this problem can be derived from the diffusivity equation

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\phi \mu c}{6.328k}\frac{\partial p}{\partial t} \quad (1)$$

and Darcy's law (neglecting gravity forces)

$$v = (-1.127)\,[(k/\mu)]\,(dp/dr) \quad (2)$$

For incompressible liquids, c = 0 In Equation 1, and[1]

$$p = p_w + \frac{Q_o \mu_o B_o \ln (r/r_w)}{7.08\, k_o h} \quad (3)$$

In equation 3 the rate of flow at every circumference from r_w to r_e is a constant. In the system for a compressible liquid no flow occurs at r_e and $\partial p/\partial t$ in Equation 1 is constant. Thus[1 2]

$$p = p_w + \frac{Q_o \mu_o B_o [\ln (r/r_w) - (r^2/2r_e^2)]}{7.08\, k_o h} \quad (4)$$

Where:

p_w, r_e, μ_o, B_o, k_o and Q_o are defined with the data and

r = radial distance from well bore to point of interest, ft

p = pressure at r, psi

ϕ = porosity, fraction

c = fluid compressibility

t = time

r_w = well radius, ft

h = net sand thickness, ft.

In such a system the pressure falls everywhere at the same rate.

Substitution of the known parameters into Equation 3 gives

$$p = 2.240 + \left[\frac{(150)(1.35)(1.44)}{(7.08)(0.010)(31)}\right] \ln (r/0.333)$$

$$p = 2,240 + 132.86 \ln 3r \quad (5)$$

Similarly Equation 4 becomes

Table 1—Calculation of steady state pressure distributions for incompressible and com

(1)	(2)	(3)	(4)	(5)	(6)	(7)
			Incompressible liquid			
Radius, r, ft	r/r_w = 3r	$\ln (r/r_w)$ ln (2)	$\frac{Q_o \mu_o B_o \ln (r/r_w)}{7.08\, k_o h}$ = 132.86×(3)	p, psia 2,240+(4)	$11.48\times10^{-7} r^2$ = $11.48\times10^{-7}\times(1)^2$	$\ln (r/r_w) - 11.48\times10^{-7} r^2$ (3) − (6)
0.333	1.0	0	0	2,240	0.000	0.000
2.5	7.5	2.015	268	2,508	0.000	2.015
5	15	2.705	359	2,599	0.000	2.705
10	30	3.400	452	2,692	0.000	3.400
30	90	4.500	598	2,838	0.001	4.499
50	150	5.005	665	2,905	0.003	5.002
100	300	5.700	757	2,997	0.011	5.689
200	600	6.400	850	3,090	0.046	6.354
300	900	6.800	903	3,143	0.103	6.697
400	1,200	7.095	943	3,183	0.184	6.911
500	1,500	7.310	971	3,211	0.287	7.023
600	1,800	7.495	996	3,236	0.413	7.082
660	1,980	7.580	1,007	3,247	0.500	7.080

$$p = 2{,}240 + 132.86$$
$$[\ln 3r - 11.48 \times 10^{-7}\, r^2] \quad (6)$$

At radius of 400 ft and for steady-state conditions

$$p = 2{,}240 + 132.86 \ln (3)(400)$$

$$p = 2{,}240 + 943 = 3{,}183 \text{ psi}$$

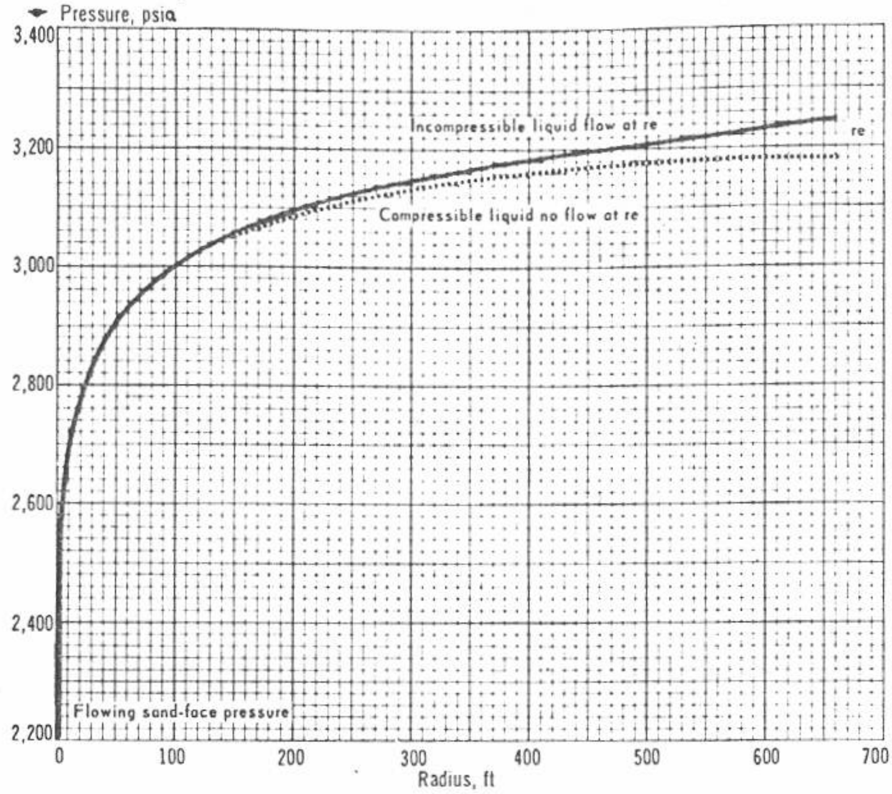
For the succession of steady states case

$$p = 2.240 + 132.86$$
$$[\ln (3)(400) - (11.48 \times 10^{-7})(400)^2]$$

$$p = 2{,}240 + 918 = 3{,}158 \text{ psi}$$

DISCUSSION: Steady-flow occurs when the fluid mass remains constant at every point within the radius of drainage. For an incompressible liquid this also means that pressure remains constant at every point. Although not strictly steady state, flow is considered steady state when the fluid mass changes at a constant rate at every point within the radius of well drainage.

To maintain this condition the pressure must decline at the same rate at all points in the reservoir. The curves of pressure versus radial distance at successive times are all parallel. Each unit volume of reservoir fluid (oil and water) expands at the same rate.



STEADY-STATE pressure distributions for incompressible and compressible liquids. Fig. 1.

Before a well is placed on production the pressures at all points within the drainage area are the same. Lowering of the pressure at the sand face during production causes a pressure disturbance that is rapidly transmitted through the reservoir fluid to points away from the well bore. A system approaches steady-state flow when such disturbances are transmitted nearly instantaneously. This requires a fluid of very low compressibility and/or the transmission of the disturbances over short distances (r_e is small).

Solution of this problem is shown in Table 1 with results plotted in Fig. 1. Pressure distributions are similar out to a radius of 200 ft. Beyond this radius deviation occurs and the incompressible liquid (steady-state flow) shows higher pressures than the compressible liquid (semisteady-state case). In the former case the pressure distribution would be maintained for all time while in the latter case the pressure distribution obtained is actually applicable for only a short period of time. A succession of steady states is used to represent unsteady state flow (which is affected by time) over a short period of time.

Conditions for the strict applicability of Equations 3 and 4 are rarely encountered in subsurface reservoirs. The conditions represented by the equations (constant flow at r_e and no flow at r_e), however, encompass most steady-state applications. These equations serve as approximate solutions in cases such as flow of oil above bubble-point pressure in a small reservoir, flow of oil and water in a water flood after fillup, and flow of water from a small aquifier. In applying these equations, conditions at or near the external boundaries are not too significant since the major part of the pressure variation occurs near the sandface.

Fig. 1 shows that two-thirds of the pressure change occurs over a radius of 50 ft from the sandface. It will be shown in subsequent problems that this pressure drop is even larger when the formation is damaged through drilling, well completion, or other practices.

essible liquids

(8)	(9)
Compressible liquid	
$\mu_o B_o [\ln (r/r_w) - (r^2/2r_e^2)]$ = 7.08 $k_o h$	
132.86×(7)	p, psia 2,240+(8)
0	2,240
268	2,508
359	2,599
452	2,692
598	2,838
665	2,905
756	2,996
844	3,084
890	3,130
918	3,158
933	3,173
941	3,181
941	3,181

References

1. Craft, B. C., and Hawkins, M. F.: "Applied Petroleum Reservoir Engineering": Prentice-Hall, Inc, 1959, Chap. 6.

2. Brownscombe, E. R., and Collins, F.: "Pressure Distribution in Unsaturated Oil Reservoirs": Trans. AIME Vol. 189, 1950, p. 371.

Part 58

How to find pressure distribution for unsteady-state flow conditions

. . . using the point-source solution and constant pressure at an infinite external boundary

GIVEN: A well has been completed in a recently discovered pool. It is being produced at a stabilized rate of 100 st-tk b/d. Other data are as follows:

Initial reservoir pressure, p_i = 3,500 psia

Oil-formation volume factor, B_o = 1.415

Reservoir-oil viscosity, μ_o = 0.38 cp

Effective permeability to oil, k_o = 19 md

Net sand thickness, h = 32 ft

Interstitial water saturation, S_w = 24.0%

Reservoir-water compressibility, $c_w = 3.4 \times 10^{-6}$ vol/vol/psi

Rock compressibility, $c_f = 3.6 \times 10^{-6}$ pore vol/pore vol/psi

Reservoir-oil compressibility, $c_o = 12.5 \times 10^{-6}$ vol/vol/psi

Porosity, ϕ = 22.0%

FIND: Pressure distributions for 0.1, 1.0, 10, and 100 days, assuming an infinite external boundary.

METHOD OF SOLUTION: It can be shown that the radial, unsteady-state flow of a slightly compressible fluid is given by the diffusivity equation[1]

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\phi \mu c}{6.328\, k}\frac{\partial p}{\partial t} \qquad (1)$$

To solve Equation 1 the boundary conditions must be designated.

A boundary is considered infinite when, over the time of interest, pressure disturbances caused by fluid production have not reached the outer limits of a system. A system can be composed of an oil reservoir surrounded or flanked by an aquifer. For infinite-boundary conditions, several exact solutions are available for Equation 1 in standard mathematical references. However, because of its simplicity the "point-source" solution is widely used.[1,2]

Where:

p_i, B_o, μ_o, k_o, h, S_w, c_w, c_f, c_o and ϕ are defined with the data and

p = reservoir pressure at radius r, psi

r = radius, ft

S_o = oil saturation, fraction of pore volume

$\phi_o = \phi S_o$ or hydrocarbon (oil) porosity

μ = viscosity, cp

c = compressibility, vol/vol/psi

c_e = effective compressibility, vol/vol/psi

k = permeability, darcies

t = time, days

Q_o = oil production rate, st-tk b/d

$$p = p_i + \frac{Q_o \mu_o B_o}{14.16\, k_o h} E_i\left(-\frac{r^2 \phi_o \mu_o c_e}{25.31\, k_o t}\right) \qquad (2)$$

Table 1—Calculations for infinite-boundary unsteady state

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	t = 0.1 days				t = 1 day			
Radius, r ft	$24.16 \times 10^{-6} r^2$	$E_i(-24.16 \times 10^{-6} r^2)$ From Fig. 1*	6.245 × (3) psi	p, psi 3,500 + (4)	$2.416 \times 10^{-6} r^2$	$E_i(-2.416 \times 10^{-6} r^2)$ From Fig. 1*	6.245 × (7)	p, psi 3,500 + (
1	24.16×10^{-6}	−10.04	−62.7	3,437.3	2.42×10^{-6}	−12.34	−77.1	3,422.9
5	60.40×10^{-5}	− 6.82	−42.6	3,457.4	6.04×10^{-5}	− 9.12	−57.0	3,443.0
10	24.16×10^{-4}	− 5.44	−34.0	3,466.0	2.42×10^{-4}	− 7.74	−48.3	3,451.7
50	60.40×10^{-3}	− 2.30	−14.4	3,485.6	6.04×10^{-3}	− 4.52	−28.2	3,471.8
100	24.16×10^{-2}	− 1.06	− 6.6	3,493.4	2.42×10^{-2}	− 3.15	−19.7	3,480.3
250	1.51	− 0.096	0.6	3,499.4	0.151	− 1.46	− 9.1	3,490.9
500	6.04	0	0.0	3,500	0.604	− 0.44	− 2.7	3,497.3
750	13.59	0	0.0	3,500	1.36	− 0.13	− 0.8	3,499.2
1,000	24.16	0	0.0	3,500	2.42	− 0.027	− 0.2	3,499.8
2,500	151.00	0	0.0	3,500	15.10	0	0.0	3,500
5,000	604.00	0	0.0	3,500	60.40	0	0.0	3,500
10,000	2,416.00	0	0.0	3,500	241.60	0	0.0	3,500

*When $\left[-2.416 \times 10^{-n} \frac{r^2}{t}\right]$ is less than 0.02, $E_i\left[-2.416 \times 10^{-n} \frac{r^2}{t}\right] = \ln\left(\frac{2.416 \times 10^{-n_R}}{t}\right) + 0.577$

In Equation 2

$$\phi_o = (22.0)\,(1.00 - 0.24) = (22.0)\,(0.760) = 16.7\%$$

$$c_e = c_o + \frac{c_w\,S_w}{1 - S_w} + \frac{c_f}{1 - S_w} \qquad (3)$$

$$= 12.5 \times 10^{-6} + \frac{(3.4 \times 10^{-6})\,(0.24)}{1.00 - 0.24} + \frac{3.6 \times 10^{-6}}{1.00 - 0.24}$$

$$= 12.5 \times 10^{-6} + 1.07 \times 10^{-6} + 4.74 \times 10^{-6}$$

$$= 18.31 \times 10^{-6} \text{ vol/vol/psi}$$

Thus

$$p = 3{,}500 + \frac{(100)\,(0.38)\,(1.415)}{(14.16)\,(0.019)\,(32)}\,E_i\left[-\frac{(r^2)\,(0.167)\,(0.38)\,(18.31 \times 10^{-6})}{(25.31)\,(0.019)\,t}\right]$$

$$= 3{,}500 + 6.245\,E_i\,[-2.416 \times 10^{-6}\,r^2/t]$$

For $t = 10$ days and $r = 250$ ft

$$p = 3{,}500 + 6.245\,E_i\,[-1.51 \times 10^{-2}]$$

From Fig. 1

$$E_i\,[-1.51 \times 10^{-2}]$$
$$= \ln 1.51 - \ln 100 + 0.577$$
$$= -3.61$$

Thus

$$p = 3{,}500 + (6.245)\,(-3.61)$$
$$= 3{,}477.5 \text{ psi}$$

DISCUSSION:

Equation 1 defines the variation of pressure with position and time. Derivation of this equation for its application to porous media requires sweeping assumptions regarding the reservoir and the fluids contained within the rock pores.[1 2 3] The reservoir is assumed to be homogeneous, horizontal, and of uniform thickness throughout. The flowing fluid is assumed to obey Darcy's law, to be present in one phase only, to have a compressibility and viscosity that remain constant over the range of temperature and pressure variation encountered.

Of these basic assumptions it would appear that the most critical is the one that requires the presence of only a single phase of reservoir fluid. Both the compressibility and permeability are sensitive to changes in fluid saturation and pressure below the bubble point. Although theoretically the equation is not supposed to, it often is found to be useful for conditions below the bubble point.

Several exact solutions of Equation 1 are given in the literature[3 4 5 6 7] for various boundary conditions. Most of these solutions involve complex integrals and Bessel functions which make them unwieldy for calculation purposes.

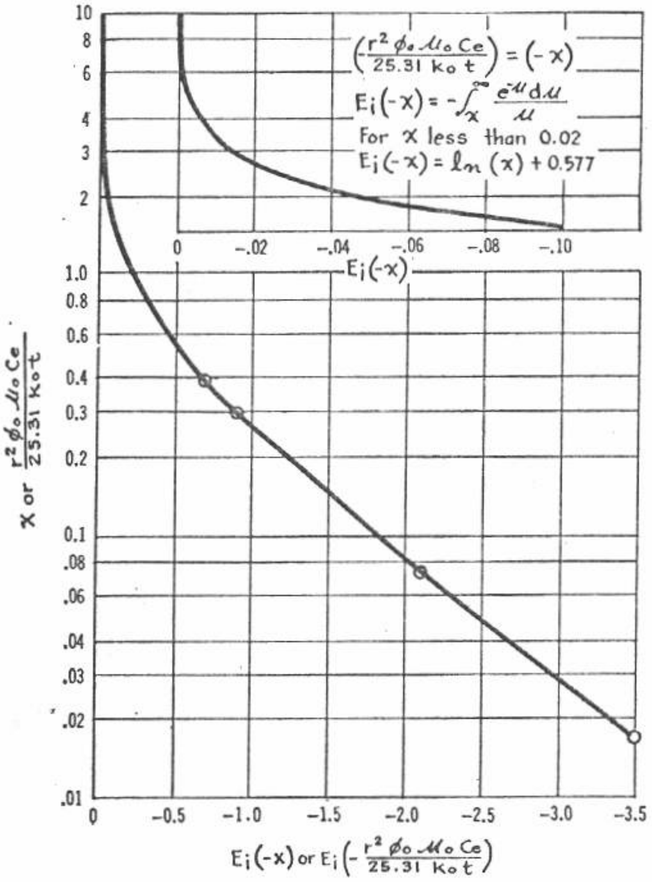
The point-source solution used in this problem is the simplest and is applicable for many reservoir-engineering problems. It assumes an infinite external boundary or radius of drainage with constant pressure, p_i; and a vanishing internal boundary (well radius) with a constant flow rate, Q_o, across it. This latter assumption introduces an error which is negligible when r is much greater than r_w (r/r_w greater than about 25) and for times greater than about 1 minute.

Table 1 gives the calculation of Equation 2 at various radii and for time periods of 0.1, 1.0, 10, and 100 days. The results are given in columns 5, 9, 13, and 17. Note in Fig. 1[8] that E_i functions for

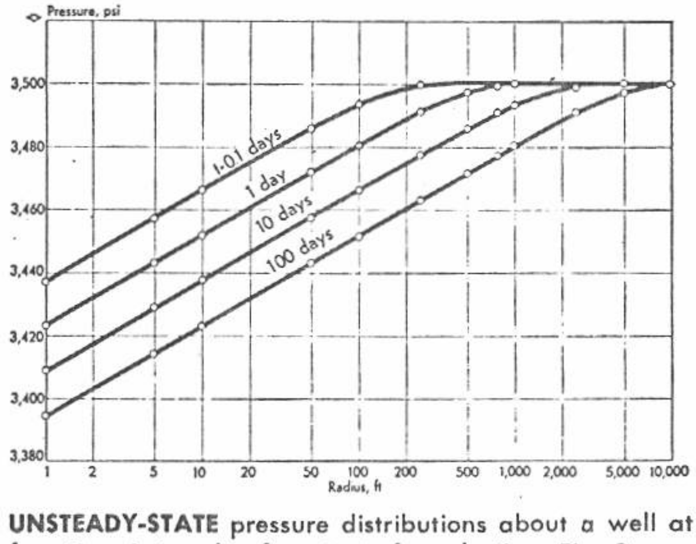
$$\frac{r^2\phi_o\mu_o c_e}{25.31\,k_o t}$$

essure distributions at 0.1, 1.0, 10, and 100 days

(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
t = 10 days				t = 100 days			
.2416 × 10⁻⁶r²	E_i (−0.2416 × $10^{-6}r^2$) From Fig. 1*	6.245 × (11)	p, psi 3,500 + (12)	0.02416 × $10^{-6}r^2$	E_i (−0.02416 × $10^{-6}r^2$) From Fig. 1*	6.245 × (15)	p, psi 3,500 + (16)
2.42×10^{-7}	−14.64	−91.4	3,408.6	2.42×10^{-8}	−16.94	−105.8	3,394.2
6.04×10^{-6}	−11.42	−71.3	3,428.7	6.04×10^{-7}	−13.72	− 85.7	3,414.3
2.42×10^{-5}	−10.04	−62.7	3,437.3	2.42×10^{-6}	−12.34	− 77.1	3,422.9
6.04×10^{-4}	− 6.82	−42.6	3,457.4	6.04×10^{-5}	− 9.12	− 57.0	3,443.0
2.42×10^{-3}	− 5.44	−34.0	3,466.0	2.42×10^{-4}	− 7.74	− 48.3	3,451.7
1.51×10^{-2}	− 3.61	−22.5	3,477.5	1.51×10^{-3}	− 5.91	− 36.9	3,463.1
6.04×10^{-2}	− 2.29	−14.3	3,485.7	6.04×10^{-3}	− 4.52	− 28.2	3,471.8
1.36×10^{-1}	− 1.56	− 9.7	3,490.3	1.36×10^{-2}	− 3.72	− 23.2	3,476.8
2.42×10^{-1}	− 1.07	− 6.7	3,493.3	2.42×10^{-2}	− 3.15	− 19.7	3,480.3
1.51	− 0.096	− 0.6	3,499.4	1.51×10^{-1}	− 1.46	− 9.1	3,490.9
6.04	0	0.0	3,500	6.04×10^{-1}	− 0.44	− 2.7	3,497.3
24.16	0	0.0	3,500	2.42	− 0.027	− 0.2	3,499.8



EXPONENTIAL integral values.[8] Fig. 1.



UNSTEADY-STATE pressure distributions about a well at four time intervals after start of production. Fig. 2.

less than about 0.02 are computed with

$$E_i\left(-\frac{r^2\phi_o\mu_o c_e}{25.31\ k_o t}\right) = \ln\left(-\frac{r^2\phi_o\mu_o c_e}{25.31\ k_o t}\right) + 0.577$$

Results from Table 1 are plotted on Fig. 2 and show pressure distributions at the four time intervals. If these data were plotted on a coordinate graph, they would show about 75% of the pressure drop occurring within a few feet of the sand face.

Since available reservoir energy causes the flow of fluids to producing wells, an understanding of pressure distributions is basic to defining fluid-flow behavior and predicting pressure-buildup behavior after a well is shut in. After a well is shut in, the reservoir-pressure behavior is an opposite image of what occurred before shut-in if the well had been produced at a stabilized rate.

References

1. Horner, D. R., "Pressure Buildup in Wells": Proc. Third World Pet. Cong., Sec. 11, pp. 503-522, 1951.
2. Craft, B. C., and Hawkins, M. F., "Applied Reservoir Engineering": Prentice-Hall, Inc., Englewood Cliffs, N.J., 1959.
3. Muskat, M., "Physical Principles of Oil Production": McGraw-Hill Book Co., New York, 1949.
4. Carslaw, H. S., and Jaeger, J. C., "Conduction of Heat in Solids": Clarendon Press, Oxford, 1959.
5. Carslaw, H. S., and Jaeger, J. C., "Operational Methods in Applied Mathematics": University Press, Oxford, 1953.
6. Hildebrand, F., "Advanced Calculus for Engineers": Prentice-Hall, Inc., Englewood Cliffs, N.J., 1949.
7. Collins, R. E., "Flow of Fluids Through Porous Media": Reinhold Publishing Corp., New York, 1961.
8. "Tables of Sine, Cosine, and Exponential Integrals, Vols. I and II": Federal Works Agency, WPA for New York City, sponsored by U.S. Natl. Bur. of Stand. Available from Supt. of Documents, Washington 25, D.C.

Part 59

How to calculate short-term shut-in effect on subsequent pressure-buildup test

GIVEN: A well is located in a field under development with 80-acre spacing. Since completion the well has produced at a constant rate of 80 st tk b/d. At the end of 10 days the well was shut in and pressure-buildup data taken, columns 1 and 2, Table 1. Other data at reservoir conditions were viscosity of oil, $\mu_o = 1.6$ cp; oil-formation-volume factor, $B_o = 1.42$.

FIND: Determine the effect on the pressure-buildup-test results of shutting the well in for 3 hours, at 6 hours ($t_2 = 237$, $t_1 = 234$), 12 hours, ($t_2 = 231$, $t_1 = 228$), 24 hours, ($t_2 = 219$, $t_1 = 216$), and 5 days ($t_2 = 123$, $t_1 = 120$) prior to the test and for 12 hours at 24 hours ($t_2 = 228$, $t_1 = 216$) prior to the test. The short-term shut-in is depicted in Fig. 1.

SOLUTION: The unsteady-state pressure behavior early in the life of a well can be computed with the point-source solution to the diffusivity equation.[1,2] For pressure-drawdown conditions

$$p_{wt} = p_i + \frac{162.6\, Q_o \mu_o B_o}{k_o h} E_i \left[- \frac{\phi_o \mu_o c_e r_w^2}{0.02531\, k_o t} \right] \quad (1)$$

while for pressure-buildup conditions

$$p_{w\Delta t} = p_i + \frac{162.6\, Q_o \mu_o \beta_o}{k_o h} \times \left\{ E_i \left[- \frac{\phi_o \mu_o c_e r_w^2}{0.02531\,(t + \Delta t)} \right] - E_i \left[- \frac{\phi_o \mu_o c_e r_w^2}{0.02531\, \Delta t} \right] \right\} \quad (2)$$

When the well is shut in a short period before the pressure-buildup test:

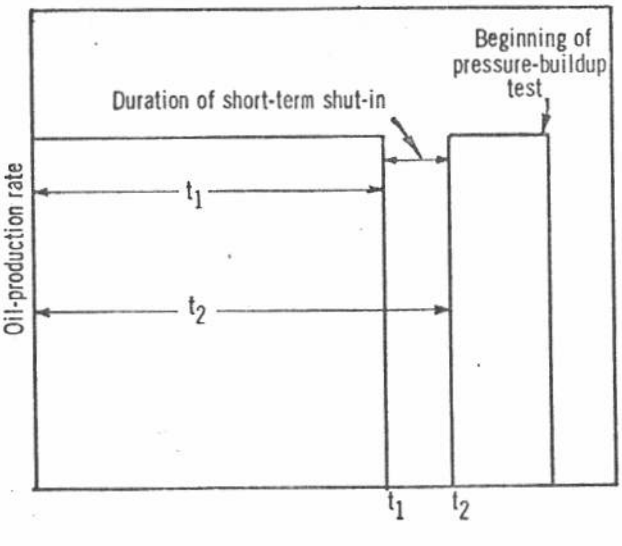
$$p_{w\Delta t} = p_i + \frac{162.6\, Q_o \mu_o \beta_o}{k_o h} \times \left\{ E_i \left[- \frac{\phi_o \mu_o c_e r_w^2}{0.02531\,(t+\Delta t)} \right] - E_i \left[- \frac{\phi_o \mu_o c_e r_w^2}{0.02531\,(t+\Delta t - t_1)} \right] + E_i \left[- \frac{\phi_o \mu_o c_e r_w^2}{0.02531\,(t+\Delta t - t_2)} \right] - E_i \left[- \frac{\phi_o \mu_o c_e r_w^2}{0.02531\, \Delta t} \right] \right\} \quad (3)$$

For values of $\frac{\phi_o \mu_o c_e r_w^2}{0.02531\, t}$ less than 0.02

$$E_i \left[- \frac{\phi_o \mu_o c_e r_w^2}{0.02531\, t} \right] \approx \ln \left[- \frac{\phi_o \mu_o c_e r_w^2}{0.02531\, t} \right] + 0.577$$

Thus Equation 3 becomes

$$p_{w\Delta t} = p_i - \frac{162.6\, Q_o \mu_o B_o}{k_o h} \left[\ln \left(\frac{t + \Delta t}{\Delta t} \right) \left(\frac{t + \Delta t - t_2}{t + \Delta t - t_1} \right) \right] \quad (4)$$



SHORT-TERM shut-in before pressure-buildup test can be represented diagramatically as shown here. Fig. 1.

