

APPENDICES

Appendix A

Units Systems and Conversions

A.1 Introduction

In any book of this nature, it is worthwhile to include a comprehensive list of units conversion factors, since data are often reported in units different from those used in the equations. Such factors are presented in this appendix. Because of the possibility of eventual conversion of engineering calculations to a metric standard, we also include information about the "SI" system of weights and measures.¹ Finally, we compare some important units and equations in five different unit systems.

A.2 The International (SI) Metric System

"SI" is the official abbreviation, *in all languages*, for the International System of Units (*les Système International d'Unités*). That system is neither the centimetre-gram-second (cgs) system nor the metre-kilogram-second (mks) system. Rather, it is a modernized version of mks. A complete description of SI is presented by Hopkins.¹ The American Petroleum Institute has proposed a set of metric standards for use in the petroleum industry.²

TABLE A.1—SI SYSTEM UNITS.

Base SI Units Used in Well Test Analysis

Quantity	Name	Symbol
length	metre	m
time	second	s
mass	kilogram	kg
temperature	kelvin	K
amount of substance	mole	mol

Units That Are Multiples or Submultiples of SI Base Units Given Special Names

Quantity	Name of Unit	Symbol	Definition	SI Term
mass	tonne	t	$1t = 10^3 \text{ kg}$	Mg
volume	litre*	l	$1l = 1 \text{ dm}^3$	dm ³

SI-Derived Units With Special Names Used in Well Test Analysis

Quantity	Name	Symbol	Expression in Terms of Other Units	Expression in Terms of SI Base Units
force	newton	N	—	$\text{m} \cdot \text{kg} \cdot \text{s}^{-2}$
pressure	pascal	Pa	N/m^2	$\text{m}^{-1} \cdot \text{kg} \cdot \text{s}^{-2}$
energy, work, quantity of heat	joule	J	$\text{N} \cdot \text{m}$	$\text{m}^2 \cdot \text{kg} \cdot \text{s}^{-2}$

*In 1964, the 12th Conférence Générale des Poids et Mesures (CGPM) redefined the litre to be $1 \text{ dm}^3 = 0.001 \text{ m}^3$. At the same time, it abrogated the 1901 definition of the litre given by the third CGPM.

Table A.1 lists the five *base* SI units encountered in well test analysis. The approved spelling is the French spelling. Names of units are never capitalized, although some of the abbreviations are. Most units are abbreviated with a single symbol. Table A.1 also lists two SI units that have been given special names and three derived units with special SI names. No units are normally used in well test analysis other than those presented in Table A.1.

SI allows prefixes to indicate multiples of the base units. The prefixes are summarized in Table A.2. Compound prefixes, such as micro-micro, are not allowed; the correct prefix, in this case pico, should always be used.

A.3 Constants and Conversion Factors

Table A.3 presents values for several physical constants useful in petroleum engineering, in several sets of units. Table A.4 summarizes useful conversion factors. SI units are indicated by boldface type. To use Table A.4, multiply the quantity given in the left-hand column by the number given in the "multiply by" column to obtain units in the second column. For simplicity, the "inverse" column may be used to reverse this procedure. Thus, to convert from square feet to acres, one would multiply the number of square feet by 2.296×10^{-5} .

Some permeability units are given under the heading "area" in Table A.4, since permeability has the units of area. Table A.5 is also supplied to simplify other conver-

TABLE A.2—SI PREFIXES.

Factor	Prefix	Symbol*
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^2	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

*Only the symbols T (tera), G (giga), and M (mega) are capital letters. Compound prefixes are not allowed — for example, use nm (nano metre) rather than μm (milli micro metre).