Where μ_o and B_o are defined with the data and:

p_{wt} = sand-face pressure at time t, psia

p_i = initial reservoir pressure, psia

Q_o = oil-production rate, st tk b/d

k_o = interwell effective permeability to oil, md

h = net sand thickness, ft

ϕ_o = hydrocarbon porosity = $\phi\,(1 - S_w)$

ϕ = porosity

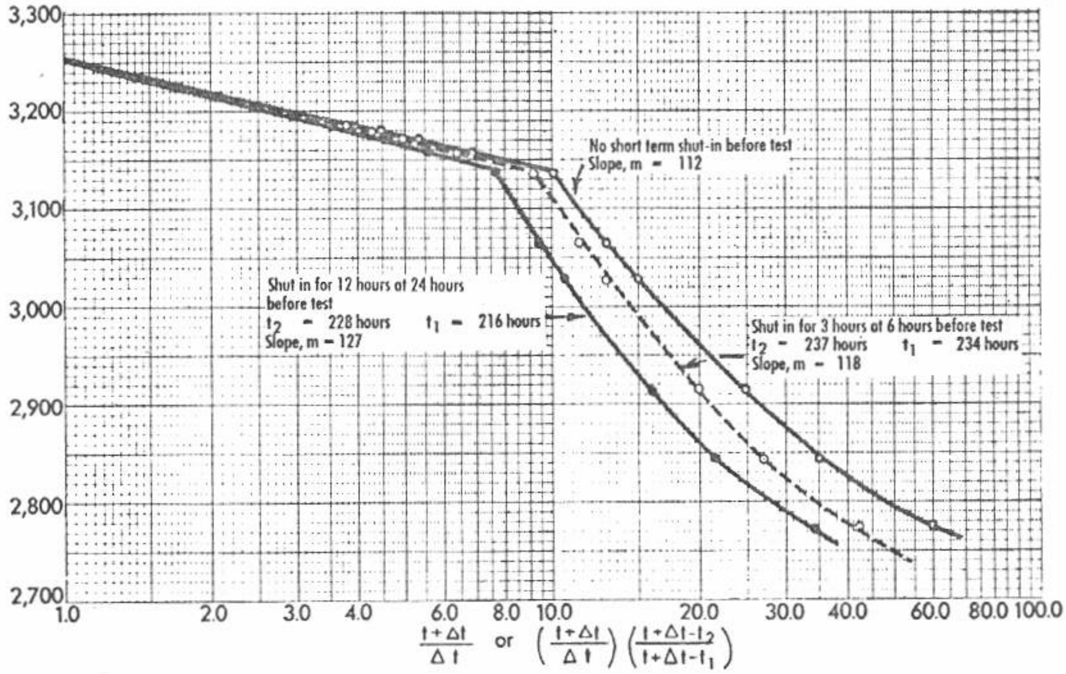
S_w = interstitial-water saturation

c_e = effective reservoir oil compressibility, vol/vol/psi

r_w = well radius, ft

Table 1—Calculations for short-time shut-in effect on

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		$\frac{t+\Delta t}{\Delta t}$	$(t+\Delta t-t_2) \div (t+\Delta t-t_1)$				
Sand-face pressure, psia	Shut-in time, Δt, hrs	$\frac{240+(2)}{(2)}$	$t_2 = 237$, $t_1 = 234$, $(3+\Delta t)/(6+\Delta t)$	$t_2 = 231$, $t_1 = 228$, $(9+\Delta t)/(12+\Delta t)$	$t_2 = 219$, $t_1 = 216$, $(21+\Delta t)/(24+\Delta t)$	$t_2 = 123$, $t_1 = 120$, $(117+\Delta t)/(120+\Delta t)$	$t_2 = 228$, $t_1 = 216$, $(12+\Delta t)/(24+\Delta t)$
2,775	4	61.0	0.700	0.813	0.893	0.976	0.571
2,845	7	35.3	0.769	0.842	0.903	0.976	0.613
2,915	10	25.0	0.813	0.864	0.912	0.977	0.647
3,030	17	15.1	0.870	0.897	0.927	0.978	0.707
3,065	20	13.0	0.885	0.906	0.932	0.979	0.727
3,135	26	10.2	0.906	0.921	0.940	0.979	0.760
3,158	41	6.85	0.936	0.943	0.954	0.981	0.815
3,170	55	5.36	0.951	0.955	0.962	0.983	0.848
3,180	70	4.43	0.961	0.963	0.968	0.984	0.872
3,185	82	3.93	0.966	0.968	0.972	0.985	0.887



SLOPE of pressure-buildup curve is altered by the length of the short-term shut-in period, and the length of time between short-term shut-in and the actual buildup test. Fig. 2.

t = time since initial completion of well, days

p_{wst} = sand-face pressure after Δt shut-in time, psia

Δt = shut-in time, days

t_1 = time from initial completion of well to beginning of short-term shut-in period, days

t_2 = time from initial completion of well to end of short-term shut-in period, days

As was true for Equation 3, Equation 2 can be written as

$$p_{wst} = p_i + \frac{162.6\, Q_o \mu_o B_o}{k_o h} \ln \left(\frac{t + \Delta t}{\Delta t} \right) \quad (5)$$

In Equations 4 and 5 other units of time, e.g. hours, can be used in place of days.

Since p_i, Q_o, μ_o, B_o, k_o, and h are assumed constant, Equations 4 and 5 represent straight lines on semi-log graphs. The necessary calculations are shown in Table 1 and the results for three sets of conditions are plotted on Fig. 2. It is noted that for small values of Δt the curves are not straight lines. This is caused by the after-flow period following shut-in.[3,4]

In Equations 4 and 5

$$\text{Slope} = m = \frac{162.6\, Q_o \mu_o \beta_o}{k_o h} \quad (6)$$

$$\text{or } k_o h = \frac{162.6\, Q_o \mu_o \beta_o}{m}$$

$$= \frac{(162.6)(80)(1.6)(1.42)}{m}$$

$$= \frac{29{,}554}{m}$$

From Fig. 1 for no short-time shut-in

$$m = 112 \text{ and}$$

$$k_o h = \frac{29{,}554}{112} = 264 \text{ md-ft}$$

For 3 hours shut-in at 6 hours before test

$$m = 118 \text{ and}$$

$$k_o h = \frac{29{,}554}{118} = 250 \text{ md-ft}$$

$$\%\ \text{error} = \frac{(264 - 250)\,100}{250} = 5.6$$

For 12 hours shut-in at 24 hours before test

ressure-buildup-test results

(9)	(10)	(11)	(12)	(13)
	$\left(\frac{t+\Delta t}{\Delta t}\right)$	$\left(\frac{t+\Delta t-t_2}{t+\Delta t-t_1}\right)$		
$t_2 = 237$ $t_1 = 234$ (3) × (4)	$t_2 = 231$ $t_1 = 228$ (3) × (5)	$t_2 = 219$ $t_1 = 216$ (3) × (6)	$t_2 = 123$ $t_1 = 120$ (3) × (7)	$t_2 = 228$ $t_1 = 216$ (3) × (8)
42.7	49.6	54.5	59.5	34.8
27.1	29.7	31.9	34.5	21.6
20.3	21.6	22.8	24.4	16.2
13.1	13.5	14.0	14.8	10.7
11.5	11.8	12.1	12.7	9.45
9.24	9.39	9.59	9.99	7.75
6.41	6.46	6.53	6.72	5.58
5.10	5.12	5.16	5.27	4.55
4.26	4.27	4.29	4.36	3.86
3.80	3.80	3.82	3.87	3.49

m = 127 and

$$k_o h = \frac{29{,}554}{127} = 233 \text{ md-ft}$$

$$\% \text{ error} = \frac{(264 - 233)\,100}{233} = 13.3$$

DISCUSSION: Applicability of the theory for pressure-buildup analysis requires that a well be produced at a constant rate for several days.[1,2] In operations, however, it often happens that before a pressure-buildup test is made, it is necessary to shut the well in for a short period to change a valve, because of a shortage of tankage, or due to another reason. This short-term shut-in is followed by placing the well back on production at the same rate as before. This problem shows the effect of such interruptions on the results of a pressure-buildup test.

Computations show that a 3-hour shut-in followed by 3 hours of production before testing results in an error of 5.6% in capacity (kh) while a 12-hour shut-in followed by 12 hours of production before testing results in an error of 13.3%. Thus the error depends not only on the ratio of duration of short term shut-in to flowing time following short term shut-in but also on the actual duration of short-term shut-in.

The results given by conditions of columns 10, 11, and 12 of Table 1 are not shown on Fig. 2. These conditions would give errors below 5.6%.

Note on Fig. 2 that the effects of a short-term shut-in are greater in the early part of the test and decrease to zero for large values of Δt. Although under many conditions the effects of short-term shut-in may be negligible, it is recommended that calculations described by Equation 4 be made.[1]

This problem shows the effect of a short-term change (from a constant rate of production to zero rate) in production rate on pressure-buildup-test results. Other less-drastic or minor variations in oil-production rate would have much less effect. For the case of finite boundary conditions the effect would be equivalent or less.

References

1. Horner, D. R., "Pressure Build-Up in Wells": Proc. Third World Petroleum Congress (1951), Section 11, pp. 503-522.

2. Nisle, R. G., "The Effect of a Short Term Shut-In on a Subsequent Pressure Build-Up Test on an Oil Well": Trans. AIME Vol. 207, 1956, pp 320-321.

3. Guerrero, E. T., and Stewart, F. M., Reservoir Engineering Part 13, "How to Determine Effective Permeability from Pressure-Buildup Data Under Infinite Boundary Conditions": The Oil and Gas Journal, Vol. 57, No. 33, Aug. 10, 1959, pp 119-120.

4. Guerrero, E. T., and Stewart, F. M.: Reservoir Engineering Part 14, "How to Determine Effective Permeability from Pressure-Buildup Data Under Finite Boundary Conditions": The Oil and Gas Journal, Vol. 57, No. 41, Oct. 5, 1959, pp 167-169.

Part 60

Two ways to find sand-face pressures

at small times and infinite boundary conditions for unsteady-state flow of a slightly compressible liquid.

GIVEN: A well is located in a field under development with 80-acre spacing. The well has been produced at a constant rate of 50 st-tk b/d. Other data are as follows:

Average permeability, k_o = 25 md
Porosity, ϕ = 22.0%
Net sand thickness, h = 35 ft
Initial reservoir pressure, p_i = 2,500 psi
Interstitial water saturation, S_w = 26.0%
At reservoir conditions
Effective oil compressibility, c_o = 15 x 10^{-6} vol/vol/psi
Oil formation-volume factor, B_o = 1.32
Oil viscosity, μ_o = 0.7 cp
Radius of well, r_w = 4 in.

FIND: Sand-face pressures at 0.00017, 0.0167, 1/6, 1/2, 1, 5, 10, 20, 30, and 60 minutes using the Van Everdingen-Hurst [1] and Horner equations.[2]

METHOD OF SOLUTION: In the development of a reservoir, newly completed wells behave as if other wells were not present or as if their radius of drainage were infinite. Pressure distributions created by production can be computed with the Van Everdingen-Hurst equation.[1]

$$p = p_i - \frac{887.4\, \mu_o B_o Q_o}{2\pi k_o h} P_T \quad (1)$$

and the Horner equation [2]

$$p = p_i + \frac{70.6\, Q_o \mu_o B_o}{k_o h} E_i \left(- \frac{r^2 \phi_o \mu_o c_e}{0.02531\, k_o t} \right) \quad (2)$$

Where: k_o, ϕ, h, p_i, S_w, c_e, B_o, and μ_o are defined with the data and

$$P_T = \frac{4}{\pi^2} \int_o^{\infty} \frac{(1 - e^k)\, dw}{w^3 [J_1^2 (w) + Y_1^2 (w)]} \quad (3)$$

P_T = dimensionless cumulative pressure drop

r = radius, ft

Q_o = oil-production rate, st-tk b/d

p = pressure at r or interior radius, psi

$\phi_o = S_o \phi$ = hydrocarbon porosity = (1 − 0.26) (0.22) = 0.1628 or 16.3%

t = time, days

r_w = well radius, ft

w = function of a complex variable

$k = w^2 t$

$J_1(w)$ = Bessel function of the first kind and order one

$Y_1(w)$ = Bessel function of the second kind and order one

Solution of Equation 3 for P_T is complex and tedious. Fortunately useful solutions have been made and are available in tabular and graphical forms.[1,3] The solutions are given as functions of dimensionless time (T) which for this problem is computed with

$$T = \frac{6.33 \times 10^{-3} k_o t}{\phi_o \mu_o c_e r_w^2} \quad (4)$$

Substitution of known parameters into Equations 1 and 4 gives

$$p = 2{,}500 - \frac{(887.4)(0.7)(1.32)(50)}{(6.28)(25)(35)} P_T$$

$$p = 2{,}500 - 7.461 P_T \quad (5)$$

The second method of solution, using the Horner equation involves substitution of parameters into Equation 2:

$$p = 2{,}500 + \frac{(70.6)(50)(0.7)(1.32)}{(25)(35)} E_i \left[- \frac{(0.333)^2 (0.1628)(0.7)(15 \times 10^{-6})}{(0.02531)(25)\, t} \right]$$

$$p = 2{,}500 + 3.728\, E_i \left[- \frac{0.300 \times 10^{-6}}{t} \right] \quad (6)$$

Substitution into Equation 4 gives

$$T = \frac{(6.33 \times 10^{-3})(25)\, t}{(0.1628)(0.7)(15 \times 10^{-6})(0.333)^2} = 8.332 \times 10^5\, t \quad (7)$$

where t is in days or

$$T = \frac{8.332 \times 10^5\, t}{1440} = 578.6\, t \quad (8)$$

where t is in minutes.

Solution. For a time of 1 minute, T = 578.6.

From Table 1 of Reference 3, P_T = 3.5891, and using Equation 5

$$p = 2{,}500 - (7.461)(3.5891)$$

$$= 2{,}500 - 27 = 2{,}463 \text{ psi.}$$

Using Equation 6

$$p = 2{,}500 + 3.728\, E_i \left[- \frac{0.300 \times 10^{-6}}{1 \div 1440} \right]$$

$$= 2{,}500 + 3.728\, E_i [-4.32 \times 10^{-4}]$$

From Fig. 1 of Reference 4

$$E_i [-4.32 \times 10^{-4}] = \ln (4.32 \times 10^{-4}) + 0.577 = -7.16$$

Thus

$$p = 2{,}500 - (3.728)(7.16) = 2{,}463 \text{ psi}$$

DISCUSSION: Equation 1 represents an exact solution of the diffusivity equation. Its derivation involves complex integrals and Bessel functions which make calculations tedious and difficult. Some practical solutions have been made for linear and radial systems and both infinite and finite boundary conditions.[1] Equation 2 is the point source solution to the diffusivity equation for infinite external boundary conditions. An error is introduced in its development when the well radius is assumed infinitely small. However, this error is considered to be negligible in pressure drawdown and pressure-buildup-test analysis.[2]

Equation 2 has the advantage of ease of solution. In fact, adequate solutions are available[5] or can be developed. Another criticism of Equation 2 is that when

$$\frac{r^2 \phi \mu c}{0.02531\, kt} \text{ is less than } 0.02$$

$$E_i \left[- \frac{r^2 \phi \mu c}{0.02531\, kt} \right] = \ln \left[\frac{r^2 \phi \mu c}{0.02531\, kt} \right] + 0.577 \quad (9)$$

Table 1—Calculation of sandface pressures at small times using Van Everdingen-Hurst and Horner solutions

(1)	(2)	(3)	(4)	(5)
	Van Everdingen-Hurst Solution[1]			
Time minutes	Dimensionless time Eq. 8	P_T Table 1 Ref. 3	$7.461\ P_T$ psi	p. psi Eq. 5
0.00017	0.1	0.3144	2	2,498
0.0167	9.7	1.6373	12	2,488
0.167	96.6	2.7058	20	2,480
0.5	289.3	3.2437	24	2,476
1	578.6	3.5891	27	2,473
5	2,893.0	4.3871	33	2,467
10	5,786.0	4.7296	35	2,465
20	11,572.0	5.0796	38	2,462
30	17,358.0	5.2846	39	2,461
60	34,716.0	5.6271	42	2,458

(6)	(7)	(8)	(9)
Horner Solution[2]			
$0.300 \times 10^{-6}/t$ $0.300 \times 10^{-6} \div \frac{(1)}{1,440}$	$E_i\,(-0.300 \times 10^{-6}/t)$ Fig. 1 of Ref. 4	$3.728 \times (7)$ psi	p, psi Eq. 6
2.54	− 0.024	0	2,500
2.59×10^{-2}	− 3.07	11	2,489
2.59×10^{-3}	− 5.37	20	2,480
8.64×10^{-4}	− 6.47	24	2,476
4.32×10^{-4}	− 7.16	27	2,473
8.64×10^{-5}	− 8.77	33	2,467
4.32×10^{-5}	− 9.46	35	2,465
2.16×10^{-5}	− 10.15	38	2,462
1.44×10^{-5}	− 10.56	39	2,461
7.20×10^{-6}	− 11.25	42	2,458

This solution is used extensively in pressure-buildup-test analysis which means that for constant values of r, ϕ, μ, c, and k, t cannot be too small.

For the conditions of this problem Equations 1 and 2 gave identical results (columns 5 and 9 of Table 1) for times of 10 seconds and greater. This verifies Horner's contention that Equation 2 is satisfactory within a matter of seconds after closing in a well.

For a much smaller permeability, the necessary time for both methods to give comparable results would be about a minute.

References

1. Van Everdingen, A. F., and Hurst, W., "The Application of the La Place Transformation to Flow Problems in Reservoirs": Trans. AIME (1949), 186, p. 305.
2. Horner, D. R., "Pressure Buildup in Wells": Proc. Third World Pet. Cong., Sec. 11, pp. 503-522, 1951.
3. Chatas, A. T., "A Practical Treatment of Nonsteady-State Flow Problems in Reservoir Systems": Petroleum Engineer, Pt. 1, May 1953, p. B-42; Pt. 2, June 1953, p. B-38; Pt. 3, August 1953, p. B-44.
4. Guerrero, E. T., Reservoir Engineering Part 58—"How to Find Pressure Distribution for Unsteady-State Flow Conditions Using the Point Source Solution and Constant Pressure at an Infinite External Boundary": The Oil and Gas Journal, Vol. 61, No. 24, June 17, 1963, p. 99-101.
5. "Tables of Sine, Cosine, and Exponential Integrals, Vols. 1 and 2": Federal Works Agency, WPA for New York City, sponsored by U.S. Nat'l. Bur. of Stand. Available from Supt. of Documents, Washington 25, D.C.

Part 61

How to find pressure distributions for unsteady-state flow conditions for finite external boundary

—using the Van Everdingen-Hurst and Horner solutions

GIVEN: A well completed in a Kansas pool has been produced at a constant rate of 100 st tk b/d. The pool was developed rapidly after initial discovery. Data pertaining to the well and pool are as follows:

Radius of drainage for well, r_e = 1,320 ft

Radius of well, r_w = 4 in.

Average effective oil permeability, k_o = 25 md

Porosity, ϕ = 22.0%

At reservoir conditions:

Water compressibility, $c_w = 3.0 \times 10^{-6}$ vol/vol/psi

Oil compressibility, $c_o = 8.0 \times 10^{-6}$ vol/vol/psi

Rock compressibility, $c_r = 3.5 \times 10^{-6}$ vol/vol/psi

Oil formation-volume factor, B_o = 1.35

Oil viscosity, μ_o = 0.4 cp

Net sand thickness, h = 45 ft

Initial reservoir pressure, p_i = 3,000 psi

Interstitial-water saturation, S_w = 25.0%

Bubble-point pressure = 1,450 psi

FIND: Pressure distributions in the well drainage area at 5, 100, and 365 days.

Method of solution: Exact and approximate solutions, based on the diffusivity equation, have been developed[1,2] for computing the effect of production and time on reservoir pressure behavior. The approximate solution

$$p = p_i + \frac{70.6\, Q_o \mu_o B_o}{k_o h}\left[E_i\left(-\frac{r^2 \phi \mu_o c_{av}}{0.02531\, k_o t}\right) - Y\left(\frac{r_e^2 \phi \mu_o c_{av}}{0.02531\, k_o t}\right)\right] \quad (1)$$

was developed by Horner[2] while a more exact solution

$$p = p_i - \frac{887.4\, \mu_o B_o Q_o}{2\pi\, k_o h} P_T \quad (2)$$

was presented by Van Everdingen and Hurst.[1] In these equations

r = radius, ft

P_T = dimensionless cumulative pressure drop

Q_o = oil-production rate, st tk b/d

$$E_i(-x) = -\int_x^{\infty} \frac{e^{-u}}{u}\, du$$

$$Y(u) = E_i(-u) + \frac{1}{u} e^{-u}$$

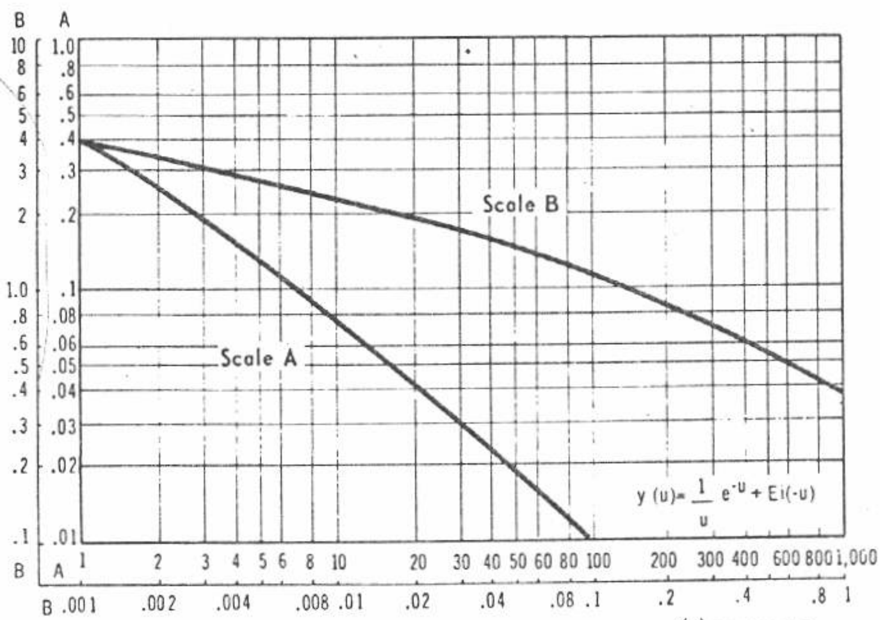
p = reservoir pressure at radius r and time t

c_{av} = average compressibility, vol/vol/psi

t = time, days

p_i, μ_w, B_o, k_o, h, ϕ, r_w and r_e are defined with the data.

In Equation 2, P_T is solved with Equations 16 and 17 of Reference 3. The solutions of these equations are rather complex and tedious. Fortunately some solutions have been made and are available in tabular and graphic forms.[1,3] Solutions are available for r_e/r_w values between 1.5 and 10 and dimensionless times from 6.0×10^{-2} to 70.0. For larger values of dimensionless time[4]



EVALUATION GRAPH for y (u) function. Fig. 1.

$$P_T = \frac{0.5 + 2T}{(r_e/r_w)^2 - 1} - \frac{3\,(r_e/r_w)^4 - 4\,(r_e/r_w)^4 \ln (r_e/r_w) - 2\,(r_e/r_w)^2 - 1}{4\,[(r_e/r_w)^2 - 1]^2} \tag{3}$$

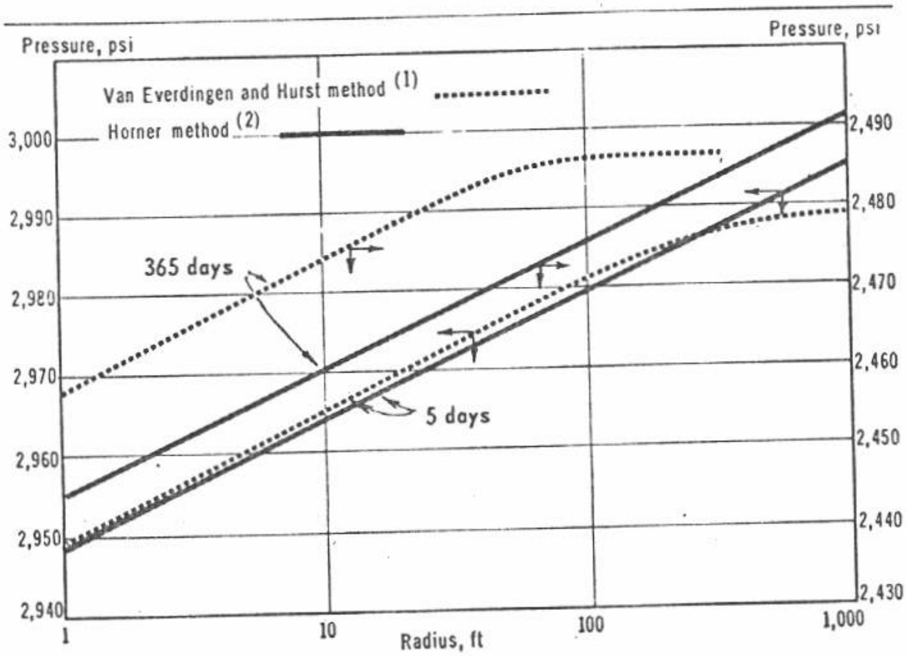
In order to arrive at pressure distributions using the P_T concept, the external radius in Equation 3 was held constant and r_w allowed to vary. This introduces minor errors in the calculations. Dimensionless time T is given by

$$T \doteq \frac{6.33 \times 10^{-3}\, k_o t}{\phi\, \mu_o\, c_{av}\, r_w^2} \tag{4}$$

In Equation 1 values for $E_i(-u)$, where

$$u = \frac{r^2\, \phi\, \mu_o\, c_{av}}{0.02531\ kt},$$

are obtained from Fig. 1 on p. 14 (also reference 5), and values of Y (u) are obtained from Fig. 1.



UNSTEADY-STATE pressure distributions for no flow at finite boundary conditions. Fig. 2.

In the oil pool

$$c_{av} = c_{oav} = S_o c_o + S_w c_w + c_r = [(0.75)\,(8.0) + (0.25)\,(3.0) + 3.5]\ 10^{-6} = 10.25 \times 10^{-6}\ \text{vol/vol/psi} \tag{5}$$

Substitution of known factors into Equation 1 gives

$$p = 3{,}000 + \frac{(70.6)\,(100)\,(0.4)\,(1.35)}{(25)\,(45)} \{E_i\,[- \frac{(r^2)\,(0.22)\,(0.4)\,(10.25 \times 10^{-6})}{(0.02531)\,(25)\,t}]$$

$$- Y\,[\frac{(1{,}320)^2\,(0.22)\,(0.4)\,(10.25 \times 10^{-6})}{(0.02531)\,(25)\,t}]\}$$

$$p = 3{,}000 + 3.39\,[E_i\,(\frac{1.426 \times 10^{-6}\, r^2}{t}) - Y\,(\frac{2.48}{t})] \tag{6}$$

Equation 2 becomes

$$p = 3{,}000 - \frac{(887.4)\,(0.4)\,(1.35)\,(100)}{(6.28)\,(25)\,(45)} P_T = 3{,}000 - 6.78\, P_T \tag{7}$$

Similarly

$$T = \frac{(6.33 \times 10^{-3})\,(25)\,t}{(0.22)\,(0.4)\,(10.25 \times 10^{-6})\, r_w^2} = 1.754 \times 10^5 \frac{t}{r_w^2} \tag{8}$$

Detailed calculations and results are shown in Table 1.

Discussion. This problem illustrates the computation of pressure distributions for a finite boundary case by two methods. The Horner method is approximate while the Van Everdingen and Hurst method is more mathematically rigorous.

Horner considered an exact solution for finite boundary conditions to be too complicated for other than theoretical interest. He derived an approximate solution by reasoning that the pressure drop caused by a well in an infinite reservoir is less than that of an identical well in a finite reservoir by an amount dependent on the quantity of fluid which flows across the external radius. It can be said that this quantity of fluid, if produced from the finite reservoir, would cause an additional pressure drop.

His solution is expressed by Equation 1 which, knowing the

Table 1—Calculation of pressure distribution for finite boundary

(1)	(2)	(3)	(4)	(5)	(6)	(7)
		t = 5 days				
radius, r ft	r_e/r_w = 1,320 ÷ (1)	$\frac{1.426 \times 10^{-6} r^2}{t}$ = $0.2852 \times 10^{-6} \times (1)^2$	$E_i\left[\frac{-1.426\times10^{-6} r^2}{t}\right]$ Ref. 5	Y (2.48/t) = Y (0.496) Fig. 1	(4) − (5)	p=3,000+3.39×(6) psi, eq. 6
⅓	3,960	0.0317×10^{-6}	−16.671	0.66	−17.33	2,941
1	1,320	0.285×10^{-6}	−14.476	0.66	−15.14	2,949
5	264	7.13×10^{-6}	−11.258	0.66	−11.92	2,960
10	132	28.52×10^{-6}	− 9.873	0.66	−10.53	2,964
50	26.4	7.13×10^{-4}	− 6.658	0.66	− 7.32	2,975
100	13.2	2.85×10^{-3}	− 5.275	0.66	− 5.94	2,980
250	5.28	1.78×10^{-2}	− 3.44	0.66	− 4.10	2,986
500	2.64	7.13×10^{-2}	− 2.14	0.66	− 2.80	2,991
1,000	1.32	0.285	− 0.94	0.66	− 1.60	2,995
1,300	1.015	0.482	− 0.57	0.66	− 1.23	2,996

(13)	(14)	(15)	(16)	(17)	(18)
365 days					
$\frac{1.426 \times 10^{-6} r^2}{t}$ = $3.907 \times 10^{-9} \times (1)^2$	$E_i\left[\frac{-1.426\times10^{-6} r^2}{t}\right]$ Ref. 5	Y (2.48/t) = Y (0.00679) Fig. 1	(14) − (15)	p = 3,000 + 3.39 × (16)	$\frac{(r_e/r_w)^2}{(2)^2}$
0.434×10^{-9}	−20.956	145	−165.956	2,437	1.568×10^7
3.907×10^{-9}	−18.761	145	−163.761	2,445	1.742×10^6
9.768×10^{-8}	−15.548	145	−160.548	2,456	69,696
3.907×10^{-7}	−14.161	145	−159.161	2,460	17,424
9.768×10^{-6}	−10.948	145	−155.948	2,471	697.0
3.907×10^{-5}	− 9.561	145	−154.561	2,476	174.2
2.442×10^{-4}	− 7.731	145	−152.731	2,482	27.88
9.768×10^{-4}	− 6.348	145	−151.348	2,487	6.97
3.907×10^{-3}	− 4.961	145	−149.961	2,492	1.74
6.603×10^{-3}	− 4.436	145	−149.436	2,493	1.03

(24)	(25)	(26)	(27)	(28)	(29)
t = 5 days					
$T = 8.770 \times 10^5/(1)^2$ Eq. 8	[0.5 + 2T] ÷ [(18) − 1]	P_T = (25) − (23)	$p = 3,000 - 6.78\ P_T$ Eq. 7	$T = 1.754 \times 10^7/(1)^2$	[0.5 + 2T] ÷ [(18) − 1]
7.893×10^6	1.007	8.517	2,942	15.786×10^7	20.135
8.770×10^5	1.007	7.457	2,949	1.754×10^7	20.138
3.508×10^4	1.007	5.828	2,960	7.016×10^5	20.133
8.770×10^3	1.007	5.137	2,965	1.754×10^5	20.134
3.508×10^2	1.009	3.537	2,976	7.016×10^3	20.162
87.70	1.016	2.870	2,981	1.754×10^3	20.257
14.03	1.063	*2.08	2,986	2.806×10^2	20.897
3.508	1.259	*1.72	2,988	70.16	23.588
0.877	3.046	*1.60	2,989	17.54	48.081
0.519	51.27	. . .	. . . .	10.38	708.67

*Obtained from Fig. 9 and Table 4 of Reference 3.

rock and fluid properties, can be solved for any radius at a certain time or for any time at a certain radius. Its principal advantage is its flexibility of solution for virtually any set of conditions.

The more rigorous method of Van Everdingen and Hurst can also be easily applied in those cases for which solutions are available. For dimensionless-time values greater than 100, Chatas recommends Equation 3 in solving for the dimensionless cumulative pressure-drop values, P_T. As can be seen for 365 days in Table 1 (for r = 250 and 500 ft) this equation is not always applicable. Solution with this equation here would have resulted in a decline in pressure (with increasing r) instead of an increase as should be true. No P_T solutions were available for the other sets of conditions that are left unsolved in Table 1.

Generally the two methods gave comparable results at 5 days and

…nditions by the Horner and Van Everdingen-Hurst methods

(8)	(9)	(10)	(11)	(12)
t = 100 days				
$\frac{1.426 \times 10^{-6} r^2}{t}$ = 1.426 × 10^{-8} × $(1)^2$	$E_i\left[\frac{-1.426 \times 10^{-6} r^2}{t}\right]$ Ref. 5	Y (2.48/t) = Y (0.0248) Fig. 1	(9) − (10)	p = 3,000 + 3.39 × (11)
0.1584 × 10^{-8}	−19.664	36.5	−56.164	2,810
1.426 × 10^{-8}	−17.468	36.5	−53.968	2,817
3.565 × 10^{-7}	−14.253	36.5	−50.753	2,828
1.426 × 10^{-6}	−12.868	36.5	−49.368	2,833
3.565 × 10^{-5}	− 9.653	36.5	−46.153	2,844
1.426 × 10^{-4}	− 8.268	36.5	−44.768	2,848
8.913 × 10^{-4}	− 6.438	36.5	−42.938	2,854
3.565 × 10^{-3}	− 5.053	36.5	−41.553	2,859
1.426 × 10^{-2}	− 3.668	36.5	−40.168	2,864
2.410 × 10^{-2}	− 3.150	36.5	−39.650	2,866

(19)	(20)	(21)	(22)	(23)
$(r_e/r_w)^4$ $(18)^2$	ln (r_e/r_w)	4 $[(r_e/r_w)^2 - 1]^2$	4 $(r_e/r_w)^4$ ln (r_e/r_w) = 4 × (19) × (20)	[3 × (19) − (22) − 2 × (18) − 1] ÷ (21)
459 × 10^{14}	8.26	9.836 × 10^{14}	81.245 × 10^{14}	−7.510
035 × 10^{12}	7.20	12.140 × 10^{12}	87.408 × 10^{12}	−6.450
858 × 10^{9}	5.57	19.428 × 10^{9}	10.824 × 10^{10}	−4.821
036 × 10^{8}	4.88	12.144 × 10^{8}	59.263 × 10^{8}	−4.130
858 × 10^{5}	3.27	19.376 × 10^{5}	63.543 × 10^{5}	−2.528
035 × 10^{4}	2.58	12.000 × 10^{4}	31.321 × 10^{4}	−1.854
777.3	1.66	2,890	5,161	−0.999
48.58	0.97	142.6	188.5	−0.405
3.028	0.28	2.19	3.39	0.554
1.061	0.015	0.0036	0.064	16.39

(30)	(31)	(32)	(33)	(34)	(35)
t = 100 days		t = 365 days			
…= (29) − (23)	p = 3,000 − 6.78 × (30)	T = 6.402 × $10^7/(1)^2$	[0.5 + 2T] ÷ [(18) − 1]	P_T = ~~(34)~~ (33) − (23)	p = 3,000 − 6.78 × ~~(35)~~ (34)
27.645	2,813	57.618 × 10^7	73.492	81.002	2,451
26.588	2,820	6.402 × 10^7	73.502	79.952	2,458
24.954	2,831	2.561 × 10^6	73.492	78.313	2,469
24.264	2,835	6.402 × 10^5	73.489	77.619	2,474
22.690	2,846	2.561 × 10^4	73.593	76.121	2,484
22.111	2,850	6.402 × 10^3	73.929	75.783	2.486
21.896	2,852	1.024 × 10^3	76.209	.....	.....
.....	..	2.561 × 10^2	85.879	.....	.....
.. .		64.02	173.70	.....	.....
		37.88	2,542	.....	.....

100 days. A deviation of 5 to 6% occurred at 365 days. Some of the results are shown plotted in Fig. 2. It can be seen that the Horner method gives slightly lower pressure values. This comparison tends to indicate that for times exceeding 100 days the more rigorous method should be used.

References

1. Van Everdingen, A. F., and Hurst, W., "The Application of the La Place Transformation to Flow Problems in Reservoirs": Trans. AIME vol. 186, 1949, p. 305.

2. Horner, D. R., "Pressure Buildup in Wells": Proc. Third World Pet. Cong. Sec. 11, 1951, pp. 503-522.

3. Chatas, A. T., "A Practical Treatment on Nonsteady-State Flow Problems in Reservoir Systems": Petrol. Engr., Pt. 1, Vol. 25, No. 5, May 1953, p. B42, Pt. 2, Vol. 25, No. 6, June 1953, p. B38; Pt. 3, Vol. 25, No. 9, August 1953, p. B44.

4. Katz, D. L., et al., "Handbook of Natural Gas Engineering": McGraw-Hill Book Co., Inc., 1959, Chapter 10.

5. Guerrero, E. T., "Reservoir Engineering Part 58—How to Find Pressure Distributions for Unsteady-state Flow Conditions": The Oil and Gas Journal, Vol. 61, No. 24, June 17, 1963, pp. 99-101.

Part 62

How to find pressure distribution for a gas reservoir in infinite system

...using steady-state and unsteady-state methods

GIVEN: A gas well completed in a field under development has produced at a constant rate, Q_g, of 4 MMscfd for 5 days. Other data pertaining to this well are as follows:

Initial reservoir pressure, p_i = 3,000 psia

Estimated average flowing pressure $= \dfrac{p_i + p_w}{2} = p_{av} = 2,739$ psia

At average flowing pressure

Viscosity of gas, $\mu_g = 0.022$ cp

Gas-compressibility factor, $Z = 0.87$

Reservoir temperature, T_r = 150° F. or 610° R.

Net sand thickness, $h = 32$ ft

Effective permeability to gas, $k_g = 11$ md

Porosity, $\phi = 19.0\%$

Well radius, $r_w = 4$ in.

Interstitial water saturation, $S_w = 26.0\%$

FIND: The pressure distribution away from the well using steady-state and unsteady-state methods.

Method of solution. The relationship among pressure, position, and time for the flow of gases through porous media is given by

$$\frac{\partial^2 p^2}{\partial r^2} + \frac{1}{r}\frac{\partial p^2}{\partial r} = \frac{\mu_g \phi}{0.00633\, k_g p}\frac{\partial p^2}{\partial t} \quad (1)$$

Under conditions (e.g. in a gas-cycling project) where time is not an important factor (steady-state flow) the right-hand side of Equation 1 is zero. For such conditions Equation 1 can be solved and the results combined with Darcy's law to give

$$Q_g = \frac{0.000703\, k_g h\,(p_i^2 - p_r^2)}{Z\, T_r\, \mu_g \ln (r_e/r)}$$

or

$$p_r^2 = p_i^2 - \frac{Q_g\, Z\, T_r\, \mu_g \ln (r_e/r)}{0.000703\, k_g h} \quad (2)$$

For unsteady-state flow conditions Cornell and Katz[1 2 3] developed a graphical solution of Equation 1 to define pressure distributions.

For infinite boundary conditions or conditions where a well or reservoir has been shut in long enough so that initially the pressure is uniform, the Cornell and Katz solutions were simplified to facilitate computations. Fig. 1 gives these results in a generalized chart for reservoir pressure gradients in terms of dimensionless groups. Values obtained from this graph apply for a regular, homogeneous, horizontal producing formation without water drive and with gravity effects neglected; formation pressure, p_f, is 1,000 psi; well radius is 6 in.; and the production period, $m = 0.010$. Values obtained from Fig. 1 can be adjusted for other conditions with

$$(p_r/p_f)^2_{corr} = 1.00 - [1.00 - (p_r/p_f)^2_{Fig.\,1}]\,100\,m \quad (3)$$

where

$$m = \frac{1{,}424\, \mu_g\, Z\, T_r\, Q_g}{k_g\, h\, p_f^2} \quad (4)$$

To read $(p_r/p_f)^2$ from Fig. 1 dimensionless time is required.

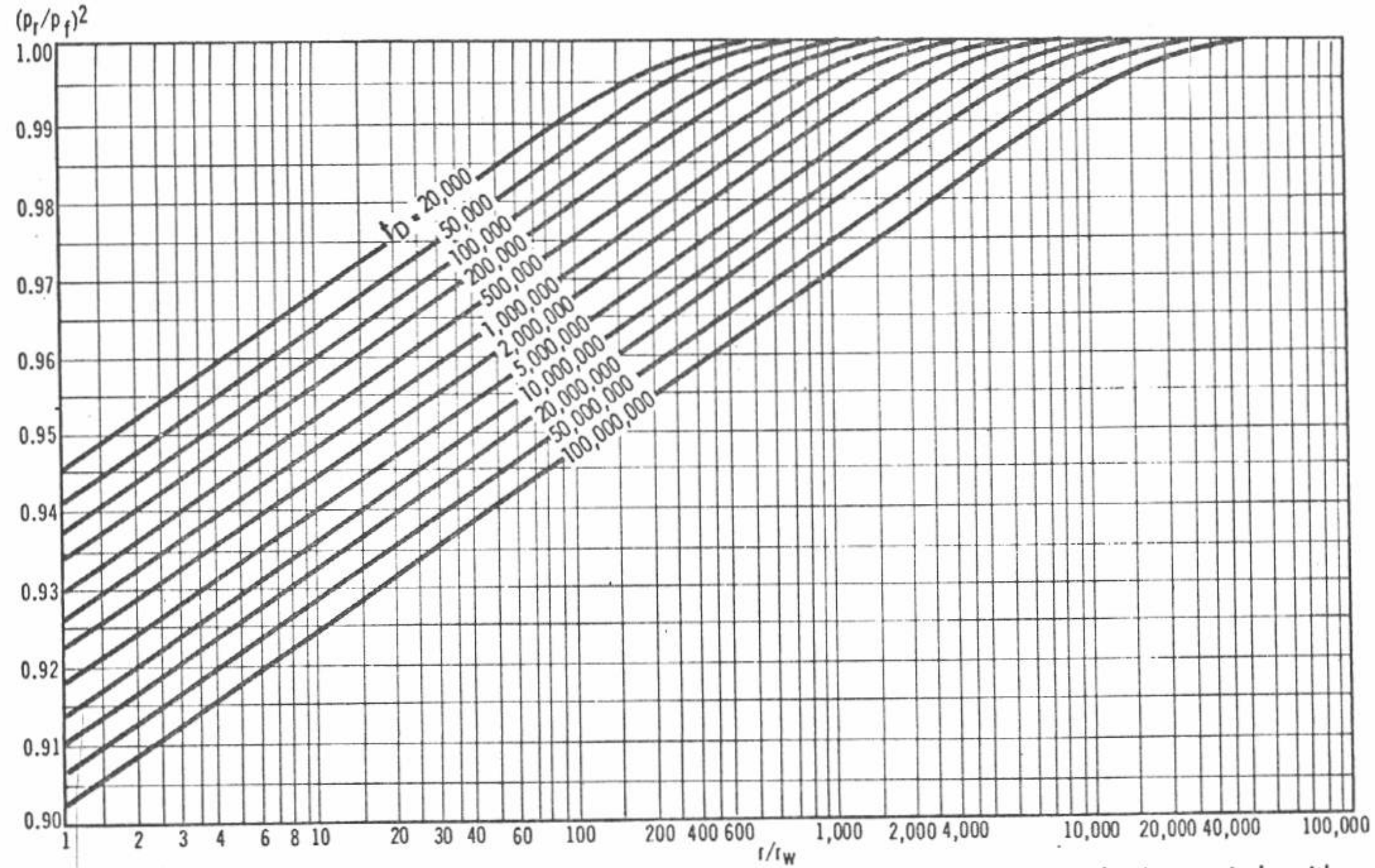
$$t_D = \frac{6.33 \times 10^{-3}\, k_g\, p_{av}\, t}{\mu_g\, \phi\, r_w^2} \quad (5)$$

In Equations 1 through 5 p_i, p_{av}, μ_g, Z, T_r, h, k_g, ϕ, r_w, Q_g are defined with the data and

$p = p_r$ = pressure at radius r, psia

Table 1—Computations for steady-state and unsteady-state pressure distributions

Steady-state flow					Unsteady-state			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
r ft	r_e/r = 933 ÷ (1)	ln (r_e/r) = ln (2)	188,695 × (3)	P_r, psia Eq. (5) (6)	r/r_w = 3 × (1)	$(p_r/p_f)^2$ = Fig. 1	1.00 − $(p_r/p_f)^2$ = 1.00 − (7)	100 m × (8) = 2.1 × (8)
⅓	2,799	7.94	1,498,222	2,739	1	0.922	0.078	0.1638
1	933	6.83	1,288,773	2,777	3	0.932	0.068	0.1428
10	93.3	4.53	854,779	2,854	30	0.956	0.044	0.0924
100	9.33	2.23	420,785	2,929	300	0.978	0.022	0.0462
300	3.11	1.134	213,978	2,964	900	0.989	0.011	0.0231
600	1.56	0.444	83,780	2,986	1,800	0.995	0.005	0.0105
900	1.037	0.036	6,793	2,999	2,700	0.997	0.003	0.0063



GENERALIZED CHART for natural-gas-reservoir pressure gradients for constant - rate production periods with m = 0.010. (After Katz et al.[1]. Courtesy McGraw-Hill Book Co.) Fig. 1.

t = time, days

p_i = initial reservoir pressure, psia

r_e = external radius of drainage for steady-state solution, ft

p_f = stabilized formation pressure = initial reservoir pressure in this problem, psia

m is defined by Equation 4

t_D = dimensionless time

r = radius, ft

Solution: Substitution of known values into Equations 2, 4, and 5, and assuming r_e = 933 ft (80-acre spacing) gives

...or gas well

...low

(10) $(p_r/p_f)_{corr}$ = 1.00 − (9) Eq. (3)	(11) $(p_r/p_f)_{corr}$	(12) p_r psia
0.8362	0.9145	2,744
0.8572	0.9259	2,778
0.9076	0.9527	2,858
0.9538	0.9766	2,930
0.9769	0.9884	2,965
0.9895	0.9947	2,984
0.9937	0.9969	2,991

$$p_r^2 = (3{,}000)^2 - \frac{(4{,}000)\,(0.87)\,(610)\,(0.022)\ln(933/r)}{(0.000703)\,(11)\,(32)}$$

$$= 9{,}000{,}000 - \frac{(3{,}480)\,(13.42)\ln(933/r)}{0.2475}$$

$$= 9{,}000{,}000 - 188{,}693\ln(933/r) \qquad (6)$$

$$m = \frac{(1{,}424)\,(0.022)\,(0.87)\,(610)\,(4{,}000)}{(11)\,(32)\,(3{,}000)^2} = \frac{(31.33)\,(2{,}122{,}800)}{(352)\,(9{,}000{,}000)}$$

$$= \frac{66{,}507{,}324}{3{,}168{,}000{,}000} = 0.021$$

$$t_D = \frac{(6.33\times10^{-3})\,(11)\,(p_{av})\,(5)}{(0.022)\,(0.19)\,(1/3)^2} = \frac{3.133\,p_{av}}{0.00418} = 749.5\,p_{av}$$

By substituting ⅓ ft for r in Equation *6*, an estimate for sand-face flowing pressure is computed as 2,739 psi. Therefore

$$p_{av} = \frac{3{,}000 + 2{,}739}{2} = 2{,}870 \text{ psi}$$

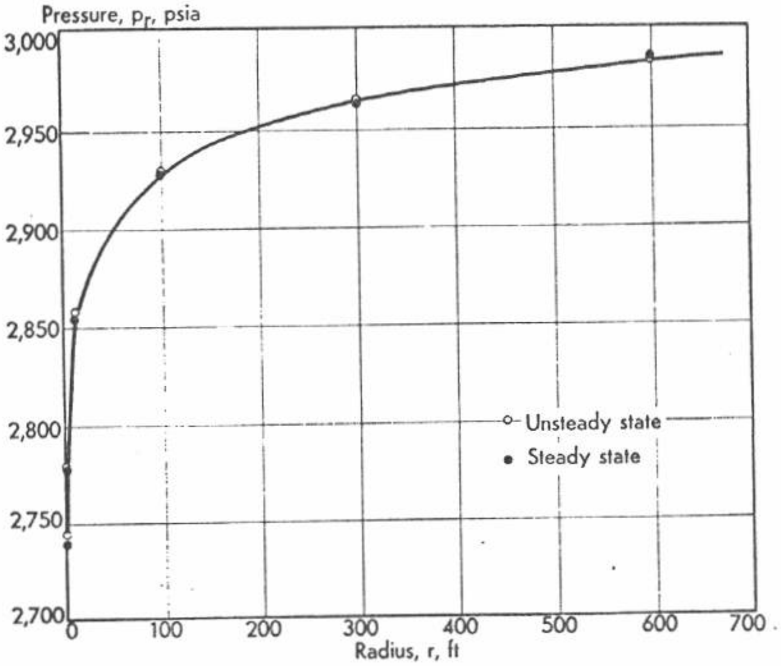
thus,

$$t_D = (749.5)\,(2{,}870) = 2.15\times10^6$$

For r = 100 ft p_r from Equation *6* is

$$p_r^2 = 9{,}000{,}000 - 188{,}693\ln(933/100)$$

$$= 9{,}000{,}000 - (188{,}693)\,(2.23)$$



PRESSURE DISTRIBUTION for gas well in infinite system. Fig. 2.

$$= 8{,}579{,}215$$

$$p_r = 2{,}929 \text{ psia}$$

Using t_D computed above and Fig. 1 ($r/r_w = 100/1/3 = 300$)

$$(p_r/p_f)^2_{Fig.\,1} = 0.978$$

From Equation 3

$$(p_r/p_f)^2_{corr} = 1.00 - [1.00 - 0.978]\,(100)\,(0.021)$$

$$= 1.00 - (0.022)\,(2.1)$$

$$= 1.00 - 0.0462$$

$$= 0.9538$$

$$p_r/p_f = 0.9766$$

$$p_r = 2{,}930 \text{ psia}$$

Discussion. In their initial work Cornell and Katz[1,2] used Equation 2 to evaluate the slope of the pressure gradient at r_w. A means was also devised for correcting for turbulence if desired. At points away from the well bore pressures are computed with Equation 6.

$$p_{r,\,t+\Delta t}^2 = \frac{1}{2}\left[\left(1 - \frac{\Delta r}{2\,r}\right) p^2_{r-\Delta r,\,t} + \left(1 + \frac{\Delta r}{2\,r}\right) p^2_{r+\Delta r,\,t}\right] \qquad (6)$$

Here pressures are related to position, r, and times, t and $t + \Delta t$. Δr is related to time increment, Δt, by

$$\Delta t = \frac{\Delta r^2\, \mu_g\, \phi}{2\, k_g p_{av}} \qquad (7)$$

Equation 6 is difficult to apply at the initial time points or until pressure gradients are developed throughout the reservoir. With this approach unsteady-state pressure distributions can be determined for both infinite and finite boundary conditions.[1] The presentation of the infinite boundary solution in Fig. 1 against the parameters t_D and r/r_w considerably simplifies this method for these boundary conditions.

In this problem the pressure distribution for a gas well is computed by two methods. It is felt that transients in gas reservoirs can be approximated by the steady-state approach.[4,5] To solve Equation 2 the average pressure [$p_{av} = (p_w + p_l) \div 2$] must be estimated to evaluate Z and μ_g. Use of these values at p_l is sufficiently accurate for most of the calculations. The major problem is estimation of a reliable value for r_e.

If the radius of drainage has a value of several hundred feet, error in the estimated value will not be critical since $\ln(r_e/r)$ is involved. Actually, r_e is a variable representing a fictitious radius since fluid may move toward a producing well from points farther away.[4] The recession of this radius cannot continue indefinitely; its upper limit is the radius of the reservoir or the interference with similar parameters from other producing wells. Jenkins and Aronofsky[4] have found that after 5% of the gas originally in place has been produced, r_e stops receding and becomes constant at a value of about one-half its maximum possible value.

The results of this problem are shown in columns 5 and 12 of Table 1 and graphically in Fig. 2. It can be seen that similar pressure distributions were obtained by the steady and unsteady state methods. More than one-half of the pressure drop necessary for the flow conditions occurs within a few feet of the well bore.

References

1. Katz, D. L., et al., "Handbook of Natural Gas Engineering": McGraw-Hill Book Co., 1959, Chap. 10.
2. Cornell, D., and Katz, D. L., "Pressure Gradients in Natural Gas Reservoirs": Trans. AIME, Vol. 198, 1953, pp. 61-70.
3. Cornell, D., "How to Determine Gas Well Interference Graphically": World Oil, Nov. 1952, pp. 187-188.
4. Jenkins, R., and Aronofsky, J. S., "Nonsteady Radial Flow of Gas Through Porous Media": Proceedings Fifth Oil Recovery Conference, TPRC, A&M College of Texas, Dec. 11-12, 1952, pp. 125-135.
5. Bruce, G. H., Peaceman, D. W., Rachford, H. H., Jr., and Rice, J. D., "Calculations of Unsteady-State Gas Flow Through Porous Media": Trans. AIME, Vol. 198, 1953, pp. 79-92.