TABLE A.3—PHYSICAL CONSTANTS AND VALUES.*

Quantity	Magnitude	Unit
Triple point of water	273.16 exactly	K
	0.01 exactly	°C
	491.688 exactly	°R
	32.018 exactly	°F
Absolute zero	0.00 exactly	K
	-273.15 exactly	°C
	0.00 exactly	°R
	-459.67 exactly	°F
Gas constant (R)	8.3143	J·mol ⁻¹ ·K ⁻¹
	8.3143 E + 07	erg·(gm mole) ⁻¹ ·°K ⁻¹
	10.732	psi·ft ³ ·(lb mole) ⁻¹ ·°R ⁻¹
Maximum density of water	999.973	kg·m ⁻³
	0.999 973	g·cm ⁻³
	62.426 1	lb _m ·ft ⁻³
Density of water at 60 °F (15.56 °C, 288.71 K)	999.014	kg·m ⁻³
	0.999 014	g·cm ⁻³
	62.366 4	lb _m ·ft ⁻³
Water gradient at 60 °F (15.56 °C, 288.71 K)	9,796.98	Pa·m ⁻¹
	979.698	dyne·cm ⁻³
	0.433 100	psi·ft ⁻¹
Standard atmosphere	1.013 25 E + 05	Pa
	1.013 25 E + 06	dyne·cm ⁻²
	14.695 9	psi
Density of air at 1 atm, 60 °F (15.56 °C, 288.71 K)	1.223 2	kg·m ⁻³
	1.223 2 E - 03	g·cm ⁻³
	0.076 362	lb _m ·ft ⁻³
Earth's gravitational acceleration, g	9.806 650	m·s ⁻²
	980.665 0	cm·s ⁻²
	32.174 05	ft·s ⁻²
g _c	1.000 000	kg·m·N ⁻¹ ·sec ⁻²
	1.000 000	g·cm·dyne ⁻¹ ·sec ⁻²
	32.174 05	lb _m ·ft·lb _f ⁻¹ ·s ⁻²
π	3.141 593	
e	2.718 282	
ln (10)	2.302 585	
γ (Euler's constant)	0.577 215 66	
°API	141.5	
	—	—
	—	—
	—	y(60 °F)

*SI values are in boldface type. All quantities are consistent with conversion factors for the current SI system.

sions using permeability. That table is similar to data given by Amyx, Bass, and Whiting³ but there is some variation in numbers. Differences occur because numerical values are shown only to the significance possible (in this case, limited by the SI agreed-on accuracy of atmospheric pressure and the density of water and mercury), and are based on conversion factors derived from the SI Standards. Some of those factors are slightly different from those used previously because of more precise definitions for certain quantities.

Table A.6 provides conversions for various temperature scales. The SI standard temperature unit is the kelvin; that unit is neither capitalized nor associated with the word "degree." One kelvin is equivalent to 1 degree Celsius (the SI system drops the use of the term Centigrade). The triple point of water is *defined* as 273.16 kelvin exactly. All other temperatures are derived from that. The normal Celsius and Fahrenheit scales are the same, as are their conversions to the other scales.

Table A.7 compares units and equations from five systems of units. The oilfield units are used exclusively throughout this monograph. The column for SI units is a coherent system (that is, one in which basic equations contain no units conversion factors). The preferred API standard SI unit system² is not a coherent system, but has the advantages of providing reasonable size values for most physical quantities. The cgs units column is the standard cgs

system used for many years in petroleum engineering. Finally, the column for groundwater units is provided for those who practice in that field. The symbols used in groundwater hydrology vary from reference to reference, so the reader should check carefully when using groundwater literature. Table A.8 shows the correspondence between some groundwater quantities and oilfield quantities for any consistent unit system.

Many of the conversion factors provided in Tables A.3 through A.7 have been calculated from SI-stated factors for other unit conversions. Depending on the approach used in performing such a calculation, the seventh significant digit may vary by a few units. The reader should be aware of that when attempting to verify the values given, or to use them in precision computations.

References

1. Hopkins, Robert A.: *The International (SI) Metric System and How It Works*, Polymetric Services, Inc., Tarzana, Calif. (1974).
2. "Conversion of Operational and Process Measurement Units to the Metric (SI) System," *Manual of Petroleum Measurement Standards*, Pub. API 2564, American Petroleum Institute (March 1974) Chap. 15, Sec. 2.
3. Amyx, James W., Bass, Daniel, M., Jr., and Whiting, Robert L.: *Petroleum Reservoir Engineering: Physical Properties*, McGraw-Hill Book Co., Inc., New York (1960) 79.

TABLE A.4—CONVERSION FACTORS USEFUL IN WELL TEST ANALYSIS.

SI conversions are in boldface type. All quantities are current to SI standards as of 1974. An asterisk (*) after the sixth decimal indicates the conversion factor is exact and all following digits are zero. All other conversion factors have been rounded. The notation E+03 is used in place of 10^3 , and so on.