Part 63

How to find pressure distribution for a gas well in a finite system

. . . using steady-state and unsteady-state methods

GIVEN: A natural-gas reservoir was developed on 80-acre spacing. After 4 years of production, the reservoir pressure declined from 3,500 to 2,600 psia. Other data on this reservoir are as follows:

Porosity, $\phi = 19.0\%$

Estimated average flowing pressure, $p_{av} = 2{,}200$ psi

Gas viscosity, $\mu_g = 0.027$ cp (at p_{av} and T_r)

Gas-deviation factor, $Z = 0.89$ (at p_{av} and T_r)

Reservoir temperature, $T_r = 140°$ F. or 600° R.

Net sand thickness, $h = 31$ ft

Effective permeability to gas, $k_g = 16$ md

Radius of drainage, $r_b = 933$ ft (80-acre spacing)

Radius of well bore, $r_w = 3$ in.

Interstitial-water saturation, $S_w = 25.0\%$

FIND: Pressure distribution about a well in this reservoir if it is produced (after shut-in and pressure stabilization) for 22 days at a rate of 10,000 Mcfd.

METHOD OF SOLUTION: The unsteady-state flow of natural gas through porous media is defined by the diffusivity equation

$$\frac{\partial^2 p^2}{\partial r^2} + \frac{1}{r}\frac{\partial p^2}{\partial r} = \frac{\mu_g \phi}{6.33 \times 10^{-3} k_g p}\frac{\partial p^2}{\partial t} \quad (1)$$

Where:

p = pressure at r, psia

r = radius, ft

μ_g = reservoir gas viscosity, cp

ϕ = porosity, fraction

k_g = effective permeability to gas, md

t = time, days

For determining unsteady-state pressure distributions in a finite system, Bruce et al.[1] developed a stable numerical procedure of solving Equation 1. A portion of their solutions is shown on Fig. 1. These graphs may be used to calculate the pressure at any point in a gas reservoir for the given dimensionless parameter mt_D which represents the product of dimensionless rate and dimensionless time.

In a discussion of the work by Bruce et al.[1], Aronofsky and Jenkins[1] made the observation that, after some time interval, the effective drainage radius will stabilize at one-half the outer radius and that it is possible to calculate, with a revised steady-state formula, the pressure distribution for any later time. Thus

$$p_r^2 = p_f^2 - \left[\frac{Q_g Z T_r \mu_g \ln(0.5 r_b / r)}{0.000703\, k_g h}\right] \quad (2)$$

Where:

Z, T_r, μ_g, r_w, k_g and h are given with the data and

p_f = formation pressure, psia

Q_g = rate of gas production, Mscfd

Other equations required for the solution are[2]

$$m = \frac{1{,}424\, \mu_g Z T_r Q_g}{k_g h p_f^2} \quad (3)$$

and

$$t_D = \frac{6.33 \times 10^{-3} k_g p_{av} t}{\mu_g \phi r_e^2} \quad (4)$$

Where:

m = dimensionless flow rate

t_D = dimensionless time

SOLUTION:

$$m = \frac{(1{,}424)(0.027)(0.89)(600)(10{,}000)}{(16)(31)(2{,}600)^2} = 0.061$$

On Fig. 1, $Q' = 2m = 0.122$

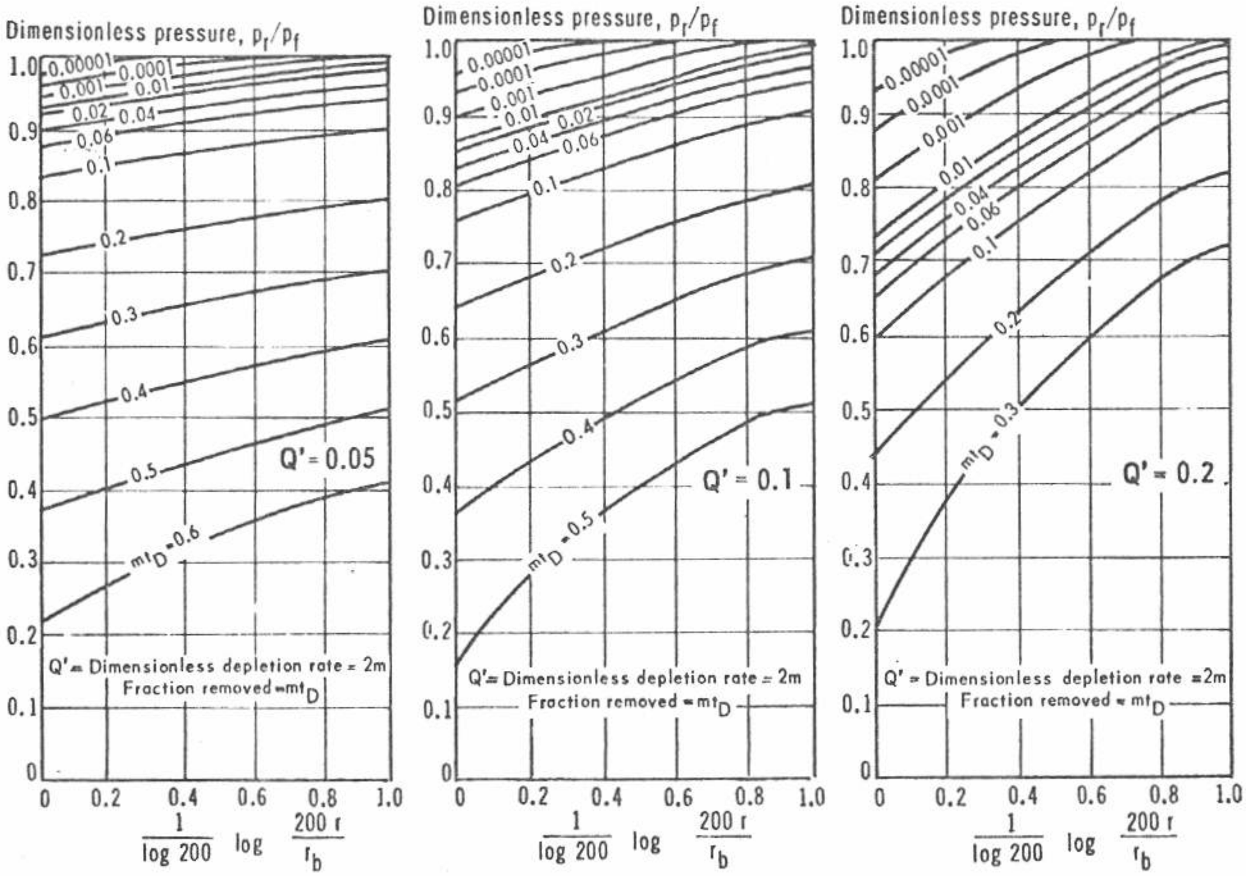
$$t_D = \frac{(6.33 \times 10^{-3})(16)(2{,}200)(22)}{(0.027)(0.19)(933)^2} = 1.098$$

$$mt_D = 0.061 \times 1.098 = 0.0670$$

The abscissa of the graphs on Fig. 1 can be reduced to

$$\frac{1}{\log 200}\log\frac{200\, r}{r_b} = \frac{1}{\ln 200}\left[\ln 200 + \ln\frac{r}{r_b}\right] = 1 + \left(\frac{1}{5.30}\right)\left(\ln\frac{r}{r_b}\right) \quad (5)$$

Substitution of known parameters into Equation 2 gives



GRAPHS for calculation of pressure distribution of radial gas-reservoir system. t_D is based on exterior radius ($r_e = r_b$). After Bruce et al.[1], courtesy AIME. Fig. 1.

$$p_r^2 = p_f^2 - \frac{(10,000)(0.89)(600)(0.027)\ln\left(\frac{0.5 r_b}{r}\right)}{(0.000703)(16)(31)}$$

$$p_r^2 = p_f^2 - 413,493 \ln \frac{467}{r} \qquad (6)$$

To obtain representative results p_f in this equation must be adjusted for 22 days of production at a rate of 10,000 Mcfd.[2] Let the adjustment be based on a volumetric hydrocarbon material balance. Thus

$$A\phi h\,(1-S_w)\,\frac{p_{av\,1}}{p_s}\,\frac{T_s}{T_r}\,\frac{1}{Z_f} - A\phi h\,(1-S_w)\,\frac{p_{av\,2}}{p_s}\,\frac{T_s}{T_r}\,\frac{1}{Z_{av}} = 220 \times 10^6$$

Where:

p_s and T_s = pressure and temperature at standard conditions

S_w = interstitial-water saturation

A = 80 acres,

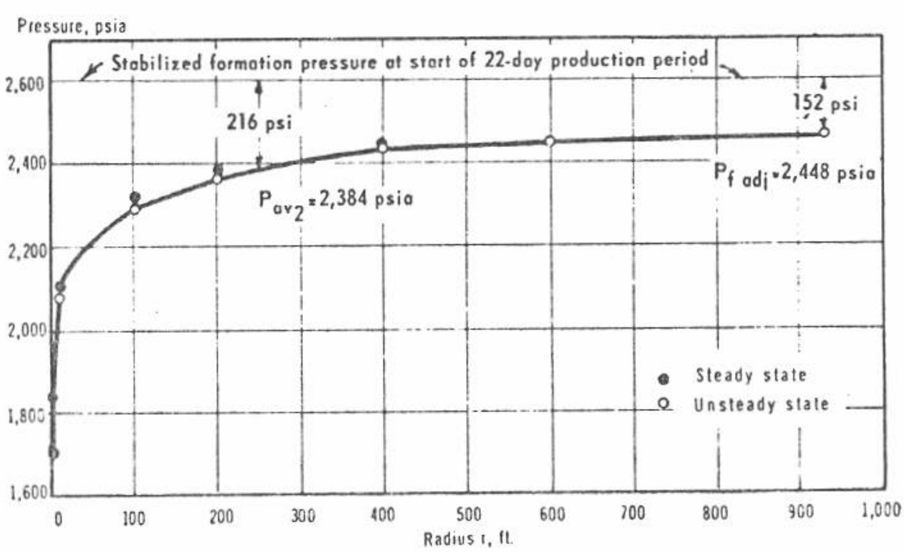
$p_{av\,2}$ = average pressure in drainage area after production of 220 × 10^6 scf of gas

$P_{av\,1}$ = average reservoir pressure of 2,600 psia.

Assuming $Z_f = Z_{av}$

$$p_{av\,1} - p_{av\,2} = \left[220 \times 10^6 \frac{p_s}{T_s} T_r Z\right] \div A\phi h\,(1-S_w)$$

Noting that $\frac{p_s}{T_s} = \frac{10.73}{379}$



PRESSURE DISTRIBUTION for well in developed gas field. Fig. 2.

$$p_{av\,1} - p_{av\,2} = \frac{(220 \times 10^6)(10.73)\,T_r\,Z}{379\,A\,\phi\,h\,(1-S_w)} \qquad (7)$$

or

$$p_{av\,1} - p_{av\,2} = \frac{(220 \times 10^6)(10.73)(600)(0.89)}{(379)(80 \times 43{,}560)(0.190)(31)(0.75)} = 216 \text{ psi}$$

$$p_{av\,2} = 2{,}600 - 216 = 2{,}384 \text{ psi}$$

This pressure is the average value for the drainage area after production of 220×10^6 scf of gas, Fig. 2. It is necessary to estimate $p_{f\,adj}$ for this time at the external radius of drainage (see Fig. 2). It can be shown that the difference between average pressure and pressure at the external boundary for a gas in a bounded system is given by[3]

$$(p_{f\,adj})^2 - (p_{av\,2})^2 = \frac{Q_g\,Z\,T_r\,\mu_g}{0.000703\,k_g\,h}\left(\frac{3}{4}\right) \qquad (8)$$

which from Equation 6

$$(p_{f\,adj})^2 - (p_{av\,2})^2 = 413{,}493 \times 0.75 = 310{,}120$$

$$(p_{f\,adj})^2 = (2{,}384)^2 + 310{,}120 = 5{,}993{,}576$$

$$p_{f\,adj} = 2{,}448 \text{ psia}$$

Thus Equation 6 becomes

$$p_r^2 = (2{,}448)^2 - 413{,}493 \ln\left(\frac{467}{r}\right)$$

$$p_r^2 = 5{,}992{,}704 - 413{,}493 \ln\left(\frac{467}{r}\right) \qquad (9)$$

Both unsteady and steady-state calculations are shown in Table 1. At a radius of 10 ft and using Equation 5

$$1 + \left(\frac{1}{5.30}\right) \ln\left(\frac{r}{r_b}\right) = 1 + \left(\frac{1}{5.30}\right) \ln\left(\frac{10}{933}\right) = 0.15.$$

Reading parts (b) and (c) of Fig. 1 for $mt_D = 0.0670$ and an abscissa value of 0.15 and interpolating for $Q' = 2\,m = 0.122$

$$\frac{p_r}{p_f} = 0.800$$

$p_r = 0.800 \times 2{,}600 = 2{,}080$ psia (unsteady-state value)

Using Equation 9

$p_r^2 = 5{,}992{,}704 - 413{,}493 \ln (467/10) = 4{,}404{,}891$

$p_r = 2{,}099$ psia (steady state)

DISCUSSION: The unsteady-state method used to solve this problem applies for the production of gas at constant rate from linear and radial systems. Bruce, et al.[1] solved Equation 1 using a computer to perform the numerical integration with an implicit form of an approximating equation. Fig. 1 gives a portion of their solutions. Other solutions can be obtained from Reference 1. Since the solutions are presented in graphical form, the method is convenient to apply.

Results obtained with this method are compared with computations using the steady-state equation. A comparison of columns 6 and 11 of Table 1 shows the computed pressures by the two methods to agree fairly well, particularly at large radii.

Boundary conditions for the Bruce, et al. method have been selected such that this method does not apply for values of r_b/r greater than 200. It is seen in Table 1 that no computations were made for the first two radii. Since there is rapid establishment of steady-state flow in the region near the well bore, the use of the radial steady-state equation is recommended. Similarly, the modified steady-state radial-flow formula (Equation 2) recommended by Aronofsky and Jenkins[1] does not apply for $r > \frac{1}{2}\,r_b$.

To solve Equation 4 the average flowing pressure, p_{av}, is required. Column 11 of Table 1 shows the average flowing pressure, to be about 2,070 psi [(1,698 + 2,434) ÷ 2]. This is sufficiently close to the estimated value of 2,200 psi. In this case use of a more correct average pressure would not affect mt_D enough to cause different readings from Fig. 1.

Fig. 2 gives the computed pressure distributions of the well. The behavior is similar to that for slightly compressible fluids (oil and water). More than two-thirds of the pressure drop occurs within a few feet of the well bore. The region near the well bore exhibits a plugging effect, or a point of high resistance to flow, due to its smaller cross-sectional area.

References

1. Bruce, G. H., Peaceman, D. W., Rachford, A. A., Jr., and Rice, J. D., "Calculations of Unsteady-state Gas Flow Through Porous Media:" Trans. AIME Vol. 198, 1953, pp 79-92.
2. Katz, D. L., et al., "Handbook of Natural Gas Engineering:" McGraw-Hill Book Co., 1959, Chapter 10.
3. Craft, B. C., and Hawkins, M. F., "Applied Petroleum Reservoir Engineering:" Prentice-Hall, Inc., 1959, Chapter 6.

Table 1—Computation of unsteady-state and steady-state pressure distributions

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	Unsteady state					Steady state				
r, ft	r_b/r, r_b = 933 ft	$\ln(r/r_b)$	$1+(1/5.30) \times \ln(r/r_b) = 1+(3)/5.30$	p_r/p_f Fig. 2	p_r = (5)×2,600 psia	467/r	ln (467/r)	413,493 × ln (467/r)	p_r^2 Eq. 9	p_r psia
1/4	3,732	−8.21	−0.55	....	....	1,868	7.52	3,109,467	2,883,237	1,698
1	933	−6.83	−0.29	....	....	467	6.14	2,538,847	3,453,857	1,858
10	93.3	−4.53	+0.15	0.800	2,080	46.7	3.84	1,587,813	4,404,891	2,099
100	9.33	−2.23	+0.58	0.881	2,291	4.67	1.54	636,779	5,355,925	2,314
200	4.67	−1.54	+0.71	0.908	2,361	2.34	0.85	351,469	5,641,235	2,375
400	2.33	−0.85	+0.84	0.933	2,426	1.17	0.16	66,159	5,926,545	2,434
600	1.56	−0.44	+0.92	0.940	2,444	......	...	....	....	....
933	1.00	0	+1.00	0.945	2,457	......	...	....	....	....

Part 64

How to find reservoir pressure

using Muskat, and Arps and Smith methods

GIVEN: The following data were obtained during a pressure-buildup test of a West Texas oil well:

Shut-in time, Δt, hours	Well pressure, $p_{w\Delta t}$, psia
0	1,615
1	1,999
3	2,038
7	2,067
10	2,079
20	2,102
30	2,116
40	2,124
50	2,131
60	2,136
70	2,140
80	2,143
90	2,146
100	2,148
110	2,150
130	2,153

FIND: Reservoir pressure in the drainage area of the well using the (1) Muskat[1] method and (2) the Arps and Smith[2] method.

METHOD OF SOLUTION: By assuming that during the buildup period the fluid entry into a well bore is a succession of steady-state operations and the fluid has a uniform density, Muskat derived the equation (shown here in oil-field units)

$$\log (p_e - p_{w\Delta t}) = \log (p_e - p_i) - 0.1 \frac{\rho_o g c \Delta t}{A} \quad (1)$$

Where:

p_e = reservoir pressure in drainage area, psia

$P_{w\Delta t}$ = well pressure at shut-in time Δt, psia

p_i = well pressure at zero shut-in time, psia

$\rho_o g$ = density of oil or oil gradient at avg. reservoir conditions, psi/ft

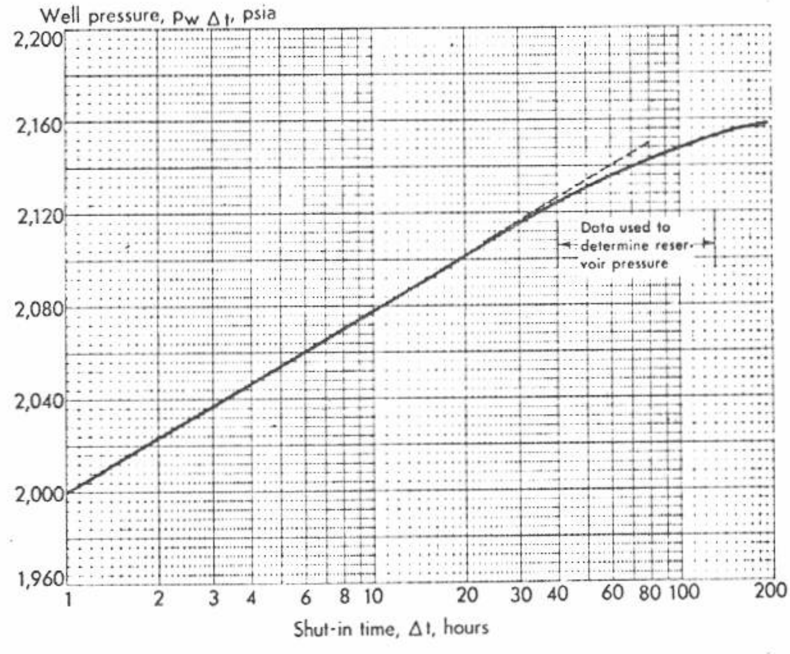
$c = 7.08\, k_o h / [\mu_o B_o \ln (r_e/r_w)]$ b/d/psi

Δt = shut-in time, hours

A = well-bore entry area, sq ft

In a study of the behavior of bounded reservoirs composed of stratified layers, Lefkovits et al.[3] recommend the following equation for finding average reservoir pressure in the drainage area:

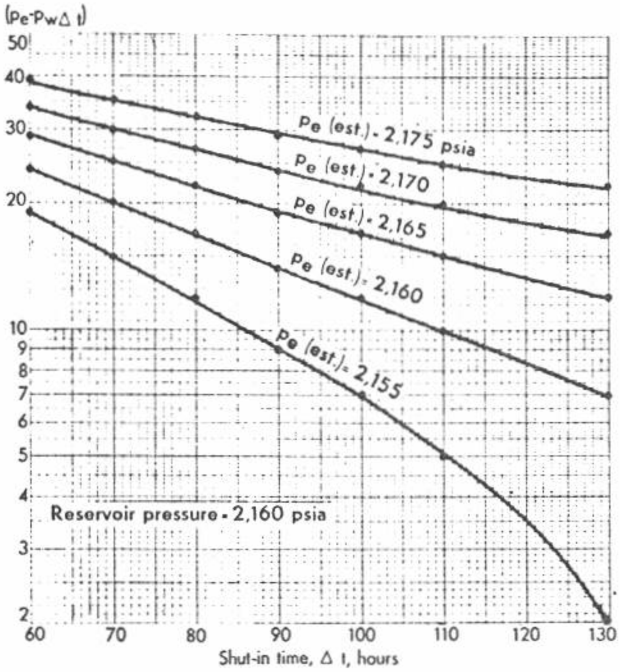
$$p_{w\Delta t} = p_e - be^{C\Delta t}$$



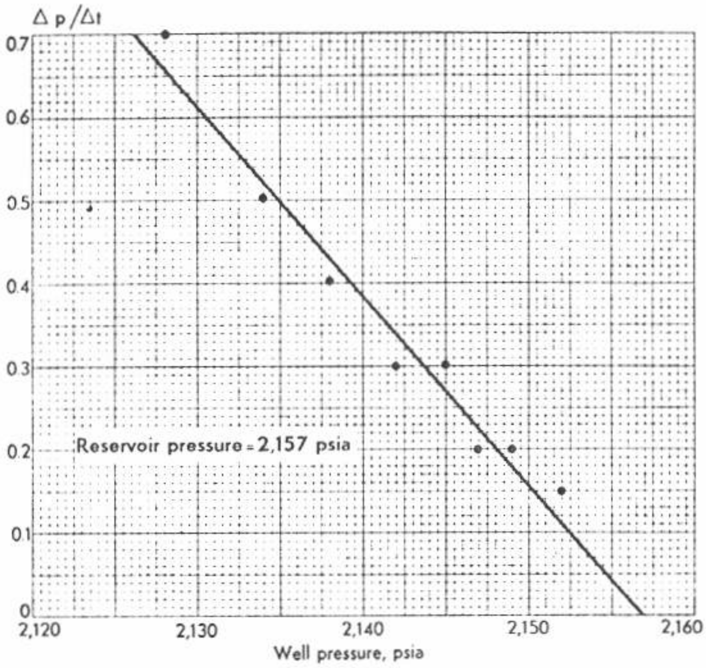
BOTTOM-HOLE PRESSURE build up curve. Fig. 1.

Table 1—Calculations for determination of reservoir pressure by (1)

(1) Shut-in time	(2) Well pressure	(3) Muskat method ($p_e - p_{w\Delta t}$)	(4)	(5)	(6)	(7)
Δt, hours	$p_{w\Delta t}$, psia	p_e = 2,160	p_e = 2,165	p_e = 2,170	p_e = 2,175	p_e = 2,155
40	2,124	36	41	46	51	31
50	2,131	29	34	39	44	24
60	2,136	24	29	34	39	19
70	2,140	20	25	30	35	15
80	2,143	17	22	27	32	12
90	2,146	14	19	24	29	9
100	2,148	12	17	22	27	7
110	2,150	10	15	20	25	5
130	2,153	7	12	17	22	2



DETERMINATION of reservoir pressure by Muskat method[1]. Fig. 2.



DETERMINATION of reservoir pressure by Arps and Smith method[1]. Fig. 3.

or

$$\log (p_e - p_{w\Delta t}) = b + C\Delta t \quad (2)$$

where b and C are constants.

This equation is applicable to the section of the buildup curve subsequent to the straight-line section (see Fig. 1), provided the well has been producing long before shut-in. Equations 1 and 2 are similar if

$$b = \log (p_e - p_i) \text{ and}$$

$$C = -\frac{0.1\, \rho_o g c}{A}$$

If on entry into the well bore, through an area A, the fluid has a uniform density ρ_o, the rate of flow may be expressed as[1 2]

$$Q_o = \frac{A}{\rho_o g} \frac{dp}{dt} \quad (3)$$

Also from Darcy's law

$$Q_o = c\,(p_e - p_{w\Delta t}) \quad (4)$$

Combining Equations 3 and 4 gives

$$\frac{dp}{dt} = \frac{\rho_o g c}{A} (p_e - p_{w\Delta t})$$

or in oil-field units

$$\frac{dp}{dt} = \frac{0.23\, \rho_o g c}{A} (p_e - p_{w\Delta t}) \quad (5)$$

Integration of Equation 5 (with $p_{w\Delta t}$ varying from p_i to $p_{w\Delta t}$) would give Equation 1.

Equation 1 is used in the Muskat method and Equation 5 in the Arps and Smith method. Since $\rho_o g$, c, and A are assumed constant, Equation 1 represents a straight line ($p_e - p_{w\Delta t}$ vs. Δt) on a semilog plot.

In applying the Muskat method, estimate a value for p_e and plot ($p_e - p_{w\Delta t}$) vs. Δt. If a straight line is obtained, the correct value of p_e was used. However, if the curve obtained is concave downward, the estimated value of p_e is too small. Conversely, if the curve is concave upward, the estimated value of p_e is too large. Thus a trial-and-error procedure is involved until a straight line is obtained.

Equation 5 represents a straight line (dp/dt vs. $p_{w\Delta t}$) on a coordinate plot. Here when $dp/dt = 0$, $p_{w\Delta t} = p_e$. Thus this equation is a modification of the Muskat method to eliminate the trial-and-error procedure.

he Muskat method and (2) the Arps and Smith method

(8)	(9)	(10)	(11)
Arps and Smith method			
Δp (2)n − (2)n−1 psi	Δt (1)n − (1)n−1 hours	Δp/Δt (8) ÷ (9)	Avg. $p_{w\Delta t}$ [(2)n + (2)n−1] ÷ 2 psia
		....	.....
7	10	0.70	2,128
5	10	0.50	2,134
4	10	0.40	2,138
3	10	0.30	2,142
3	10	0.30	2,145
2	10	0.20	2,147
2	10	0.20	2,149
3	20	0.15	2,152

SOLUTION: Columns 4 through 7 of Table 1 show computations necessary for the Muskat method.[1] Each of these columns is plotted vs. column 1 in Fig. 2. For $p_e = 2{,}160$ psia a straight line is obtained showing this to be the correct reservoir pressure.

Columns 8 through 11 of Table 1 show computations necessary for the Arps and Smith method. Here increment of pressure, Δp, and time, Δt, are used to approximate

$$\frac{dp}{dt} \approx \frac{\Delta p}{\Delta t}.$$ These values are plotted in Fig. 3 against the average

pressure, p_{wst} for each respective increment.

DISCUSSION: The two methods used in this problem to determine reservoir pressure from pressure-buildup data differ from the Horner and Miller, Dyes, and Hutchinson methods.[4 5 6]

Muskat's method is applicable for equal or unequal time intervals. It is only valid when there is steady-state flow into the well bore so that the productivity index is constant. Use of data from the latter portion of the pressure-buildup curve (Fig. 1) tends to assure such conditions provided the well has been producing long enough before shutin for the pressure waves to reach the exterior boundary (interference with surrounding wells or the sealed boundary of the reservoir). Note on Fig. 1 that this is the section of the buildup curve following the straight line portion.[3]

Arps and Smith have attempted to use the simpler relationship, Equation 5, to obtain a linear relationship and eliminate the necessity of trial-and-error calculations. The intervals of pressure, Δp and time, Δt, must be made small so that $\frac{\Delta p}{\Delta t} \approx \frac{dp}{dt}$ or Δt must be constant when larger time intervals are used.

In this example comparable results were obtained with the two methods (2,160 psia with Muskat method compared to 2,157 psia with Arps and Smith method.) Since both methods are based on the same theory, they should be equally applicable. Lefkovits et al.[3] recommend similar procedures (Equation 2) for determining the average reservoir pressure in the drainage area of wells for stratified (multilayer) reservoirs, fractured dolomite reservoirs, and hydraulically fractured reservoirs. In these reservoirs the initial portion of the pressure build-up curve (following the afterflow period) is linear on a plot of P_{wst} vs. log Δt. Next a slight flattening of the curve may occur, then a rise, and, after a long time, a final flattening.[3]

References

1. Muskat, Morris, "Use of Data on the Buildup of Bottom-Hole Pressure": Trans. AIME, Vol. 123, 1937, p. 44.
2. Arps, J. J., and Smith, A. E., "Practical Use of Bottom-Hole-Pressure Buildup Curves": API Drilling and Production Practice, 1949, p. 155.
3. Lefkovits, H. C., Hazebroek, P., Allen, E. E., and Matthews, C. S., "A Study of the Behavior of Bounded Reservoirs Composed of Stratified Layers": Trans. AIME Vol. 222, 1961, pp. II-43—II-58.
4. Guerrero, E. T., and Stewart, F. M., Reservoir Engineering Part 13 c, "How to Determine Effective Permeability from Pressure Buildup Data Under Infinite Boundary Conditions": The Oil and Gas Journal, Vol. 57, No. 33 (Aug. 10) 1959, p. 119.
5. Guerrero, E. T., and Stewart, F. M., Reservoir Engineering Part 14 d, "How to Determine Effective Permeability from Pressure Buildup Data Under Finite Boundary Conditions": The Oil and Gas Journal, Vol. 57, No. 41 (Oct. 5) 1959, p. 167.
6. Guerrero, E. T., and Stewart, F. M., Reservoir Engineering Part 15 e, "How to Determine Effective Permeability from Multiphase Pressure-Buildup Data Under Finite Boundary Conditions": The Oil and Gas Journal, Vol. 57, No. 44 (Oct. 26) 1959, p. 87.

Part 65

How to find average reservoir pressure of a well-drainage area

by the Matthews et al. method

GIVEN: A pressure-buildup test on a Kansas well yielded the following data:

Buildup time, Δt, hours	$\frac{t_e + \Delta t}{\Delta t}$	Well pressure, $P_{\Delta t}$, psia
0		1,534
1	769	1,942
2	335	1,966
4	193	1,992
6	129	2,005
8	97.0	2,015
10	77.8	2,024
14	55.9	2,036
20	39.4	2,048
30	26.6	2,062
40	20.2	2,072
50	16.4	2,080
80	10.6	2,083
120	7.4	2,085

Other data for this well in a finite system are:

Net sand thickness, $h = 8.5$ ft

Porosity, $\phi = 0.09$

Avg. oil compressibility, $c_{av} = 50 \times 10^{-6}$ vol/vol/psi

External radius of drainage, $r_e = 1{,}000$ ft

Well radius, $r_w = 1/4$ ft

Oil-producing rate prior to shut in, $Q_o = 95$ st-tk b/d.

Oil-formation volume factor, $B_o = 1.31$

Reservoir oil viscosity, $\mu_o = 0.7$ cp

Oil produced since last pressure survey, $N_p = 3{,}040$ bbl

$t_e = N_p/Q_o = 3{,}040/95 = 32$ days or 768 hours

FIND: Average reservoir pressure in the drainage area of the well using the Matthews et al. method.

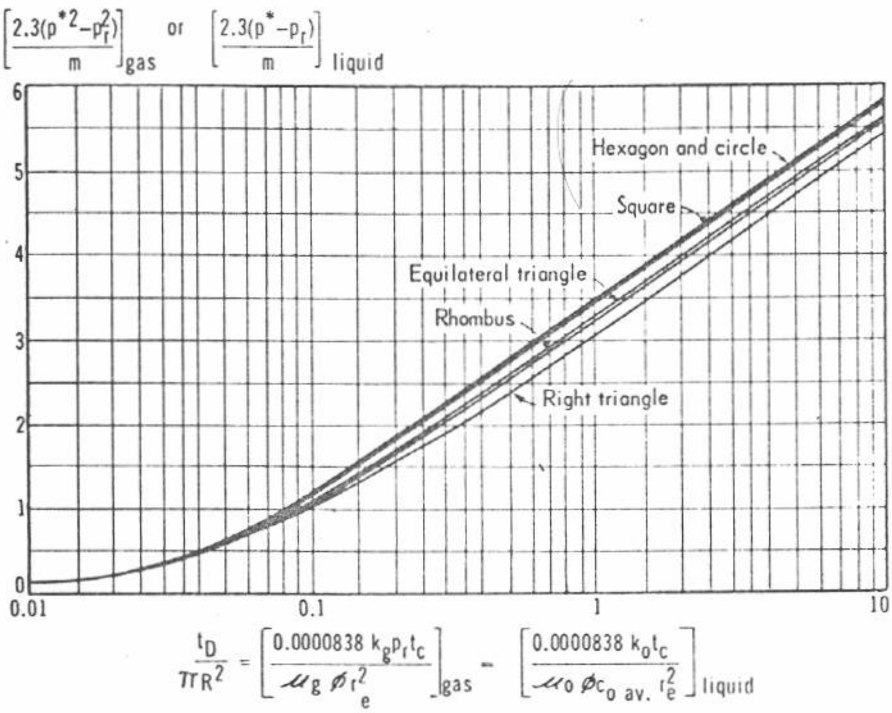
METHOD OF SOLUTION: The basic theory of the Matthews et al. method is similar to the Horner method.[2] To calculate the average pressure in a well-drainage area, a correction is applied to the extrapolated pressure, p* obtained by extrapolating to infinite time

$$\left(\frac{\Delta t}{t_e + \Delta t} \text{ or } \frac{t_e + \Delta t}{\Delta t} = 1.0\right)$$

the proper linear portion of the graph of

$$p_{\Delta t} \text{ vs. } \log\left(\frac{\Delta t}{t_e + \Delta t}\right)$$

The correction, which is a function of t_e, is presented in Fig. 1 for different shapes of the drainage area. These graphs are based on solutions of the diffusivity equation similar to those of References 2, 3, and 4.



PRESSURE FUNCTIONS of single well in center of equilateral enclosures. After Matthews, Brons, and Hazebroek,[1] courtesy AIME. Fig. 1.

$t_c = N_p/Q_o$ = time well has produced from last pressure survey

p^* = pressure at log $[\Delta t/(t_c+\Delta t)]$ = 1.0 on plot of $p_{\Delta t}$ vs. log $[\Delta t/(t_c+\Delta t)]$, psia

p_r = average reservoir pressure in drainage area, psia

m = slope of proper portion of pressure-buildup curve, psi/cycle

t = time, hours

r_w = well radius, ft

The procedure is to compute $T_D/\pi R^2$ (from Equation 1) and read the value of Equation 2 from Fig. 1. Since p^* and m can be obtained from the pressure-buildup plot, p_r can be computed.

The reservoir pressure sought is the equalized pressure that will exist in the reservoir after sufficient time has passed (large or infinite time), that flow of fluid to the depleted area has ceased, and that for practical purposes, no pressure gradients exist in the reservoir. The method is applicable to gases as well as liquids. The abscissa of Fig. 1 for an oil well is

$$t_D/\pi R^2 = (0.0000838\ k_o\ t_c)/(\mu_o\ \phi\ c_{av}\ r_e^2) \quad (1)$$

while the ordinate is

$$2.3\ (p^* - p_r)/m \quad (2)$$

Permeability, k_o, is obtained with the equation[5]

$$k_o = 162.6\ Q_o\mu_o B_o/mh \quad (3)$$

Where:

μ_o, ϕ, c_{av}, r_e, Q_o, B_o, N_p, and h are defined with the data and

t_D = dimensionless time =

$$\frac{2.634 \times 10^{-4}\ k_o t}{\mu_o\ c_{av}\ \phi\ r_w^2}$$

$R = r_e/r_w$

k_o = effective permeability to oil in interwell area, md

SOLUTION:
From Fig. 2

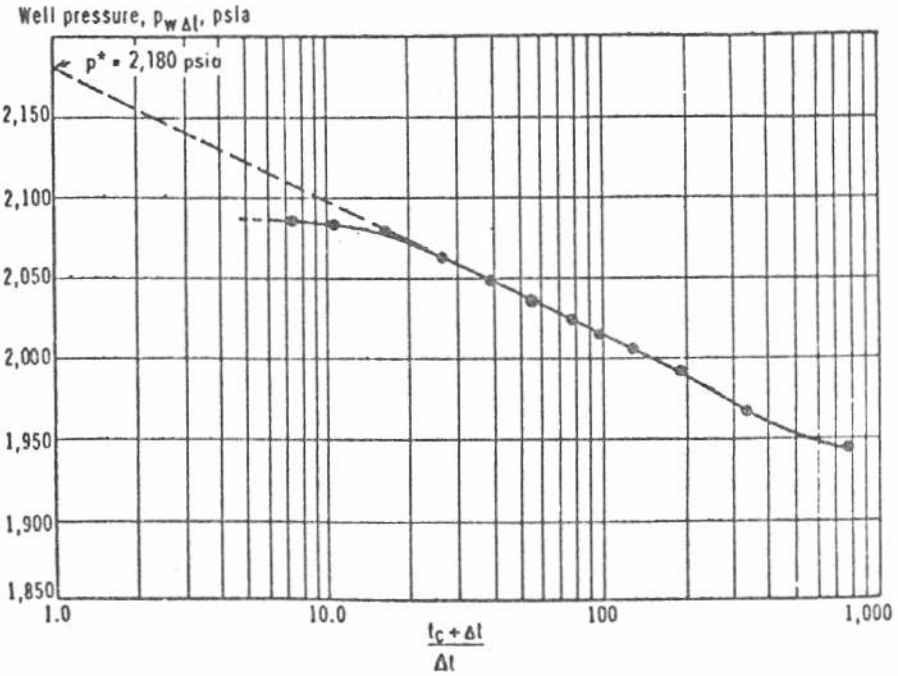
$$\text{Slope} = \frac{p_2 - p_1}{\log\left(\frac{t_c+\Delta t}{\Delta t}\right)_2 - \log\left(\frac{t_c+\Delta t}{\Delta t}\right)_1} = \frac{2{,}097 - 2{,}015}{\log 100 - \log 10} = 82$$

Solving Equation 3 for effective permeability to oil,

$$k_o = \frac{(162.6)(95)(0.7)(1.31)}{(82)(8.5)} = 20.3 \text{ md}$$

Solution of Equation 1 gives

$$\frac{t_D}{\pi R^2} = \frac{(0.0000838)(20.3)(768)}{(0.7)(0.09)(50 \times 10^{-6})(1{,}000)^2} = 0.415$$



PRESSURE BUILDUP curve; slope = 82. Fig. 2.

From Fig. 1 using the curve for hexagon and circle

$$\frac{2.3\,(p^* - p_r)}{m} = 2.6$$

Thus

$$\frac{(2.3)\,(2{,}180 - p_r)}{82} = 2.6$$

$$2{,}180 - p_r = 93$$

$$p_r = 2{,}087 \text{ psia}$$

DISCUSSION: The method used to solve this problem assumes that the reservoir is horizontal, homogeneous, isotropic, and of uniform thickness. The reservoir fluid is assumed to be in a single phase, to have small and constant compressibility, and to have constant viscosity. In spite of these limitations the method is believed applicable[1] to many bounded reservoirs and reservoir fluids. The presence of formation damage at the well bore does not affect the determination of average pressure by this method.

The pressure-buildup behavior of a single oil well in the center of a circular reservoir is given by

$$p_w = p_i + \frac{Q_o\mu_o B_o}{4\pi k_o h}\left[\ln\frac{\Delta t}{t + \Delta t} + Y(t + \Delta t) - Y(\Delta t)\right] \quad (4)$$

Where:

p_w = buildup pressure
p_i = initial reservoir pressure
t = time since well was initially put on production

Based on a modification of this equation the graph for the hexagon and circle, Fig. 1, was developed. The graphs for a well located in the center of a square, equilateral triangle, rhombus, and right triangle were obtained by the method of images.[1]

Fig. 1 shows that in a symmetrically located well, the shape of the surrounding boundary has little effect on the pressure correction function, $(p^* - p_r)/m$. The graphs differ little, regardless of whether the outer boundaries form a hexagon, square, rhombus, equilateral triangle, right triangle, or circle. This tends to support the often-used assumption that the behavior of a circular drainage area may be used to represent the behavior of a square drainage area.

References

1. Matthews, C. S., Brons, F., and Hazebroek, P., "A Method of Determination of Average Pressure in a Bounded Reservoir": Trans. AIME, Vol. 201, 1954, p. 182.

2. Horner, D. R., "Pressure Buildup in Wells": Proc. Third World Petroleum Congress, Section II, W. J. Brill, Leiden, Holland (1951).

3. Miller, C. C., Dyes, A. B., and Hutchinson, C. A., Jr., "The Estimation of Permeability and Reservoir Pressure from Bottom-Hole Pressure-Buildup Characteristics": Trans. AIME Vol. 189, 1950, p. 91.

4. Muskat, M., "The Flow of Homogeneous Fluids Through Porous Media": McGraw-Hill Book Co., Inc., New York, 1937.

5. Guerrero, E. T., and Stewart, F. M., Reservoir Engineering Part 14d, "How to Determine Effective Permeability from Pressure-Buildup Data Under Finite Boundary Conditions": The Oil and Gas Journal, Vol. 57, No. 41, 1959, p. 167.

Part 66

How to find static reservoir pressure for gas well

in infinite system with adaptation of the Horner[1 2] method

GIVEN: A pressure-buildup test was performed on a well located in a gas field under development on 640-acre spacing. The data obtained are shown in Table 1.

Other data were as follows:

Net sand thickness, h = 22 ft.

Reservoir gas viscosity, μ_g = 0.025 cp.

Reservoir temperature, T_r = 125° F. or 585° R.

Gas-deviation factor at reservoir conditions, Z = 0.87.

Producing rate, Q_g = 9,500 Mscfd.

Cumulative gas production, G_p = 75,212 Mscf.

FIND: (1) Initial reservoir pressure assuming infinite boundary conditions. (2) Effective permeability to gas in interwell area.

METHOD OF SOLUTION: Aronofsky and Jenkins[3 4] have shown that the performance of a gas well producing at constant rate is similar to that for an oil reservoir. Based on this work and that of Van Everdingen and Hurst,[5] Tracy[2] has shown that for infinite boundary conditions and an oil reservoir, the pressure at time t and radius r_w is related by

$$p_r = p_i - \frac{Q_o\mu_o B_o}{4\pi k_o h}\left[\ln\left(\frac{k_o t_e}{\phi_{HC}\mu_o c_{oe} r_w^2}\right) + 0.80907\right] \quad (1)$$

while for a gas reservoir

$$p_r^2 = p_i^2 - \frac{Q_g\mu_g T_r Z}{2\pi k_g h T}\left[\ln\left(\frac{k_g t_e p_i}{\phi\,\mu_g r_w^2}\right) + 0.80907\right] \quad (2,$$

Table 1—Data obtained on pressure-buildup test

Build-up time Δt, hours	Well pressure $p_{w\Delta t}$, psia	$p_{w\Delta t}^2$	$\frac{t_e + \Delta t}{\Delta t} = \frac{190 + \Delta t}{\Delta t}$
0	1,597	2,550,400	...
½	1,650	2,722,500	381
1	1,720	2,958,400	191
2	1,805	3,258,000	96
4	1,852	3,429,900	48.5
8	1,896	3,594,800	24.8
12	1,922	3,694,100	16.8
16	1,939	3,759,700	12.9
24	1,963	3,853,400	8.92
36	1,987	3,948,200	6.28
48	2,004	4,016,000	4.96
72	2,027	4,108,700	3.64

Where:

p_r = reservoir pressure, atm

p_i = initial reservoir pressure, atm

Q_o = oil-production rate, cc/sec (st-tk conditions)

μ_o = reservoir-oil viscosity, cp

B_o = oil formation-volume factor

k_o = effective permeability to oil, darcies

h = net sand thickness, cm

t_e = apparent time well produced before shut-in, seconds

ϕ_{HC} = hydrocarbon porosity, fraction

ϕ = porosity, fraction

c_{oe} = effective oil-compressibility factor, vol/vol/atm

r_w = well radius, cm

Q_g = gas-production rate, std cc/second

μ_g = reservoir gas viscosity, cp

T_r = reservoir temperature, °R.

T_s = standard temperature, °R.

k_g = effective permeability to gas, darcies

Z = gas-deviation factor

Equations 1 and 2 are similar except that in Equation 2 (1) p_r has been replaced by p_r^2 and p_i by p_i^2, (2) in the denominator of the coefficient of the bracket, 4 has been replaced by 2, (3) Q_oB_o has been replaced by $Q_gZ\ T_r/T_s$ and (4) the compressibility factor c_{oe} has been replaced by the reciprocal of p_i.

Following this analogy the procedures used for oil reservoirs[1] can be adapted for gas reservoirs.[2] Thus the sand-face pressure for Δt time after shut-in is given by the equations grouped in Table 2, below.

The method of obtaining the initial reservoir pressure is the same as that for oil reservoirs[1,6] except that the pressure-buildup data are plotted as $p_{w\Delta t}^2$ vs. log $(t_e + \Delta t)/\Delta t$. Such a plot for this problem is Fig. 1. For infinite shut-in time $(t_e + \Delta t)/\Delta t = 1.0$, $p_{w\Delta t}^2$ is read and its square root is the initial reservoir pressure.

The effective permeability to gas for the area away from the well bore is computed with Equation 5. The slope m is obtained from Fig. 1.

Another equation recommended by the API[7] for determining effec-

Table 2—Equations showing determination of sand-face pressure

$$p_{w\Delta t}^2 = p_i^2 - \frac{1{,}637\ Q_g\mu_g T_r Z}{k_g h} \log\left(\frac{t_e + \Delta t}{\Delta t}\right) \qquad (3)$$

Where:

$$t_e = \frac{24\ G_p}{Q_g} = \frac{(24)\ (75{,}212)}{9{,}500} = 190 \text{ hours} \qquad (4)$$

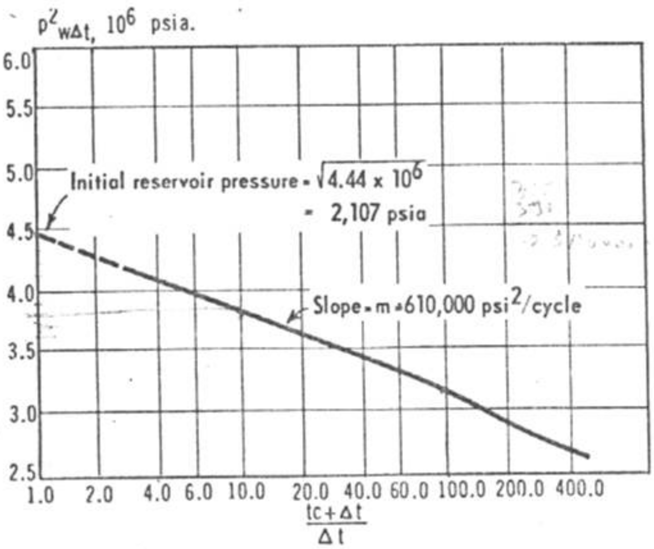
$$m = \frac{1{,}637\ Q_g\mu_g T_r Z}{k_g h} = \frac{(p_{w\Delta t}^2)_2 - (p_{w\Delta t}^2)_1}{\log[(t_e+\Delta t)/\Delta t]_1 - \log[(t_e+\Delta t)/\Delta t]_2} \qquad (5)$$

The parameters are as defined above except the units are now p_i, psia; Q_g, Mscfd; μ_g, cp; T_r, °R; k_g, md; h, ft; G_p, Mscf; t_e, hours; and

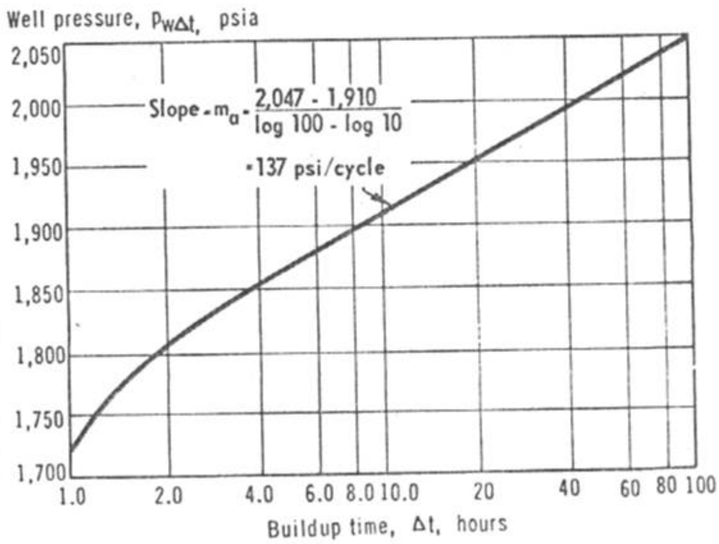
$p_{w\Delta t}$ = well sand-face pressure for Δt time after shut-in, psia

Δt = time well has been shut in, hours

m = slope of proper portion of pressure-buildup curve, psi²/cycle



INITIAL RESERVOIR PRESSURE of a gas field can be found by using same procedure usually applied to oil reservoirs, with slight modifications. Fig. 1.



PRESSURE BUILDUP DATA for gas wells, plotted in the same manner as is done for oil wells. But applicability is less broad than in Fig. 1. Fig. 2.

tive permeability to gas from pressure-buildup data is

$$k = \frac{815\, Q_g \mu_g T_r Z}{m_a\, h\, p_r} \qquad (6)$$

where m_a = slope of proper portion of pressure-buildup curve (psi/cycle) when p_{wst} is plotted vs. log Δ t. This equation is obtained from Equation 3 which can be written as

$$(p_i - p_{wst})(p_i + p_{wst}) = \frac{1{,}637\, Q_g \mu_g T_r Z}{k_g h}\left[\log(t_e + \Delta t) - \log \Delta t\right]$$

If $p_i + p_{wst} \approx 2\, p_r = 2\, p_i$

and $\log(t_e + \Delta t) \approx \log t_e$

then

$$p_{wst} = p_i - \frac{818\, Q_g\, \mu_g\, T_r\, Z}{k_g h\, p_i}\left(\log \frac{t_e}{\Delta t}\right) \qquad (7)$$

Equation 7 shows that plotting p_{wst} vs. log Δ t (Fig. 2) results in a straight line with the slope

$$m_a = \frac{818\, Q_g\, \mu_g\, T_r\, Z}{k_g\, h\, p_i} = \frac{(p_{wst})_2 - (p_{wst})_1}{\log \Delta t_2 - \log \Delta t_1} \qquad (8)$$

SOLUTION: from Fig. 1, p_{wst}^2 is read at infinite time $(t_e + \Delta t)/\Delta t = 1.0$ and $p_i^2 = \sqrt{4.44 \times 10^6} = 2{,}107$ psia.

Using abcissa values of 1.0 and 10.0 and Equation 5

$$m = \frac{4.44 \times 10^6 - 3.83 \times 10^6}{\log 10 - \log 1.0} = 610{,}000$$

Also using Equation 5

$$k_g = \frac{(1{,}637)(9{,}500)(0.025)(585)(0.87)}{(610{,}000)(22)} = 14.7 \text{ md}$$

Using the slope obtained on Fig. 2 (m = 137 psi/cycle) and Equation 8

$$k_g = \frac{(818)(9{,}500)(0.025)(585)(0.87)}{(137)(22)(2{,}107)} = 15.6 \text{ md}$$

Discussion: This problem shows that with slight modifications the procedure used to obtain initial reservoir pressure of an oil reservoir can be applied to gas reservoirs (Fig. 1). In fact, Fig. 2 shows the pressure-buildup data for a gas well plotted in the same manner as is done for oil wells. Here applicability is less broad than in Fig. 1. The plot is based on Equation 7 which applies when the reservoir pressure changes little from point to point during production or when the reservoir pressure is high (more than 1,000 psia). Also, the apparent producing time before shut-in, t_e must be large compared to shut-in times, Δ t. This latter assumption is not too critical since time is treated as a logarithmic function in Equation 7. As an example, at a buildup time of 72 hours, $\log(t_e + \Delta t)/\Delta t = \log(190 + 72)/72 = 0.56$, while $\log t_e/\Delta t = \log 190/72 = 0.42$.

Figs. 1 and 2 show that the pressure-buildup performance of a gas well is similar to that of an oil well. At small shut-in times the buildup behavior is influenced by gas afterflow into the well and differences in formation permeability near and away from the well bore. Tracy[2] reports that studies made to determine the duration of time that gas-well pressure-buildup data are affected by fillup of the well bore show that the afterflow effect cannot be considered negligible until 60 minutes' buildup time has elapsed. This does not include the effects of formation damage which may result in even a longer time required to reach the straight line or proper buildup portion of the data.

For gas wells whose pressure disturbances have reached a boundary or interfered with pressure disturbances from adjacent wells, the straight-line portion of the pressure-buildup data is followed with the curve tending to level off. This tendency is less prevalent in gas wells than oil wells because of the larger spacings (320 and 640 acres/well) used in gas fields.

References

1. Horner, D. R., "Pressure Buildup in Wells": Proc. Third World Petroleum Congress, Section II, E. J. Brill, Leiden, Holland (1951).

2. Tracy, G. W., "Diagnosing Productivity Problems in Gas Wells": Ninth Oil Recovery Conference, TPRC, Mar. 26, 1956; also The Oil and Gas Journal, Aug. 6, 1956, p. 84.

3. Aronofsky, J. S., and Jenkins, R., "Nonsteady Radial Flow of Gas Through Porous media": Proc. Fifth Oil Recovery Conference, TPRC pp. 125-135, A. & M. College of Texas, 12/11-12/52.

4. Aronofsky, J. S., and Jenkins, R., "A Simplified Analysis of Unsteady Radial Gas Flow": Trans. AIME, Vol. 201, 1954, p. 149.

5. Van Everdingen, A. F., and Hurst, W., "The Application of the La Place Transformation to Flow Problems in Reservoirs": Trans. AIME, Vol. 186, 1949, pp. 305-324.

6. Guerrero, E. T., and Stewart, F. M., "How to Determine Effective Permeability from Pressure-Buildup Data Under Infinite Boundary Conditions": The Oil and Gas Journal, Vol. 57, No. 33, Aug. 10, 1959, pp. 119-120.