To Convert From	To	Multiply by	Inverse
AREA			
acre	metre² (m²)	4.046 856 E+03	2.471 054 E-04
	foot ²	4.356 000* E+04	2.295 684 E-05
darcy	metre² (m²)	9.869 23 E-13	1.013 25 E+12
	centimetre ² (cm ²)	9.869 23 E-09	1.013 25 E+08
	micrometre ² (μm ²)	9.869 23 E-01	1.013 25 E+00
	millidarcy	1.000 000* E+03	1.000 000*E-03
	cm ² ·cp·sec ⁻¹ ·atm ⁻¹	1.000 000* E+00	1.000 000*E+00
foot ²	metre² (m²)	9.290 304* E-02	1.076 391 E+01
	centimetre ²	9.290 304* E+02	1.076 391 E-03
	inch ²	1.440 000* E+02	6.944 444 E-03
hectare	metre² (m²)	1.000 000* E+04	1.000 000*E-04
	acre	2.471 054 E+00	4.046 856 E-01
mile ²	metre² (m²)	2.589 988 E+06	3.861 022 E-07
	acre	6.400 000* E+02	1.562 500*E-03
DENSITY			
gram/centimetre ³	kilogram/metre³ (kg·m⁻³)	1.000 000* E+03	1.000 000*E-03
	pound-mass/foot ³	6.242 797 E+01	1.601 846 E-02
	pound-mass/gallon	8.345 405 E+00	1.198 264 E-01
	pound-mass/barrel	3.505 070 E+02	2.853 010 E-03
pound-mass/foot ³	kilogram/metre³ (kg·m⁻³)	1.601 846 E+01	6.242 797 E-02
	pound-mass/gallon	1.336 805 E-01	7.480 520 E+00
	pound-mass/barrel	5.614 583 E+00	1.781 076 E-01
pound-mass/gallon	kilogram/metre³ (kg·m⁻³)	1.198 264 E+02	8.345 406 E-03
	pound-mass/barrel	4.200 000 E+01	2.380 952 E-02
FORCE			
dyne	newton (N)	1.000 000* E-05	1.000 000*E+05
	pound-force	2.248 089 E-06	4.448 222 E+05
kilogram-force	newton (N)	9.806 650* E+00	1.019 716 E-01
	pound-force	2.204 622 E+00	4.535 924 E-01
pound-force	newton (N)	4.448 222 E+00	2.248 089 E-01
LENGTH			
angstrom	metre (m)	1.000 000* E-10	1.000 000*E+10
centimetre	metre (m)	1.000 000* E-02	1.000 000*E+02
foot	metre (m)	3.048 000* E-01	3.280 840 E+00
	centimetre	3.048 000* E+01	3.280 840 E-02
inch	metre (m)	2.540 000* E-02	3.937 008 E+01
	centimetre	2.540 000* E+00	3.937 008 E-01
micron	metre (m)	1.000 000* E-06	1.000 000*E+06
mile (U.S. statute)	metre (m)	1.609 344* E+03	6.213 712 E-04
	foot	5.280 000* E+03	1.893 939 E-04
MASS			
gram-mass	kilogram (kg)	1.000 000* E-03	1.000 000*E+03
ounce-mass (av)	kilogram (kg)	2.834 952 E-02	3.527 397 E+01
	gram	2.834 952 E+01	3.527 397 E-02
pound-mass	kilogram (kg)	4.535 923 7*E-01	2.204 623 E+00
	ounce-mass	1.600 000* E+01	6.250 000*E-02
slug	kilogram (kg)	1.459 390 E+01	6.852 178 E-02
	pound-mass	3.217 405 E+01	3.108 095 E-02
ton (U.S. short)	kilogram (kg)	9.071 847 E+02	1.102 311 E-03
	pound-mass	2.000 000* E+03	5.000 000*E-04
ton (U.S. long)	kilogram (kg)	1.016 047 E+03	9.842 064 E-04
	pound-mass	2.240 000* E+03	4.464 286 E-04
ton (metric)	kilogram (kg)	1.000 000* E+03	1.000 000*E-03
tonne	kilogram (kg)	1.000 000* E+03	1.000 000*E-03

TABLE A.4—CONT'D.