7. Bulletin D6, "Selection and Evaluation of Well-Completion Practices": API, Dallas, July 1955.

Part 67

How to find static reservoir pressure for gas well

by adaptation of Miller, Dyes, and Hutchinson method[1 2 3]

GIVEN: A pressure-buildup test was run on a gas well in a field on 640 - acre spacing. The following data were obtained:

Shut-in time Δt, hours	Well pressure, $p_{w\Delta t}$, psia	$(p_{w\Delta t})^2$
0	1,847	3,411,400
1	1,970	3,880,900
3	2,084	4,343,100
6	2,128	4,528,400
10	2,159	4,661,300
15	2,184	4,769,900
22	2,207	4,870,800
34	2,233	4,986,300
45	2,246	5,044,500
65	2,260	5,107,600

Other data were as follows:

Approximate reservoir pressure, $p_r = 2{,}430$ psia.

Net sand thickness, $h = 27$ ft.

Reservoir temperature, $T_r = 145°$ F. or 605° R.

Gas deviation factor at reservoir conditions, $Z = 0.85$.

Producing rate, $Q_g = 10{,}000$ Mscfd.

Porosity, $\phi = 18\%$.

FIND: Compute the reservoir pressure assuming finite boundary conditions.

METHOD OF SOLUTION: For gas reservoirs dimensionless time is defined by the equation:

$$t_D = \frac{0.000528\, k_g t}{\phi\, c_{g\,av.}\, \mu_g\, r_e^2} \qquad (1)$$

considering t equivalent to Δt then:

$$\Delta t = \frac{1{,}900\, t_D\, \phi\, c_{g\,av.}\, \mu_g\, r_e^2}{k_g} \qquad (2)$$

letting $c_{g\,av.} = 1/p_r$

$$\Delta t = \frac{1{,}900\, t_D\, \phi\, \mu_g\, r_e^2}{k_g\, p_r} \qquad (3)$$

also

$$\Delta t = \frac{1{,}900\, t_D\, \phi\, \mu_g\, r_e^2}{k_g\, p_r}\; \frac{\pi h}{\pi h}$$

$$\times \frac{Q_g T_r Z\, 1{,}637}{Q_g T_r Z\, 1{,}637} = 1{,}900\, t_D\, (\pi r_e^2)\, h\, \phi$$

$$\times \frac{1{,}637\, Q_g\, T_r\, Z\, \mu_g}{k_g\, h}$$

$$\div\ 2\, Q_g\, T_r\, Z\, p_r\, 1{,}637\, \pi$$

since

$$43{,}560\ A = \pi\, r_e^2 \text{ and}$$

$$m = \frac{1{,}637\, Q_g\, T_r\, Z\, \mu_g}{k_g\, h} \qquad (4)$$

$$\Delta t = 1{,}900\, t_D \frac{A\, h\, \phi}{Q_g}\, \frac{m}{T_r\, Z\, p_r}$$

$$\times \frac{1}{1{,}637\, \pi}\,(43{,}560)$$

$$\Delta t = 16{,}090\, t_D \frac{A\, h\, \phi}{Q_g}\, \frac{m}{T_r\, Z\, p_r} \qquad (5)$$

Where:

p_r, h, T_r, Z, Q_g and ϕ are defined with data and

t_D = dimensionless time

k_g = effective permeability to gas, md

t = time, hours

Δt = shut-in or buildup time, hours

$c_{g\,av.}$ = average compressibility to gas at reservoir conditions, 1/psi

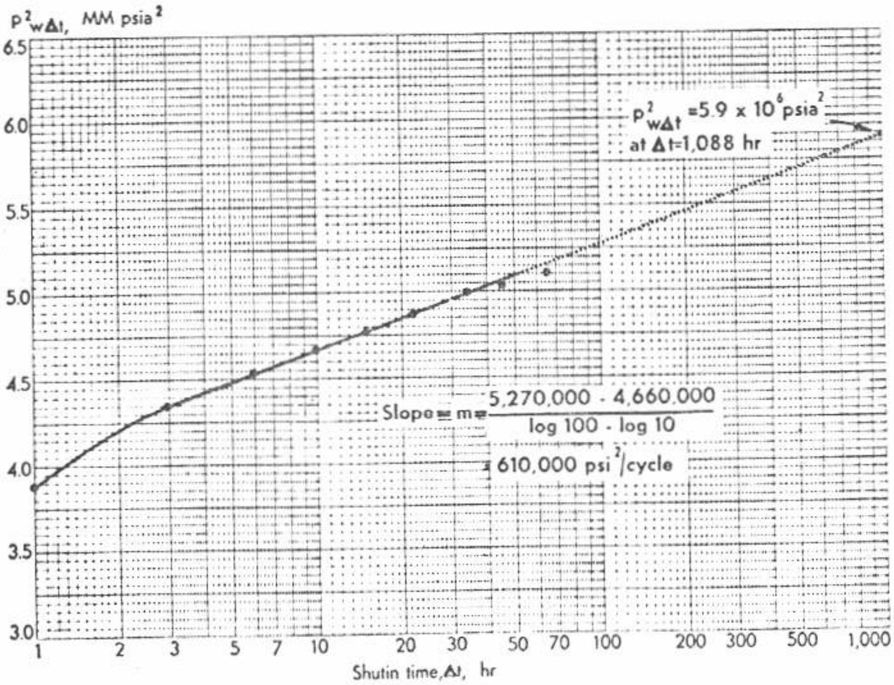
μ_g = reservoir gas viscosity, cp

r_e = radius of drainage, ft

A = area in acres

m = slope of proper portion of pressure buildup curve.

If the dimensionless time, t_D, necessary for a well to reach static reservoir pressure can be estimated, then Equation 5 can be used to compute the actual time, Δt. For condi-



PRESSURE-BUILDUP CURVE for gas well. Fig. 1.

tions of no flow across the exterior boundary, r_e, the t_D necessary for pressure buildup to static conditions is about 0.1 while for conditions of constant pressure at the exterior boundary, the value to t_D is 0.445.

The pressure-buildup data are plotted as $(p_{w\Delta t})^2$ vs. log Δt, and the proper straight-line portion of the plot is extrapolated to the Δt value computed by Equation 5. At this point the static reservoir pressure is read. This method is trial and error, since p_r must be estimated to solve Equation 5. The value estimated should be within 1% of the value obtained through computation of Δt, otherwise the procedure is repeated.

For gas-well spacings of about 320 acres or greater, the static reservoir pressure can be determined assuming a constant pressure at the external boundary, or $t_D = 0.445$. For such conditions Equation 5 becomes:

$$\Delta t = 7{,}160 \frac{A\,h\,\phi}{Q_g}\,\frac{m}{T_r\,Z\,p_r} \qquad (6)$$

where A is now well spacing in acres per well.

SOLUTION: The pressure-buildup data for this problem are plotted on Fig. 1. The slope, m, of the straight-line portion of the curve is 610,000. Reservoir pressure has been approximated as 2,430 psia. Thus with Equation 6

$$\Delta t = 7{,}160 \frac{(640)\,(27)\,(0.18)}{10{,}000} \times \frac{610{,}000}{(605)\,(0.85)\,(2{,}430)}$$

$$= (7{,}160)\,(0.311)\,(0.488)$$

$$= 1{,}088 \text{ hours}$$

From Fig. 1

$(p_{w\Delta t})^2$ at $\Delta t = 1{,}088$ hours

$= 5{,}900{,}000$

$p_{w\Delta t} = 2{,}429$ psia

DISCUSSION: The method used to solve this problem is based on ability to compute the time needed for pressure to build up to static reservoir conditions. Equation 5 is developed from the equation for dimensionless time (Equation 1). Fig. I-2 in Reference 2 gives dimensionless pressure drop versus dimensionless time for conditions of (1) no flow across the drainage radius, and (2) constant pressure at the drainage radius.

This plot indicates that at dimensionless times of 0.28 and 0.75 for the two cases, complete pressure buildup has occurred (dimensionless pressure drop equals zero). That is, on a plot such as Fig. 1, the curve would become horizontal at shut-in times equivalent to these dimensionless times, respectively. The tailoff portions of most pressure-buildup curves are not sufficiently defined for reliable extrapolation and thus cannot be used in reservoir-pressure determination.

The straight-line portions of the curves on Fig. I-2 of Reference 2 are equivalent to the proper pressure-buildup portions of pressure-buildup curves. Extrapolation of these straight-line portions shows that a dimensionless pressure drop of zero occurs at dimensionless times of 0.1 to 0.445, respectively.

These data are useful since interpretation of pressure-buildup curves requires that the proper pressure-buildup portion be measured and defined. This portion of the pressure-buildup curve can easily be extrapolated to the times given by Equations 5 or 6.

This method has the limitation that it must be decided if a particular well best fits the case of "no flow across the drainage radius," or "constant pressure at the drainage radius." For gas fields where well spacings normally exceed 300 acres per well, the latter case is most applicable. In addition to this limitation, the method requires knowledge of net sand thickness, porosity, reservoir temperature, and gas deviation factor.

References

1. Miller, C. C., Dyes, A. B., and Hutchinson, C. A., Jr., "The Estimation of Permeability and Reservoir Pressure from Bottom-Hole Pressure Buildup Characteristics": Trans. AIME, Vol. 189, 1950, p. 98.
2. Perrine, R. L., "Analysis of Pressure-Buildup Curves": API Drilling and Production Practice, 1956, pp. 482-495.
3. Guerrero, E. T., and Stewart, F. M., "Reservoir Engineering Part 14—How to Determine Effective Permeability from Pressure-Buildup Data Under Finite Boundary Conditions": The Oil and Gas Journal, Vol. 57, No. 41, Oct. 5, 1959, pp. 167-169.

Part 68

How to find static reservoir pressure for gas well in finite system

by Horner[1,2] and Matthews et al.[3] methods

GIVEN: A pressure-buildup test was performed on a well located in a gas field on 640-acre spacing. Data obtained were as follows:

Shut-in time, Δt, hours	$\frac{t_c+\Delta t}{\Delta t}$ (t_c=2,400 hours)	Well pressure, $p_{w\Delta t}$ psia	$p_{w\Delta t}^2$
0	......	1,742	3,034,564
1	2,401	1,865	3,478,225
3	801	1,979	3,916,441
6	401	2,023	4,092,529
10	241	2,054	4,218,916
15	161	2,079	4,322,241
22	110	2,102	4,418,404
34	71.6	2,128	4,528,384
45	54.3	2,145	4,601,025
65	37.9	2,170	4,708,900
126	20.0	2,190	4,796,100

Net sand thickness, h = 54 ft

Reservoir temperature, T_r = 145° F. or 605° R.

Gas-deviation factor at reservoir conditions, Z = 0.85

Producing rate, Q_g = 10,000 Mscfd

Porosity, ϕ = 18.0%

Cumulative gas production since last pressure survey, G_p = 1,000,000 Mscf

Well radius, r_w = 1/3 ft

Estimated reservoir pressure, p_r = 2,320 psia

Reservoir gas viscosity, μ_g = 0.12 cp

$t_c = G_p/Q_g = 1{,}000{,}000 \div 10{,}000 = 100$ days or 2,400 hours

Reservoir pressure at last pressure survey, p_1 = 2,390 psia

FIND: Compute the reservoir pressure in the drainage area of the well assuming finite boundary conditions: (1) using the Horner method, and (2) using the Matthews et al. method.

METHOD OF SOLUTION: In the problem starting on p. ~~34~~ 160 it was shown that the pressure-buildup analysis techniques developed by Horner[1] for liquids could easily be adapted for gases. The necessary equations for finite-boundary-condition cases are

$$p^{*2} = p_1^2 - (m/2.3)\, y(\mu_1) \quad (1)$$

$$p_r^2 = p_1^2 - (m/2.3\, \mu_1) \quad (2)$$

Where:

p^* = pressure at $\log\left(\frac{t_c + \Delta t}{\Delta t}\right)$ = 1.0 on plot of $p_{w\Delta t}^2$ vs. $\log \frac{t_c + \Delta t}{\Delta t}$

p_1 = reservoir pressure at last pressure survey, psia

m = slope of proper portion of pressure-buildup curve, psi²/cycle

$\mu_1 = (r_e^2\, \phi\, \mu_g)/(4\, k_g\, t_c\, p_r)$

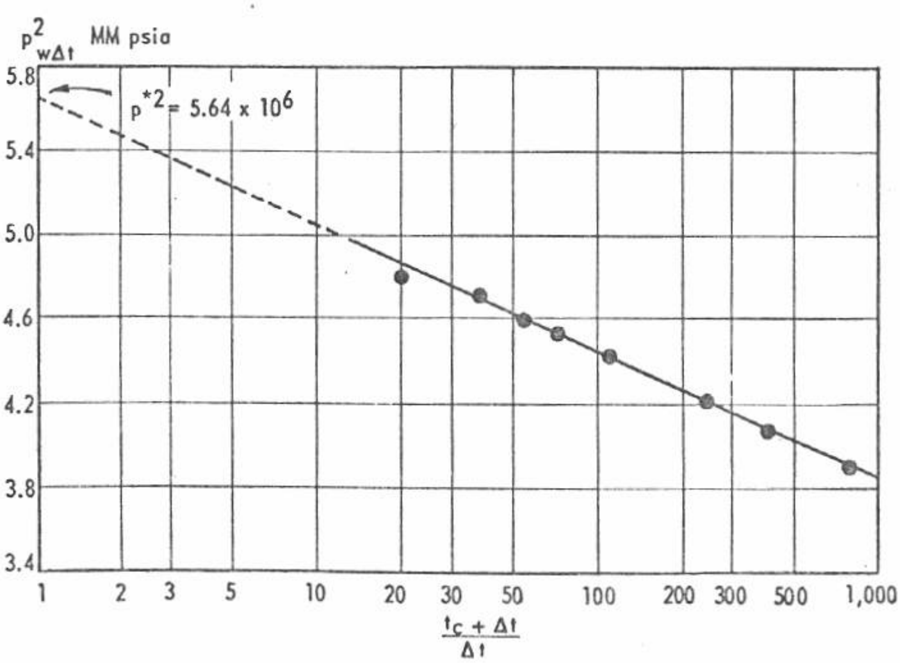
$y(\mu)$ = independent variable = $(1/\mu)\, e^{-\mu} + E_1(\mu)$

p_r = reservoir pressure, psia

k_g = effective permeability to gas, darcys; and r_e, ϕ, μ_g and t_c are defined with the data. Here, these factors are expressed in c-g-s units, while in the solutions they are expressed in oil-field units.

The solution procedure for the Horner method is

1. Plot $p_{w\Delta t}^2$ vs. $\log[(t_c + \Delta t)/\Delta t]$ (Fig. 1) and extrapolate the proper pressure-buildup portion to $(t_c+\Delta t)/\Delta t$ = 1.0.
2. Read p^{*2} at $(t_c+\Delta t)/\Delta t$ = 1.0.
3. Determine the slope, m, from the proper portion of the pressure-buildup curve.



PROBLEM SOLUTION depends on data from this pressure-buildup curve, Fig. 1.

4. Compute y (μ_1) using Equation 1.

5. Read μ_1 from Fig. 1, p. 20 (see also reference 6).

6. Compute p_r using Equation 2.

The use of the Matthews et al. method for liquids was covered on p. 32 (see also reference 5). Fig. 1, p. 33 shows that the method is also applicable for gases. The abscissa of this figure for a gas well is

$$\frac{t_D}{\pi R^2} = \left(\frac{0.0000838\ k_g\, p_r\, t_c}{\mu_g\, \phi\, r_e^2}\right)_{gas} \quad (3)$$

while the ordinate is

$$[2.3\,(p^{*2} - p_r^2)/m]_{gas} \quad (4)$$

Permeability k_g is obtained with the equation[1]

$$k_g = (1{,}637\ Q_g\, T_r\, Z\, \mu_g)/m\,h \quad (5)$$

Where t_c, μ_g, ϕ, r_e, Q_g, T_r, Z, r_w, and h are defined with the data and

t_D = dimensionless time

$R = r_e/r_w$

r_e = external radius of drainage, ft

k_g = effective permeability to gas, md

The procedure is to compute Equation 3 for $t_D/\pi R^2$ using an estimated value for p_r and read the ordinate value from the proper curve of Fig. 1, p. 33 Since p^* and m can be obtained from the pressure-buildup curve, Equation 4 can be solved for p_r. The computed value of p_r should be within 0.5% of the estimated value of p_r. Otherwise use the computed value as an estimate of p_r and repeat the calculations adjusting Z and μ_g to this value.

SOLUTION:

Horner's method

From Fig. 1, $p^{*2} = 5.64 \times 10^6$ and

$$m = \frac{(p_{wst}^2)_1 - (p_{wst}^2)_2}{\left(\log \dfrac{t_c + \Delta t}{\Delta t}\right)_2 - \left(\log \dfrac{t_c + \Delta t}{\Delta t}\right)_1} \quad (6)$$

for $(t_c + \Delta t)/\Delta t = 10$,

$p_{wst}^2 = 5.05 \times 10^6$ and

for $(t_c + \Delta t)/\Delta t = 100$,

$p_{wst}^2 = 4.45 \times 10^6$

Thus

$$m = \frac{5.05 \times 10^6 - 4.45 \times 10^6}{\log 100 - \log 10}$$

$$= 600{,}000$$

Equation 1 can now be solved for $y(\mu_1)$

$$5.64 \times 10^6 = (2{,}390)^2 - [0.6 \times 10^6\ y(\mu_1)]/2.3$$

$$y(\mu_1) = \frac{(5.71 \times 10^6 - 5.64 \times 10^6)\,2.3}{0.60 \times 10^6} = 0.27$$

From Fig. 1 of Part 61,[6] μ_1 = 0.80.

Now solving Equation 2

$$p_r^2 = 5.71 \times 10^6 - (0.60 \times 10^6)/(2.3)(0.80)$$

$$= 5.71 \times 10^6 - 0.33 \times 10^6$$

$$= 5.38 \times 10^6$$

$$p_r = 2{,}320 \text{ psia}$$

Matthews et al. method

To solve Equation 3, Equation 5 must first be solved for k_g.

$$k_g = \frac{(1{,}637)\,(10{,}000)\,(605)\,(0.85)\,(0.12)}{(600{,}000)\,(54)} = \frac{(990{,}385)\,(0.102)}{3{,}240} = \frac{101{,}019}{3{,}240}$$

$$= 31 \text{ md}$$

Also r_e^2 =

$$[(43{,}560\ ft^2/acre)\ (640\ acres)]/\pi$$

$$= 27{,}878{,}400/3.1416 = 8{,}873{,}900$$

or

$$r_e = 2{,}980 \text{ ft}$$

Substitution into Equation 3 gives

$$\frac{t_D}{\pi R^2} = \frac{(0.0000838)\,(31)\,(2{,}320)\,(2{,}400)}{(0.12)\,(0.18)\,(8{,}873{,}900)} = \frac{(25.98)\,(556.8)}{191{,}676}$$

$$= \frac{14{,}466}{191{,}676} = 0.075$$

From Fig. 1 on p. 33 using $t_D/\pi R^2$ = 0.075, the ordinate is read as 0.85 and thus

$$2.3\,(5.64 \times 10^6 - p_r^2)/600{,}000 = 0.85$$

$$5.64 \times 10^6 - p_r^2 = 510{,}000/2.3 = 222{,}000$$

$$p_r^2 = 5.418 \times 10^6$$

$$p_r = 2{,}327 \text{ psia}$$

This value compares well with that (2,320 psia) obtained with the Horner method.

DISCUSSION: the Horner method for a finite system has been applied to an oil reservoir.[7] A comparison of that problem with this problem shows that the equations and the procedures used are similar except that for gases, pressures are used as square functions in the equations and plots.[1] The theory supporting such usage was discussed in the problem starting on p. 34 (see also reference 4).

Similar principles are involved in a comparison of the equations for liquids (Page 32) and gases (Equations 3 and 4) of the Matthews et al. method. Equation 4 contains the pressures as square functions where the analogous equation of the earlier problem uses the pressure to the first power. Also in Equation 3 the average compressibility of the gas has been replaced with the reciprocal of reservoir pressure.

The two methods used in this problem and the method in the immediate preceeding problem[8] represent three methods for determining reservoir pressure from gas wells in finite systems with pressure-buildup data. Although less mathematically rigorous, the Horner method gives equally good results for most field applications. It should be used when sufficient data are not available for application of the two other methods.

In addition to pressure-buildup data, the Horner method requires knowledge of gas-production rate prior to shut-in, cumulative gas production since last pressure survey, and reservoir pressure at last pressure survey.

Except for reservoir pressure at last survey, the Matthews et al. method requires the same data plus gas viscosity, porosity, radius of drainage, reservoir temperature, and net sand thickness. Considering data required, the principal difference between the Matthews et al. method and the adaption of Miller, Dyes, and Hutchinson[8] method is that the latter does not require knowledge of cumulative gas production since last pressure survey.

References

1. Tracy, G. W., "Diagnosing Productivity Problems in Gas Wells": ninth oil-recovery conference, TPRC, Mar. 26, 1956; also The Oil and Gas Journal, Aug. 6, 1956, p. 84.

2. Horner, D. R., "Pressure Buildup in Wells": Proc. Third World Petroleum Congress, Section III, E. J. Brill, Leiden, Holland, 1951.

3. Matthews, C. S., F. Brons, and P. Hazebroek, "A Method of Determination of Average Pressure in a Bounded Reservoir": Trans. AIME Vol. 201, 1954, p. 182.

4. Guerrero, E. T., "Reservoir Engineering, Part 66—How to Find Static Reservoir Pressure for Gas Well": The Oil and Gas Journal, Vol. 62, No. 15, Apr. 13, 1964, pp. 101-103.

5. Guerrero, E. T., "Reservoir Engineering, Part 65—How to Find Average Reservoir Pressure of a Well-Drainage Area": The Oil and Gas Journal, Vol. 62, No. 10, Mar. 9, 1964, pp. 101-102.

6. Guerrero, E. T., "Reservoir Engineering, Part 61—How to Find Pressure Distributions for Unsteady-State Flow Conditions for Finite External Boundary": The Oil and Gas Journal, Vol. 61, No. 42, Oct. 21, 1963, pp. 92-95.

7. Guerrero, E. T., and Stewart, F. M., "Reservoir Engineering, Part 14d—How to Determine Effective Permeability from Pressure-Buildup Data Under Finite Boundary Conditions": The Oil and Gas Journal, Vol. 57, No. 41, Oct. 5, 1959, pp. 167-169.

8. Guerrero, E. T., "Reservoir Engineering, Part 67—How to Find Static Reservoir Pressure for Gas Well": The Oil and Gas Journal, Vol. 62, No. 20, May 18, 1964, pp. 137-139.

Part 69

How to find capacity, productivity ratio, and skin effect

for oil well from pressure-buildup data

GIVEN: A well is located in an oil field with 40-acre well spacing. Production since last pressure survey was 4,270 st-tk bbl. These pressure-buildup test data were obtained on this well:

Shut-in time Δt hours	Well pressure, p_{wst}, psia
0	1,570
1	2,199
3	2,238
5	2,256
9	2,276
15	2,293
25	2,310
35	2,320
45	2,328
60	2,336
80	2,343
100	2,348
130	2,353

Other data for the well are:

Porosity, $\phi = 0.16$

Reservoir oil viscosity, $\mu_o = 0.7$ cp

Average oil compressibility at reservoir conditions, $c_{o\,av} = 17.9 \times 10^{-6}$ vol/vol/psi

Oil formation-volume factor, $B_o = 1.29$

Well radius, $r_w = 3$ in.

Stabilized production rate prior to shut-in, $Q_o = 50$ b/d

Net sand thickness, $h = 21$ ft

Static well pressure, $p_s = 2,362$ psia

$$t_e = \frac{24\,N_p}{Q_o} = \left(\frac{(24)(4,270)}{50}\right) = 2,050 \text{ hours}$$

t_e = time well has produced from last pressure survey, hours

N_p = oil produced since last pressure survey, st-tk bbl

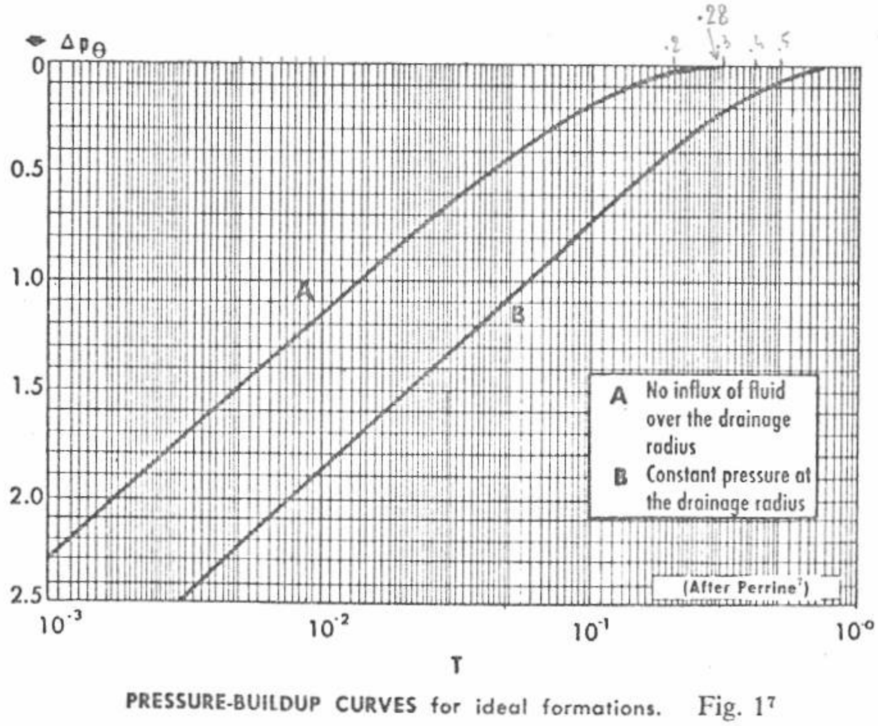
FIND: Compute the capacity, productivity ratio, and skin effect for the well.

METHOD OF SOLUTION: The capacity of a well is computed with the equation

$$k_o h = (162.6\,Q_o \mu_o B_o)/m \quad (1)$$

For an infinite system, m is obtained from a plot of p_{wst} vs. log $[(t_e + \Delta t)/\Delta t]$, while for a finite system, m is obtained from a plot of p_{wst} vs. log Δt.[1,2]

Pressure behavior of a well located in a finite system, (producing well surrounded by other producing



PRESSURE-BUILDUP CURVES for ideal formations. Fig. 1[7] wells) will be the same as if the system were infinite until the pressure disturbances reach the boundaries or interfere with similar disturbances of surrounding wells.

Time required for pressure disturbances to reach the exterior boundary can be computed with the dimensionless time equation.

$$t_b = 3{,}800\ \phi \mu_o c_{o\,av} r_e^2 T_b / k_o \quad (2)$$

where h, Q_o, μ_o, B_o, ϕ, and $c_{o\,av}$ were defined with the data and

k_o = effective permeability to oil, md

m = slope of proper buildup portion of pressure-buildup curve, psi per cycle.

t_b = time for pressure disturbances to reach the exterior boundary, r_e, hours

r_e = radius of exterior boundary, ft

T_b = dimensionless time for pressure disturbances to reach exterior boundary

Pressure behavior of the finite system with constant pressure at the exterior radius, curve B of Fig. 1[7] can be used to compute T_b.

The straight-line portion of this curve applies for an infinite system, and its extrapolation to an ordinate value of zero shows that complete pressure buildup would occur at T = 0.445, or that it would take this much dimensionless time for a disturbance to be transmitted to r_e. Thus T_b = 0.445 and Equation 2 becomes

$$t_b = (1{,}690\ \phi \mu_o c_{o\,av} r_e^2)/k_o \quad (3)$$

With an estimate of k_o, solve Equation 3, and if t_b is less than t_e, obtain m from plot of $p_{w\Delta t}$ vs. log Δt, if t_b is greater than t_e (indicating pressure disturbance has not reached r_e) obtain m from plot of $p_{w\Delta t}$ vs. log $[(t_e + \Delta t)/\Delta t]$.

The ratio of $k_o h$ from PI data to $k_o h$ from pressure-buildup data is defined as the productivity ratio.[4] Thus

$$\frac{(k_o h)_{PI}}{(k_o h)_{BU}} = \frac{\dfrac{Q_o \mu_o B_o \ln (r_e/r_w)}{0.00708\,(p_e - p_w)}}{\dfrac{162.6\, Q_o \mu_o B_o}{m}} = \text{C.F.}$$

$$= \frac{m \ln (r_e/r_w)}{1.15\,(p_e - p_w)}$$

$$= \frac{2m \log (r_e/r_w)}{(p_e - p_w)} \quad (4)$$

Where.

p_e = pressure at radius r_e, psia

p_w = sandface pressure before shut in, psia

r_w = well radius, ft

Pressure drop for flow into a well per unit rate of flow is controlled by the resistance of a formation, the viscosity of the fluid, and the resistance (formation damage or skin effect) concentrated about the well bore resulting from drilling, well completion, and production practices.

This latter effect is not considered in any of the solutions of the diffusivity equation for pressure distribution or pressure buildup. Since the effect is concentrated at or near the well bore, Van Everdingen[5] uses a modification of the infinite-source solution of the diffusivity equation for pressure distribution to account for formation damage.

$$p_w = p_i - \frac{Q_o \mu_o B_o}{4\pi k_o h} \left[\ln \left(\frac{k t_e}{\phi \mu_o c_{o\,av} r_w^2}\right) + 0.809 + 2S\right] \quad (5)$$

where all units in this equation are in the c-g-s system and

p_i = initial reservoir pressure

S = skin effect

It has also been shown[6] that when t_e is much larger than Δt

$$p_{w\Delta t} = p_i - \frac{Q_o \mu_o B_o}{4\pi k_o h} \ln \left(\frac{t_e}{\Delta t}\right) \quad (6)$$

By subtracting Equation 5 from Equation 6

$$p_{w\Delta t} - p_w = \frac{Q_o \mu_o B_o}{4\pi k_o h} \left[\ln \Delta t + \ln \left(\frac{k_o}{\phi \mu_o c_{o\,av} r_w^2}\right) + 0.809 + 2S\right] \quad (7)$$

Here $Q_o \mu_o B_o / 4\pi k_o h = m$ and $k_o = Q_o \mu_o B_o / 4\pi\ m\ h$. By making these substitutions into Equation 6, converting to oil-field units, letting Δt = 1 hour, and solving for S, an API committee[4] obtained for an oil reservoir

$$S = 1.151\, \frac{p_{w\,1\,hr} - p_w}{m} - 1.151 \log \left(\frac{Q_o B_o}{10.4\, m\, h\, \phi\, c_{o\,av} r_w^2}\right) \quad (8)$$

where S is dimensionless and

$p_{w\,1\,hr}$ = pressure at 1 hour shut-in on straight-line portion of pressure-buildup curve with slope m.

SOLUTION: Fig. 2 shows the pressure-buildup data plotted as $p_{w\Delta t}$ vs. log Δt. Using Equation 1

$$k_o = (162.6)(50)(0.7)(1.29)/(76)(21) = 7{,}341.4/1{,}596 = 4.6 \text{ md}$$

$$r_e = \sqrt{(40)(43{,}560)/3.14} = 745 \text{ ft}$$

Solution of Equation 3 gives

$$t_b = (1{,}690)(0.16)(0.7)(17.9 \times 10^{-6})(745)^2/4.6 = 409 \text{ hours}$$

Since t_b (409 hours) is smaller than t_e (2,050 hours) m is obtained from a plot of $p_{w\Delta t}$ vs. log Δt as shown on Fig. 2. Thus $k_o h = 4.6 \times 21 = 97$ md ft, and

$$\text{Productivity ratio} = \frac{(2)(76) \log (745/0.25)}{2{,}362 - 1{,}570} = 0.67$$

and

$$S = 1.151 \left(\frac{2{,}202 - 1{,}570}{76}\right) - 1.151 \log \left[\frac{(50)(1.29)}{(10.4)(76)(21)(0.16)(17.9 \times 10^{-6})(0.25)^2}\right] = 4.58.$$

DISCUSSION: The procedures given for determination of capacity, productivity ratio, and skin effect are simple to apply after determination of manner to plot the pressure-buildup data. For infinite systems and finite systems where pressure disturbances have not reached the boundaries, best results are obtained from plots of $p_{w\Delta t}$ vs. log $(t_e + \Delta t)/\Delta t$.[1]

For finite systems where the pressure disturbances have reached the boundaries, plots of $p_{w\Delta t}$ vs. log Δt should be used. When h is not available to allow computation of t_b then an estimate is made on the type of prevailing system. Even if the wrong plot is selected, useful results are obtained, since the slopes, m, from the two types of plots tend to approach each other in value.

Productivity ratio represents the ratio of actual productivity of a well to the productivity that would exist if the permeability in the vicinity of the well bore were the same as that in drainage area. This ratio is also called by various other names: condition ratio, improvement ratio, damage factor.

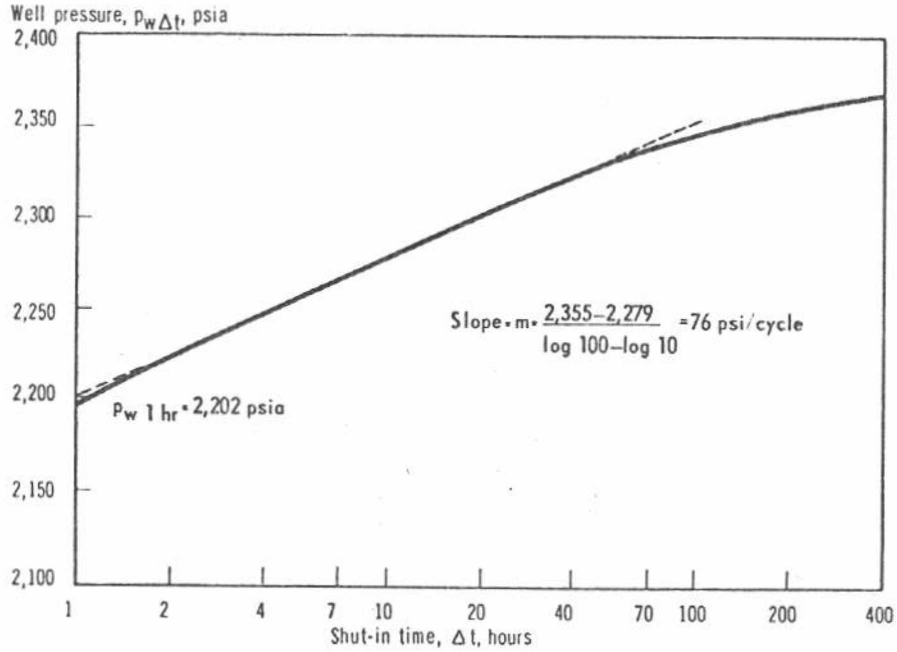
Equation 4 shows that productivity ratio is also the ratio of theoretical pressure drop, with no formation damage, to actual pressure drop. Low values of these ratios (less than 0.8) indicate formation damage or that the permeability near the well bore is less than that away from the well bore.

Values of 0.8 to 1.2 indicate little or no damage, and larger values indicate that the permeability near the well bore is higher than that away from the well bore. These features make productivity ratios useful in evaluating well-stimulation treatments. The value of the ratios before and after the stimulation treatments indicates the success achieved.

In the computation of productivity ratios, the effective radius of drainage must be estimated, and thus the well must have been producing at a stabilized rate long enough so that the logarithm of the ratio of drainage to well radius (r_e/r_w) is practically constant.

Skin effect, S, is another method of expressing the restriction to flow in the vicinity of the well bore. A positive value of S indicates a restriction (skin) exists, while a negative value indicates that the skin has been removed. A value near zero

Wells": Proc. Third World Petroleum Congress," Sec. II, E. J. Brill, Leiden, Netherlands, 1951.
2. Miller, C. C., Dyes, A. B., and Hutchinson, C. A., Jr, "The Estimation of Permeability and Reservoir Pressure from Bottom-



PRESSURE-BUILDUP behavior of oil well in a finite system. Fig. 2

(0.5 to −0.5) indicates little or no skin and that the permeability near the well bore is the same as that away from the well bore.

Two advantages of the skin-effect method are: (1) an estimate of the fully builtup pressure is not required and (2) the drainage radius, r_e, need not be determined. However, porosity, net formation thickness, and effective fluid compressibility must be known, although their effect is dampened by their being grouped in a logarithmic term.

References

1. Horner, D. R., "Pressure Buildup in Wells": Proc. Third World Petroleum Congress," Sec. II, E. J. Brill, Leiden, Netherlands, 1951.

2. Miller, C. C., Dyes, A. B., and Hutchinson, C. A., Jr, "The Estimation of Permeability and Reservoir Pressure from Bottom-Hole Pressure-Buildup Characteristics": Trans. AIME Vol. 189, 1950, p. 91.

3. Guerrero, E. T., and Stewart, F. M., "Reservoir Engineering Part 14—How to Determine Effective Permeability from Pressure-Buildup Data Under Finite Boundary Conditions": The Oil and Gas Journal, Vol. 57, No. 41, Oct. 5, 1959, pp. 167-169.

4. Bulletin D6, "Selection and Evaluation of Well Completion Practices": API, Dallas, July 1955.

5. VanEverdingen, A. F., "The Skin Effect and Its Influence on the Production Capacity of a Well": Trans. AIME, Vol. 198, 1953, p. 171.

6. Guerrero, E. T., "Reservoir Engineering Part 66—How to Find Static Reservoir Pressure for Gas Well in Infinite System with Adaptation of the Horner Method": The Oil and Gas Journal, Vol. 62, No. 15, Apr. 13, 1964, pp. 101-103.

7. Perrine, R. L., "Analysis of Pressure-Buildup Curves": API Drilling and Production Practice, 1956, pp. 482-495.

Part 70

How to find capacity, productivity ratio, and skin effect

for gas well from pressure-buildup data

GIVEN: The following pressure-buildup data were obtained on a gas well in the Oklahoma Panhandle area.

Shut-in time Δt hours	Well pressure, $p_{w\Delta t}$, psia	$p_{w\Delta t}^2$
0	1,693	2,866,200
1	1,925	3,705,600
2	2,165	4,687,200
5	2,215	4,906,200
13.6	2,260	5,106,600
21.5	2,281	5,203,000
35.0	2,301	5,294,600
45.0	2,312	5,345,300
60.0	2,323	5,396,300

Other data for this well are:

Stabilized gas production rate prior to shut-in, $Q_g = 5,000$ Mscfd

Reservoir temperature, $T_r = 175°$ F. or 635° R.

Net sand thickness, $h = 63$ ft

Static reservoir pressure, $p_r = 2,374$ psia

Reservoir gas viscosity, $\mu_g = 0.02$ cp

Gas deviation factor at reservoir conditions, $Z = 0.83$

Radius of drainage, $r_e = 2,100$ ft (well spacing is 320 acres per well)

Well radius, $r_w = 4$ in.

Porosity, $\phi = 18.4\%$

Cumulative gas production since last pressure survey, $G_p = 900,000$ Mscf

$t_p = 24\ G_p/Q_g = (24)(900,000)/5,000 = 4,320$ hours

FIND: Determine the flow capacity, productivity ratio, and skin effect for the well.

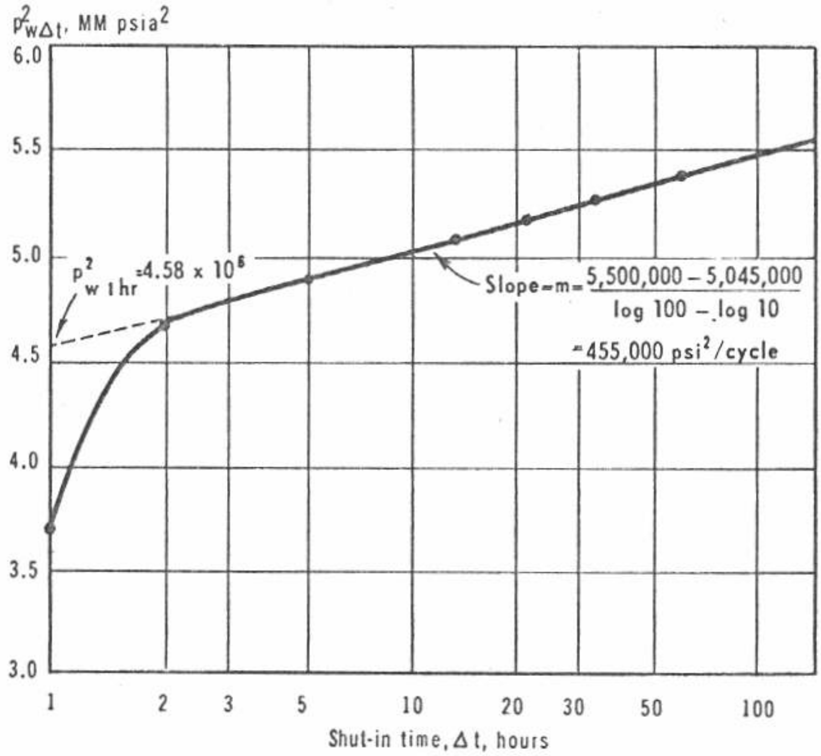
METHOD OF SOLUTION: The flow capacity of a gas well is determined with the equation

$$k_g h = \frac{1,637\ Q_g \mu_g Z T_r}{m} \qquad (1)$$

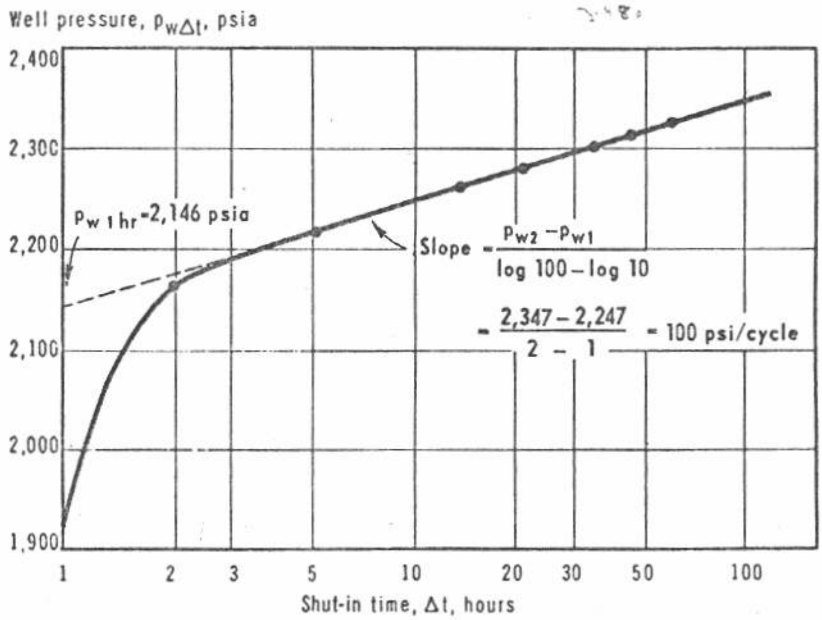
For an infinite system, or a finite system in its early life when it behaves like an infinite system, m is obtained from a plot of $p_{w\Delta t}^2$ vs. log $(t_p + \Delta t)/\Delta t$.

For finite systems in which the pressure-disturbance waves have reached the exterior boundary m is obtained prom a plot of $p_{w\Delta t}^2$ vs. log (Δt). Exterior boundaries are represented by the reservoir limit (pinchout or fault) or the point of interference with pressure-disturbance wave from adjacent wells.

Time needed for pressure disturbances to reach the exterior boundary determines which plot to



PRESSURE-BUILDUP curve for an Oklahoma Panhandle gas well, with pressure plotted as a squared function. Fig. 1.



OKLAHOMA PANHANDLE gas-well pressure-buildup curve, with pressure plotted as a linear function. Fig. 2.

use in obtaining m. This time is computed with the dimensionless-time equation which, for gas wells, is

$$t_b = \frac{1{,}900\ \phi\ \mu_g\ r_e^2\ T_b}{k_g\ p_r} \quad (2)$$

Where:

ϕ, μ_g, r_e, p_r, h, Q_g, Z, and T_r were defined with the data and

k_g = effective permeability to gas, md

m = slope of proper buildup portion of pressure-buildup curve, psi^2 per cycle

t_b = time required for pressure disturbances to reach the exterior boundary, r_e, hours

T_b = dimensionless time for pressure disturbances to reach exterior boundary

t_e = apparent time well has produced since last pressure survey, hours

From the preceeding problem[1] $T_b = 0.445$ and Equation 2 becomes

$$t_b = \frac{845\ \phi\ \mu_g r_e^2}{k_g\ p_r} \quad (3)$$

With an estimate of k_g, Equation 3 is solved. If t_b is less than t_e, then obtain m from plot of $p_{w\Delta t}^2$ vs. log (Δ t); if t_b is greater than t_e (indicating pressure disturbances have not reached r_e), obtain m from plot of $p_{w\Delta t}^2$ vs. log $(t_e + \Delta t)/\Delta t$.

Tracy[2] has defined PI for gas wells as

$$PI = \frac{Q}{p_r^2 - p_w^2} \quad (4)$$

Where:

PI = productivity index

p_w = flowing sandface pressure, psi

$$SE = 1.151\ \frac{p_{w\ 1\ hr}^2 - p_w^2}{m} - 1.151 \log \left(\frac{Q_g\ T_r\ Z\ p_r}{1.033\ mh\phi r_w^2}\right) \quad (6)$$

Where SE = skin effect.

If M is the slope of proper portion of pressure-buildup curve in psi per cycle from plot of $p_{w\Delta t}$ vs. log (Δ t), then using M instead of m, PR could be computed with[4]

$$PR = \frac{4M\ p_r \log (r_e/r_w)}{(p_r^2 - p_w^2)} \quad (7)$$

and SE from

$$SE = 1.151\ \frac{p_{w\ 1\ hr} - p_w}{M} - 1.151 \log \left(\frac{Q_g\ T_r\ Z}{2.07\ M\ h\ \phi\ r_w^2}\right) \quad (8)$$

and $k_g h$ from

$$k_g h = \frac{818\ Q_g \mu_g\ T_r\ Z}{M\ p_r} \quad (9)$$

SOLUTION: Solving Equation 1 where m is obtained from Fig. 1 gives

$$k_g h = \frac{(1{,}637)\ (5{,}000)\ (0.02)\ (0.83)\ (635)}{455{,}000} = 189.6 \text{ md ft}$$

or

$$k_g = \frac{189.6}{h} = \frac{189.6}{63} = 3 \text{ md}$$

Using Equation 9 and obtaining M from Fig. 2

$$k_g h = \frac{(818)\ (5{,}000)\ (0.02)\ (635)\ (0.83)}{(100)\ (2{,}374)} = 181.8 \text{ md ft}$$

Now that a value of k_g has been computed, Equation 3 can be solved to determine if the proper plot of the pressure-buildup data has been used.

The ratio of $k_g h$ from PI data to $k_g h$ from pressure-buildup data (Equation 1) is defined as the productivity ratio (PR). Thus

$$PR = \frac{(k_g h)_{PI}}{(k_g h)_{BU}} = \frac{\dfrac{Q_g\ \mu_g\ T_r\ Z \ln (r_e/r_w)}{0.000704\ (p_r^2 - p_w^2)}}{\dfrac{1{,}637\ Q_g \mu_g T_r Z}{m}} = \frac{m \ln (r_e/r_w)}{1.15\ (p_r^2 - p_w^2)} = \frac{2\ m \log (r_e/r_w)}{p_r^2 - p_w^2} \quad (5)$$

The previous problem[1] (p. 41) showed that the formation damage, or skin effect, resulting from drilling, well-completion, and production practices can be evaluated for an oil well using an equation developed by Van Everdingen[3] and modified by an API committee.[4] The same principles can be applied to gas wells. The equation for computation of skin effect in gas wells[4] is:

$$t_b = \frac{(845)\ (0.184)\ (0.02)\ (2{,}100)^2}{(3)\ (2{,}374)}$$

$$= \frac{(3.11)\ (4{,}410{,}000)}{7{,}122}$$

$$= \frac{13{,}715{,}100}{7{,}122} = 1.926 \text{ hours}$$

t_b is less than t_e (1,926 hours compared to 4,320 hours) and thus $p_{w\Delta t}^2$ should be plotted vs. log (Δ t) as shown in Fig. 1.

For productivity ratio, Equation 5 gives

$$PR = \frac{(2)(455{,}000)\log(2{,}100/0.333)}{(2{,}374)^2 - (1{,}693)^2} = 1.25$$

Or using Equation 7

$$PR = \frac{(4)(100)(2{,}374)\log(2{,}100/0.333)}{(2.374)^2 - (1{,}693)^2} = 1.30$$

Skin effect is computed with Equation 6 where $p_{w\,1\,hr}^2$ is obtained from Fig. 1

$$SE = 1.151\left[\frac{4.58\times10^6-(1{,}693)^2}{455{,}000}\right] - 1.151\log\left[\frac{(5.000)(635)(0.83)(2{,}374)}{(1.033)(455{,}000)(63)(0.184)(1/3)^2}\right] = -0.28$$

Or using Equation 8 and data ($p_{w\,1\,hr}$ and M) from Fig. 2

$$SE = 1.151\left(\frac{2{,}146-1{,}693}{100}\right) - 1.151\log\left[\frac{(5{,}000)(635)(0.83)}{(2.07)(100)(63)(0.184)(1/3)^2}\right] = 0.61$$

DISCUSSION: A comparison of the procedures used in this problem for determination of flow capacity, productivity ratio, and skin effect of a gas well with the procedures used in the previous problem[1], for an oil well shows much similarity. As discussed in the problem starting on p. 34 (see also reference 5), the equations for oil wells were modified and adapted to gas wells. The primary modification involved allowance for gas compressibility. Also the modifications are such that for gases, computations can be made with slope, m, obtained from a plot of $p_{w\Delta t}^2$ vs. log (Δ t) or a plot of $p_{w\Delta t}$ vs. log (Δ t).

Although the former plot is mathematically more rigorous, both plots give satisfactory results. This is supported by the similar results obtained for flow capacity with Equations 1 and 9, productivity ratio with Equations 5 and 7, and skin effect with Equations 6 and 8.

A value of unity for productivity ratio, and of zero for skin effect, indicates no formation damage. That is, the permeability near the well bore is the same as that away from the well bore. Values of productivity ratio greater than one, and values of skin effect less than zero (negative values) indicate favorable well conditions. That is, the permeability near the well bore is higher than that in the interwell area.

Conversely, values of productivity ratio between zero and one and values of skin effect greater than zero indicate formation damage.

Within the accuracy of the data involved and equations used, the results of this problem indicate no formation damage even though one method gives a slightly positive value for SE, and the other method gives a slightly negative value for SE. The permeability near the well bore is about the same as that away from the well bore.

References

1. Guerrero, E. T., "Reservoir Engineering Part 69—How to Find Capacity, Productivity Ratio, and Skin Effect": The Oil and Gas Journal, Vol. 62, No. 29, July 20, 1964, p. 107.

2. Tracy, G. W., "Diagnosing Productivity Problems in Gas Wells": ninth oil-recovery conference, TPRC, Mar. 26, 1956; also The Oil and Gas Journal, Aug. 6, 1956, p. 84.

3. Van Everdingen, A. F., "The Skin Effect and Its Influence on the Production Capacity of a Well": Trans. AIME Vol. 198, 1953, p. 171.

4. Bulletin D 6, "Selection and Evaluation of Well-Completion Practices," API, Dallas, July 1955.

5. Guerrero, E. T., "Reservoir Engineering Part 66—How to Find Static Reservoir Pressure for Gas Well in Infinite System with Adaption of the Horner Method": The Oil and Gas Journal, Vol. 62, No. 15, Apr. 13, 1964, pp. 101-103.

Part 71

How to find injection capacity and skin effect

from pressure-falloff data from water-injection wells

GIVEN: A water-injection well with a stabilized injection rate (Q_{wi}) of 525 b/d was shut in for a pressure-drawdown test. The following data were obtained:

Shut-in time Δt, hours	Well pressure (bottom-hole) $p_{w\Delta t}$, psia	$\frac{\Delta t}{\Delta t + 708}$
0	5,240	.....
¼	3,960	0.000353
½	3,850	0.000706
1	3,685	0.00141
3	3,062	0.00422
5	2,750	0.00701
7	2,535	0.00979
10	2,288	0.0139
15	2,012	0.0207
20	1,815	0.0275
25	1,683	0.0341
30	1,577	0.0407
35	1,495	0.0471
40	1,435	0.0535
45	1,389	0.0598
50	1,351	0.0660

Other data are as follows:

At reservoir conditions,

Water formation-volume factor, $B_w = 1.03$

Viscosity of water, $\mu_w = 0.9$ cp

Compressibility of water, $c_w = 3 \times 10^{-6}$ psi^{-1}

Compressibility of oil, $c_o = 8 \times 10^{-6}$ psi^{-1}

Compressibility of formation, $c_f = 4 \times 10^{-6}$ psi^{-1}

Porosity, $\phi = 0.17$

Water-injection rate prior to shut-in, $Q_{wi} = 525$ b/d

Cumulative water injected, $W_i = 15{,}500$ bbl

Net sand thickness, $h = 27$ ft

Well radius, $r_w = 3$ in.

Residual-gas saturation, $S_{gr} = 0$

Residual-oil saturation, $S_{or} = 30\%$

Apparent time of injection, $t_e = W_i/Q_{wi} = 15{,}500/525 = 29.5$ days

Static reservoir pressure, $p_s = 1{,}240$ psia

FIND: (1) Injection capacity and (2) Skin effect

METHOD OF SOLUTION: The fluid-flow equations that have been developed for producing wells are applicable to injection wells if modifications are made for direction in flow and properties of the fluids.[1]

For an extensive reservoir of uniform thickness and permeability, the theoretical injection pressure at the well bore as a function of injection time for a given constant injection rate is given by:

$$p_w = p_s + \frac{70.6\, Q_{wi}\, B_w\, \mu_w}{k_w\, h}\left[-E_i\left(-\frac{948.2\, r_w^2\, \phi\, \mu_w\, c_{av}}{k_w\, t}\right)\right] \quad (1)$$

Since $\frac{948.2\, r_w^2\, \phi\, \mu_w\, c_{av}}{k_w\, t}$ is small for the case of a well bore, then

$$p_w = p_s + \frac{162.6\, Q_{wi}\, B_w\, \mu_w}{k_w\, h}\left[\log t - \log\left(\frac{1{,}689\, r_w^2\, \phi\, \mu_w\, c_{av}}{k_w}\right)\right] \quad (2)$$

Where Q_{wi}, B_w, μ_w, h, ϕ, and r_w are defined with the data and

p_w = sand-face pressure at time t, psia

p_s = static reservoir pressure, psia

k_w = effective permeability to water, md

c_{av} = average reservoir-water compressibility factor, psi^{-1}

t = time from start of injection, hours

$p_{w\Delta t}$ = sand-face pressure at shut-in time Δt, psia.