To Convert From	To	Multiply by	Inverse
PRESSURE			
atmosphere (normal—760 mm Hg)	pascal (Pa)	1.013 25 E+05	9.869 23 E-06
	mm Hg (0 °C)	7.600 000*E+02	1.315 789 E-03
	feet water (4 °C)	3.389 95 E+01	2.949 90 E-02
	psi	1.469 60 E+01	6.804 60 E-02
	bar	1.013 25 E+00	9.869 23 E-01
bar	pascal (Pa)	1.000 000*E+05	1.000 000*E-05
	psi	1.450 377 E+01	6.894 757 E-02
centimetre of Hg (0 °C)	pascal (Pa)	1.333 22 E+03	7.500 64 E-04
	psi	1.933 67 E-01	5.171 51 E+00
dyne/centimetre ²	pascal (Pa)	1.000 000*E-01	1.000 000*E+01
	psi	1.450 377 E-05	6.894 757 E+04
feet of water (4 °C)	pascal (Pa)	2.988 98 E+03	3.345 62 E-04
	psi	4.335 15 E-01	2.306 73 E+00
kilogram-force/centimetre ²	pascal (Pa)	9.806 650*E+04	1.019 716 E-05
	bar	9.806 650*E-01	1.019 716 E+00
	psi	1.422 334 E+01	7.030 695 E-02
psi	pascal (Pa)	6.894 757 E+03	1.450 377 E-04
TIME			
day	second (s)	8.640 000*E+04	1.157 407 E-05
	minute	1.440 000*E+03	6.944 444 E-04
	hour	2.400 000*E+01	4.166 667 E-02
hour	second (s)	3.600 000*E+03	2.777 778 E-04
	minute	6.000 000*E+01	1.666 667 E-02
minute	second (s)	6.000 000*E+01	1.666 667 E-02
VISCOOSITY			
centipoise	pascal-second (Pa·s)	1.000 000*E-03	1.000 000*E+03
	dyne-second/centimetre ²	1.000 000*E-02	1.000 000*E+02
	pound-mass/(foot-second)	6.719 689 E-04	1.488 164 E+03
	pound-force-second/foot ²	2.088 543 E-05	4.788 026 E+04
	pound-mass/(foot-hour)	2.419 088 E+00	4.133 789 E-01
centistoke	metre²/second (m²/s)	1.000 000*E-06	1.000 000*E+06
	centipoise/(gram/centimetre ³)	1.000 000*E+00	1.000 000*E+00
poise	pascal-second (Pa·s)	1.000 000*E-01	1.000 000*E+01
pound-mass/(foot-second)	pascal-second (Pa·s)	1.488 164 E+00	6.719 689 E-01
pound-mass/(foot-hour)	pascal-second (Pa·s)	4.133 789 E-04	2.419 088 E+03
pound-force-second/foot ²	pascal-second (Pa·s)	4.788 026 E+01	2.088 543 E-02
VOLUME			
acre-foot	metre³ (m³)	1.233 482 E+03	8.107 131 E-04
	foot ³	4.356 000*E+04	2.295 684 E-05
	barrel	7.758 368 E+03	1.288 931 E-04
barrel	metre³ (m³)	1.589 873 E-01	6.289 811 E+00
	foot ³	5.614 583 E+00	1.781 076 E-01
	gallon	4.200 000*E+01	2.380 952 E-02
foot ³	metre³ (m³)	2.831 685 E-02	3.531 466 E+01
	inch ³	1.728 000 E+03	5.787 037 E-04
	gallon	7.480 520 E+00	1.336 805 E-01
gallon	metre³ (m³)	3.785 412 E-03	2.641 720 E+02
	inch ³	2.310 001 E+02	4.329 003 E-03
litre	metre³ (m³)	1.000 000*E-03	1.000 000*E+03
VOLUMETRIC RATE			
barrel/day	metre³/sec (m³/s)	1.840 131 E-06	5.434 396 E+05
	metre ³ /hour (m ³ /h)	6.624 472 E-03	1.509 554 E+02
	metre ³ /day (m ³ /d)	1.589 873 E-01	6.289 810 E+00
	centimetre ³ /second	1.840 131 E+00	5.434 396 E-01
	foot ³ /minute	3.899 016 E-03	2.564 750 E+02
	gallon/minute	2.916 667 E-02	3.428 571 E+01
foot ³ /minute	metre³/sec (m³/s)	4.719 474 E-04	2.118 880 E+03
foot ³ /second	metre³/sec (m³/s)	2.831 685 E-02	3.531 466 E+01
gallon/minute	metre³/sec (m³/s)	6.309 020 E-05	1.585 032 E+04

TABLE A.5—AUXILIARY PERMEABILITY CONVERSIONS.*

To Convert From md	To	Multiply by	Inverse
darcy		1.000 000*E-03	1.000 000*E+03
metre ² (m ²)		9.869 23 E-16	1.013 25 E+15
centimetre ² (cm ²)		9.869 23 E-12	1.013 25 E+11
micrometre ² (μm ²)		9.869 23 E-04	1.013 25 E+03
(cm ³ /s) cp		1.000 000* E-03	1.000 000* E+03
cm ² (atm/cm)			
(cm ³ /s) cp		9.869 23 E-10	1.013 25 E+09
cm ² [(dyne/cm ²)/cm]			
(ft ³ /s) cp		7.324 41 E-08	1.365 30 E+07
ft ² (psi/ft)			
(ft ³ /s) cp		3.417 80 E-11	2.925 85 E+10
cm ² [(cm water)/cm]			
(B/D) cp		1.127 12 E-03	8.872 17 E+02
ft ² (psi/ft)			
(gal/min) cp		1.425 15 E-05	7.016 81 E+04
ft ² [(ft water)/ft]			
ft ²		1.062 32 E-14	9.413 40 E+13

*Conversion factor is exact; all following digits are zero.