For a well in which injection has taken place for apparent time, t_e, that is then shut in for Δt time, the pressure drawdown can be expressed by

$$p_{w\Delta t} = p_s + \frac{162.6\, Q_{wi}\, B_w\, \mu_w}{k_w\, h}\log\frac{t_e + \Delta t}{\Delta t} \quad (3)$$

This equation is derived from Equation 2 using the superimposition theorem. A similar equation was developed by Horner[2] for pressure buildup. Equation 3 can also be written as

$$p_{w\Delta t} = p_s - \frac{162.6\, Q_{wi}\, B_w\, \mu_w}{k_w\, h}\log\frac{\Delta t}{t_e + \Delta t} \quad (4)$$

This equation indicates that a plot of $p_{w\Delta t}$ vs. $\Delta t/(t_e + \Delta t)$ on a semilog graph gives a linear relationship with

$$\text{Slope} = m = \frac{162.6\, Q_{wi}\, B_w\, \mu_w}{k_w\, h} \quad (5)$$

$$\text{Or injectivity capacity} = k_w\, h = (162.6\, Q_{wi}\, B_w\, \mu_w)/m \quad (6)$$

Fig. 1 gives the pressure-drawdown curve for the problem. A large portion of the curve is linear.

Nowak and Lester[1] determine skin effect for a water-injection well following a procedure similar to that used by Van Everdingen[3] for oil wells. The difference between the observed injection pressure and the theoretical injection pressure is a measure of the skin effect. Thus

$$\text{Skin effect}_{\text{Nowak and Lester}} = p_w \text{ observed} - p_{wtc} \quad (7)$$

p_{wtc} is computed with Equation 2 using $t = t_e$. Using similar theory and procedures Arps[4] defines the skin effect as a completion factor, CF:

$$CF = \frac{\text{theoretical drawdown}}{\text{actual drawdown}} \times 100$$

$$= [\frac{m}{p_w - p_s}] [\log (\frac{k\, t_c}{1{,}689\, \phi\, \mu_w\, c_{av}\, r_w^2})] \times 100 \qquad (8)$$

Groeneman and Wright[5] define the skin effect of an injection well with an equation similar to that used in Part 69[6] for an oil well. The following is a slight modification of their equation, which is believed to be more exact:

$$SE = 1.151\, \frac{p_w - p_{w\,1\,hour}}{m} - 1.151 \log [\frac{Q_{wi}\, B_w}{10.4\, m\, h\, \phi\, c_{av}\, r_w^2}] \qquad (9)$$

Another similar form of the skin-effect equation has been given by Arps:[4]

$$SE = 1.151\, \frac{p_w - p_s}{m} - 1.151 \log \frac{k_w\, t}{1{,}689\, r_w^2\, \phi\, \mu_w\, c_{av}} \qquad (10)$$

where p_w = sand-face pressure at time of shut-in = 5,240 psia.

SOLUTION: From Fig. 1, m = 1,570 psi/cycle. With Equation 6,

$$k_w h = \frac{(162.6)(525)(1.03)(0.9)}{1{,}570} = \frac{(85{,}365)(0.927)}{1{,}570}$$

$$= \frac{79{,}133}{1{,}570} = 50.4 \text{ md ft}$$

or

$$k_w = \frac{k_w h}{h} = \frac{50.4}{27} = 1.9 \text{ md}$$

The theoretical injection pressure is computed with Equation 2 where

$$c_{av} = c_o S_o + c_w S_w + c_g S_g + c_f \qquad (11)$$

$$= [(8.0\times10^{-6})(0.30) + (3\times10^{-6})(0.70) + 4\times10^{-6}] = 8.5\times10^{-6} \text{ psi}^{-1}$$

Thus,

$$p_{wtc} = 1{,}240 + \frac{(162.6)(525)(1.03)(0.9)}{50.4}$$

$$\times \{ \log 708 - \log [\frac{(1{,}689)(1/4)^2(0.17)(0.9)(8.5\times10^{-6})}{1.9}] \}$$

$$= 1{,}240 + 10{,}970 = 12{,}210 \text{ psia}$$

And

$$\text{Skin effect}_{\text{Nowak and Lester}} = 5{,}240 - 12{,}210 = -6{,}970 \text{ psi}$$

According to Equation 8,

$$CF = [\frac{1{,}570}{5{,}240 - 1{,}240}] \{ \log [\frac{(1.9)(708)}{(1{,}689)(0.17)(0.9)(8.5\times10^{-6})(1/4)^2}] \} 100$$

$$= 274\%$$

With Equation 9, skin effect,

$$SE = 1.151 (\frac{5{,}240 - 3{,}830}{1{,}570})$$

$$- 1.151 \log [\frac{(525)(1.03)}{(10.4)(1{,}570)(27)(0.17)(8.5\times10^{-6})(1/4)^2}] = -3.72$$

$$CF = \frac{\Delta P_T}{\Delta P_A} \times 100 = \frac{12210 - 1240}{5240 - 1240} = 274\%$$

For $t = t_c$ in Equation 10, p_w = 5,240 psia. Thus,

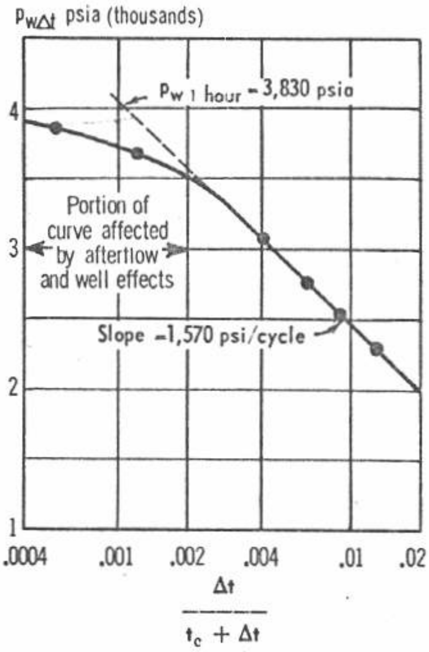
$$SE = 1.151 \frac{5{,}240 - 1{,}240}{1{,}570} - 1.151 \log \frac{(1.9)(708)}{(1{,}689)(1/4)^2(0.17)(0.9)(8.5\times10^{-6})}$$

$$= 2.94 - 8.04 = -5.10$$

Conversion of this skin effect to a pressure-drop equivalent gives

$$\frac{2\, m\, SE}{2.3} = \frac{(2)(1{,}570)(-5.10)}{2.3} = -6{,}963 \text{ psi}$$

which is the same as Skin effect $_{\text{Nowak and Lester}}$



PRESSURE DRAWDOWN CURVE for a water-injection well. Fig. 1.

DISCUSSION: An analysis of pressure-falloff curves involves consideration of two distinct phenomena: pressure rise during water injection and pressure decline after shut-in. In the first of these processes, pressure rises analogous to pressure decline of producing oil wells. In the second process pressure declines analogous to pressure buildup in a shut-in oil well.

The principal assumptions of equations used are the same as when these equations were applied for pressure-buildup analysis of oil wells.[1 2 6]

Equation 1 is the point-source solution to the diffusivity equation. From it can be developed Equations 2, 3, 4, 8, 9, and 10. These equations can also be applied when immobile fluids are present in the rock in addition to the flowing fluid.

Under such conditions effective permeability and average compressibility of the flowing fluid are used. The latter allows for expansion of rock and immobile fluids present.

This problem illustrates four methods for obtaining a measure of formation damage. The method of Nowak and Lester[1] (Equation 7) is the same as that of Equation 10, except that the former gives the pressure drop required to overcome the skin effect while the latter is a dimensionless measure.

The method of Equation 9 is

similar but not identical nor as rigorous as these two methods. In its development it has been assumed that the shut-in time, Δt, is small compared to t_c.

The method of Arps[4] (Equation 8) is based on the same theory as the other methods but represents a ratio of drawdowns rather than a pressure drop. A ratio of 100% indicates no formation damage, while one less than 100% indicates damage with the damage increasing as the ratio decreases.

Under conditions where the permeability near the well bore is higher than that away from the well bore, the completion factor would be greater than 100%.

For the other methods, positive results indicate formation damage while negative results indicate higher permeability near the well bore than away from it. A value of zero or near zero, for skin effect, indicates no formation damage. In this problem all the methods show favorable conditions near the well bore.

The formation has low permeability (1.9 md) and apparently the well has been fractured. Thus the permeability near the well bore is much higher than in the interwell area. The permeability obtained with Equation 5 is that of the interwell area.

The method of Equation 9 has the advantage over the other methods that static reservoir pressure is not required. Nowak and Lester[1] claim that extrapolation of the proper straight-line portion of the pressure-drawdown curve to $(\Delta t)/(t_c + \Delta t) = 1.0$ gives the static reservoir pressure.

This is true under conditions where interference with adjacent wells or impermeable reservoir boundaries has not occurred or where the average fluid compressibility is high (i.e. greater than 30×10^{-6} psi^{-1}).

In this problem the static reservoir pressure obtained from Fig. 1 would be −630 psi which, of course, is not possible. Fig. 1 shows the pressure tending to level out at about 1,250 psia. In spite of the evidence of interference, the slope of the drawdown curve is valid and can be used as shown in the problem.

Pressure-drawdown curves are similar to pressure-buildup curves in that there exists an afterflow effect after shut-in. This effect is shown on Fig. 1. The proper straight-line portion of the curve normally occurs some period of time after shut-in. The condition of the well bore can be studied from the initial portion of the drawdown curve.

If it lies above the extrapolated linear portion, then formation damage exists and vice versa. In Fig. 1 the initial portion of the pressure-drawdown curve lies below the extrapolated linear portion and thus no formation damage exists but, rather, the permeability near the well bore is higher than that in the interwell area.

References

1. Nowak, T. J., and Lester, G. W., "Analysis of Pressure Fall-off Curves Obtained in Water Injection Wells to Determine Injective Capacity and Formation Damage": Trans. AIME, Vol. 204, 1955, pp. 96-102.
2. Horner, D. R., "Pressure Buildup in Wells": Proc. Third World Petroleum Congress, 1951, Sec. II, E. J. Brill, Leiden, Netherlands.
3. Van Everdingen, A. F., "The Skin Effect and Its Influence on the Productive Capacity of a Well": Trans. AIME (1953) Vol. 198, p. 171.
4. Arps, J. J., "How Well-Completion Damage Can be Determined Graphically": World Oil, Vol. 140, No. 5, Apr. 1955, p. 225.
5. Groeneman, A. R., and Wright, F. F., "Analysis of Fluid Input Wells by Shut-in Pressures": Jour. of Pet. Tech., Vol. 8, No. 7, July 1956, p. 21.
6. Guerrero, E. T., Reservoir Engineering, Part 69, "How to Find Capacity, Productivity Ratio, and Skin Effect for Oil Well from Pressure-Buildup Data": The Oil and Gas Journal, Vol. 62, No. 29, July 20, 1964, pp. 107, 110-111.

Part 72

Skin effect and static pressure

from pressure falloff data

GIVEN: The condititons and data for the preceding problem are used in this problem. Pressure-drawdown data are shown in Table 1, and other data are given below.

Depth from surface to middle of sand face = 6,500 ft

Water injection rate prior to shut-in Q_{wi} = 525 b/d

Mobility ratio = 2.5

At reservoir conditions,

Water formation volume factor, B_w = 1.03

Viscosity of water, μ_w = 0.9 cp

Compressibility of water, $c_w = 3 \times 10^{-6}$ psi^{-1}

Compressibility of oil, $c_o = 6 \times 10^{-6}$ psi^{-1}

Compressibility of formation, $c_f = 4 \times 10^{-6}$ psi^{-1}

Density of water, ρ_w = 1.0 gm/cc = 0.43 psi/ft

Porosity, ϕ = 17%

Cumulative water injected, W_i = 15,500 bbl

Net sand thickness, h = 27 ft

Well radius, r_w = 3 in.

Gas saturation at start of flooding, $S_g = 16\%$

Residual-gas saturation, $S_{gr} = 0$

Oil saturation at start of flooding, $S_o = 48\%$

Residual-oil saturation, $S_{or} = 30\%$

Static reservoir pressure, p_s = 1,240 psia

Pressure at wellhead, p_h—which at time of closing in = 2,445 psia

Sand-face injection pressure at time of closing in, p_{wo} = 5,240 psia

Diameter of casing (no tubing in well), d = 6.366 in.

Pressure at wellhead, p_h did not fall to zero soon after the well was shut in.

FIND: (1) Injection capacity
(2) Skin effect and
(3) Static Reservoir pressure

METHOD OF SOLUTION: Hazebrock et al.[1] have developed two procedures for determining injection capacity and skin effect from pressure-falloff data:

1. Physical constants of water and oil are assumed equal.
2. Physical constants of water and oil are assumed different.

For Case 1, solutions were derived for two sets of conditions: (A) Surface pressure decreases slowly and well stays filled to top for long closed-in times, and (B) surface pressure drops to zero a short time after closing in and fluid level starts to sink.

For Case 1

$$k_w h = \left(\frac{Q_{wi} B_w \mu_w}{b_1}\right)\left(\frac{1-C_1-C_2}{1-C_3}\right)[f(\theta)] \quad (1)$$

$$S.E. = \frac{0.00708\, k_w h\,(p_{wo}-p_s)}{Q_{wi} B_w \mu_w} - \ln\left(\frac{r_e}{r_w}\right) \quad (2)$$

where

$$r_e = \sqrt{[(W_i)(5.615)B_w]/[\pi \phi (S_g - S_{gr})h]} \quad (3)$$

For conditions A

$$C_1 = 0.0539\,\frac{d^2 \beta_1 b_1 c_w (p_{wo}-p_h)}{Q_{wi} B_w \rho_w} \quad (4)$$

$$C_2 = 0 \quad (5)$$

$$C_3 = \frac{p_{wo}-p_s}{b_1} C_1 \quad (6)$$

and for conditions B

$$C_1 = 0.0539\,\frac{d^2 \beta_1 b_1}{Q_{wi} B_w \rho_w} \quad (7)$$

$$C_2 = \frac{p_h}{b_1} C_1 \quad (8)$$

$$C_3 = \frac{p_{wo}-p_s}{b_1} C_1 \quad (9)$$

For both conditions A and B

$$\theta = \frac{C_1(1-C_3)}{2(1-C_1-C_2)} \quad (10)$$

Often for conditions A and B, C_1, C_2, C_3, and θ are small such that $f(\theta)$ can be taken equal to 181. Consequently

$$k_w h \approx 181\,\frac{Q_{wi} B_w \mu_w}{b_1} \quad (11)$$

For Case 2, the equations[1] assume that after closing in, no water flows from the well into the formation. Here injection capacity and skin effect are obtained with

$$k_w h = \frac{2\, Q_{wi} B_w \mu_w F}{b_1} \quad (12)$$

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$$S.E. = \frac{0.00708\, k_w h\,(p_{wo}-p_s)}{Q_{wi} B_w \mu_w} - \frac{M-1}{2}\ln\left[\frac{V_o}{V_w}+1\right] - \ln\left(\frac{r_e}{r_w}\right) \quad (13)$$

where

$$\frac{V_o}{V_w} = \frac{S_o - S_{or}}{S_g - S_{gr}} = \frac{\text{vol. oil bank}}{\text{vol. water bank}} \quad (14)$$

F is obtained from Figs. 4, 5, and 6 of reference 1. On these figures

$$R_o = \frac{1}{\sqrt{(V_o/V_w)+1}} = \frac{1}{\sqrt{[(S_o-S_{or})/(S_g-S_{gr})]+1}} \quad (15)$$

$$\text{and } \gamma = c_o/c_w \quad (16)$$

In the above equations Q_{wi}, B_w, μ_w, c_w, c_o, W_i, h, r_w, S_o, S_{or}, S_g, S_{gr}, θ, ρ_w, p_h, p_{wo}, and d are given with the data and

k_w = permeability to water, md

b_1 = constant obtained from plot of $(p_{wst} - p_{s\,est.})$ vs. Δt as shown on Fig. 1, psi

p_{wst} = sand-face pressure at shut-in time, Δt psia

p_s = static reservoir pressure, psia

$p_{s\,est.}$ = estimated static reservoir pressure, psia

$f(\theta)$ = function for calculating $k_w h$ in Case 1

S.E. = skin effect

r_e = outer radius of oil bank, ft

β_1 = constant given by slope of plot of $(p_{wst} - p_{s\,est.})$ vs. Δt as shown on Fig. 1, hours^{-1}

c_w = water compressibility, psi^{-1}

p_w = sand-face injection pressure, psia

M = water-to-oil mobility ratio = $(k_w \mu_o)/(k_o \mu_w)$

F = function for calculating $k_w h$ in Case 2

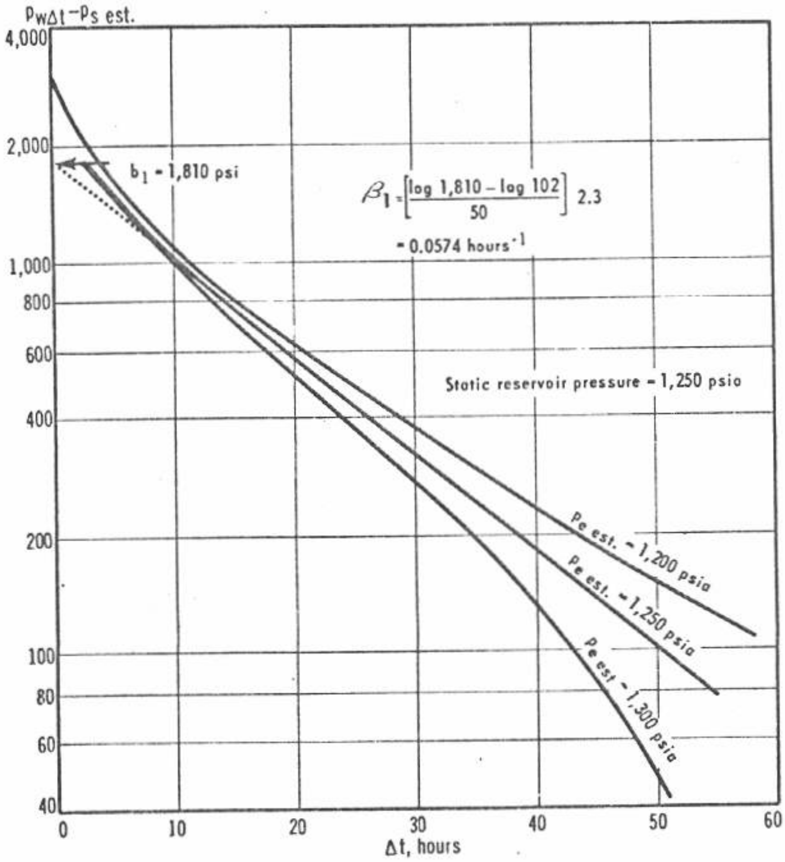
$R_o = r_o/r_e$

r_o = outer radius of water bank, ft

Static reservoir pressure is determined following a procedure similar to that used in the problem on p. 30 (also reference 3). A value of static pressure, p_s, is assumed, and a plot of log ($p_{w\Delta t} - p_{s\ est.}$) vs. shut-in time, Δt, in hours is made. If a straight line at large times is not obtained, a new value for p_s is assumed, and another curve is plotted. The p_s that gives a straight-line plot at large values of time is the static reservoir pressure. The intercept of this plot at $\Delta t = o$ gives b_1 and the slope is β_1. Table 1 contains the calculations for static reservoir pressure. Fig. 1 shows the plotted data.

In this problem the static reservoir pressure was determined to be 1,250 psia.

SOLUTION: Hazebroek[1] et al. recommend that Case 2 not be used unless $C_1 \lesseqgtr 0.1$ and θ is small (less than 0.04). Thus for conditions A and obtaining $b_1 = 1{,}810$ psi and $\beta_1 = 0.0574$ hours^{-1} from Fig. 1



DETERMINATION of static reservoir pressure from pressure drawdown data. Fig. 1.

$$C_1 = \frac{(0.0539)\,(6.366)^2\,(0.0574)\,(1{,}810)\,(3\times10^{-6})\,(5{,}240-2{,}445)}{(525)\,(1.03)\,(1.0)} = 0.0035$$

$$C_2 = 0$$

$$C_3 = \left(\frac{5{,}240-1{,}250}{1{,}810}\right)(0.0035) = 0.0077$$

According to Equation 10

$$\theta = \frac{(0.0035)\,(1-0.0077)}{2\,(1-0.0035)} = 0.00174$$

Thus the values of C_1 and θ verify that Case 2 is applicable. From Equation 15

$$R_o = \frac{1}{\sqrt{[(48-30)/(16-0)]+1}} = 0.68$$

Since $\gamma = \dfrac{6.0\times10^{-6}}{3.0\times10^{-6}} = 2.0$ and using Fig. 5 of reference 1

F = 110. Thus with Equation 12

$$k_w h = \frac{(2)\,(525)\,(1.03)\,(0.9)\,(110)}{1{,}810} = 59.2 \text{ md ft}$$

or $k_w = 59.2/27 = 2.2$ md

And from Equations 3 and 13

$$r_e = \sqrt{[(15{,}500)\,(5.615)\,(1.03)]/[(3.14)\,(0.17)\,(0.16-0)\,(27)]} = 197 \text{ ft}$$

$$\text{S.E.} = \frac{(0.00708)\,(59.2)\,(5{,}240-1{,}250)}{(525)\,(1.03)\,(0.9)} - \frac{2.5-1}{2}\ln\left[\frac{48-30}{16-0}+1\right] - \ln\left(\frac{197}{1/4}\right)$$

$$= -3.77$$

On many occasions sufficient data will not be available to apply this method as recommended.[1] It may still be possible to obtain approximate answers with Equations 11 and 2 which require fewer data than the procedure used above. Thus

$$k_w h = \frac{(181)\,(525)\,(1.03)\,(0.9)}{1{,}810}$$

$$= 48.7 \text{ md ft}$$

$$k_w = 48.7/27 = 1.8 \text{ md}$$

and

$$\text{S.E.} = -3.77+0.56 = -3.21$$

Table 2 compares these approximate results with those obtained with the more rigorous procedures using Equations 11, 12, and 13. The results obtained by the two methods agree within engineering accuracy.

It is felt that the approximate method is acceptable and much easier to apply in many cases.

DISCUSSION: The same data were used for this problem as for the preceeding one to afford a comparison of different methods for finding injection capacity, skin effect, and static reservoir pressure. Note that the results are comparable: for skin effect −3.72 and −5.10 compared to −3.77 and −3.21 in this problem; for injection capacity 50.4 md ft compared to 59.2 and 48.7 md ft in this problem.

Static reservoir pressures do not compare well because that obtained earlier is not correct—the theory there applies to a formation saturated with a slightly compressible fluid. It can be used for water injection wells when the residual gas saturation in the oil bank and water bank is nearly zero, and the compressibilities of the oil and water are nearly the same.

It is only applicable for times before interference with oil banks of adjacent wells or for a sealed reservoir boundary. This theory is usable as long as infinite boundary conditions prevail before interference of oil banks. Note in Fig. 1 of the preceding problem that at large shut-in times, Δt, the pressure is beginning to level out, indicating that interference has occurred.

In spite of interference, the procedures used earlier can be used for determination of injection capacity and skin effect assuming the proper straight-line portion of the drawdown curve can be reliably defined.

On many water-injection wells, it is difficult or impossible to determine this portion of the drawdown curve. Such behavior is often encountered in waterfloods with close well spacing or where average water compressibility is low. An advantage of the method used in this problem is that it does not require the conventional pressure-drawdown slope. Also it is more rigorously applicable for such conditions than the methods used in the previous problem.

Conversely it has the disadvantage of being more complex and requiring more different types of data. It may often be difficult to determine whether Case 1 or Case 2 should be used.

Reliable relative permeability data (or effective permeability data) are seldom available for determination of mobility ratio. It is also doubtful if reliable oil and gas-saturation data can be obtained.

In spite of these limitations, the Hazebroek et al. method[1] can be as reliable or more reliable than the methods used earlier. It is applicable to infinite-boundary conditions (before fillup or interference) and believed applicable to finite-boundary conditions (after fillup or interference).

It can be simple to apply when mobility ratio is taken equal to unity and only Equations 11 and 2 are used. It is believed that this simplification often will give satisfactory results, and that a more reliable value of static reservoir pressure is obtained with the procedures used in this problem.

The method of Hazebroek et al. is based on steady-state, succession of steady states, and unsteady-state flow solutions of the diffusivity equation. It assumes that: (1) the formation is horizontal, homogeneous, of constant thickness, and contains oil, gas, and water; (2) an oil bank is built ahead of the water with oil replacing expelled gas; (3) fluid saturations within each bank are constant; (4) the water and oil bank boundaries are circular and concentric with the well; and (5) the pressure at the outer oil boundary remains constant and equal to the static reservoir pressure.

Determination of static reservoir pressure
Table 1

Shut-in time Δt, hours	BHP $p_{w\Delta t}$, psia	$(p_{w\Delta t} - p_{s\,est.})$ $p_{s\,est.} = 1{,}200$	$p_{s\,est.} = 1{,}250$	$p_{s\,est.} = 1{,}300$
1	3,685	2,485	2,435	2,385
3	3,062	1,862	1,812	1,762
5	2,750	1,550	1,500	1,450
7	2,535	1,335	1,285	1,235
10	2,288	1,088	1,038	988
15	2,012	812	762	712
20	1,815	615	565	515
25	1,683	483	433	383
30	1,577	377	327	277
35	1,495	295	245	195
40	1,435	235	185	135
45	1,389	189	139	89
50	1,351	151	101	51

Comparison of capacity, permeability, and skin effect determined by rigorous and approximate methods
Table 2

Factor	Rigorous method	Approximate method
$k_w h$	59.2 md ft	48.7 md ft
k_w	2.2 md	1.8 md
S.E.	−3.77	−3.21

References

1. Hazebroek, P., Rainbow, H., and Matthews, C. S.: "Pressure Falloff in Water Injection Wells," Trans. AIME Vol. 213, 1958, pp. 250-260.

2. Guerrero, E. T., Reservoir Eng. Part 71—"How to Find Injection Capacity and Skin Effect from Pressure Falloff Data from Water-Injection Wells": The Oil and Gas Journal, Vol. 62, No. 38, Sept. 21, 1964, pp. 206, 208-210.

3. Guerrero, E. T., Reservoir Eng. Part 64—"How to Find Reservoir Pressure Using Muskat and Arps and Smith Methods": The Oil and Gas Journal, Vol. 62, No. 3, Jan. 20, 1964, pp. 82-84.