TABLE A.6—TEMPERATURE SCALE CONVERSIONS.*

To Convert	To	Solve
degree Fahrenheit	kelvin	$T_K = (T_F + 459.67)/1.8$
degree Rankine	kelvin	$T_K = T_R/1.8$
degree Fahrenheit	degree Rankine	$T_R = T_F + 459.67$
degree Fahrenheit	degree Celsius	$T_C = (T_F - 32)/1.8$
degree Celsius	kelvin	$T_K = T_C + 273.15$

*The SI standard, the kelvin (K), is defined so the triple point of water is 273.16 K exactly. The SI temperature symbol is written K, without a degree symbol. The cgs (and common) temperature unit is the degree Celsius, °C; the common oilfield unit is the degree Fahrenheit, °F.

TABLE A.8—RELATIONSHIP OF COMMON GROUNDWATER AND OILFIELD QUANTITIES.

A consistent-unit system is assumed. Variable definitions for each system are given in Table A.7.

Groundwater Quantity		Oilfield Quantity
Coefficient of permeability	= $P = K$	$= \frac{k}{\mu} \left(\frac{\rho g}{g_c} \right)$
Transmissivity	= $T = Km$	$= \frac{kh}{\mu} \left(\frac{\rho g}{g_c} \right)$
Coefficient of storage	= S	$= \phi c_h \left(\frac{\rho g}{g_c} \right)$
Drawdown	= s	$= \frac{p_i - p}{(\rho g/g_c)}$
Head	= h	$= \frac{p}{(\rho g/g_c)}$
Dimensionless drawdown	= $W(1/4\alpha)$	$= 2p_D(t_D)$

TABLE A.7.—COMPARISON OF UNITS AND EQUATIONS IN VARIOUS UNIT SYSTEMS.*

SI Units		Preferred API Standard SI Units		CGS Units*		Groundwater Units	
q	— production rate, STB/D	dm^3/s	cm^3/s	Q	— production rate, Gal/min	Q	Q
h	— production thickness, ft	m	cm	m	— formation thickness, ft	m	m
k	— permeability, md	m^2	darcy	P or K	— coefficient of permeability,	P	P
μ	— viscosity, cp	$\text{Pa}\cdot\text{s}$	cp		gal/day ft^{3+*}		
k/μ	— mobility, md/cp	$\text{m}^2/(\text{Pa}\cdot\text{s})$	darcy/cp		gal/(day ft^3)		
kh/μ	— mobility-thickness product, $\text{md ft}/\text{cp}$	$\text{m}^3/(\text{Pa}\cdot\text{s})$	$\text{m}(\mu\text{m}^3)/(\text{Pa}\cdot\text{s})$		— coefficient of transmissivity,	T	T
Δp	— pressure difference, psi	Pa	kPa		s — drawdown, ft of water,	s	s
p	— pressure, psi	Pa	kPa		>0 for pressure drawdown**	h	h
r	— radius, ft	m	m		— head of water, ft of water	r	r
t	— time, hours	s	s		— radius, ft	t	t
ϕ	— porosity, fraction	Pa $^{-1}$	kPa $^{-1}$		— time, days	S	S
c_r	— total system compressibility, psi^{-1}	Pa $^{-1}$	m \cdot Pa $^{-1}$		— coefficient of storage, fraction**		
$\phi c_r h$	— porosity-compressibility-thickness product, ft psi^{-1}	cm \cdot atm $^{-1}$	cm \cdot atm $^{-1}$				
<u>DIMENSIONLESS TIME</u>							
t_D	$= \frac{0.000263679 \frac{kt}{\phi \mu c_r f_w^2}}$	$t_D = \frac{kt}{\phi \mu c_r f_w^2}$	$t_D = \frac{kt}{\phi \mu c_r f_w^2}$	$\alpha = 0.1336805$	$\frac{Tt}{Sr_w^2}$	$\alpha = 0.1336805$	$\frac{Tt}{Sr_w^2}$
<u>DARCY'S LAW FOR INCOMPRESSIBLE, RADIAL FLOW</u>							
$q = \frac{0.00708188 kh(p_e - p_w)}{B\mu \ln(r_e/r_w)}$	$q = \frac{2\pi \times 10^{-6} kt(p_e - p_w)}{B\mu \ln(r_e/r_w)}$	$q = \frac{2\pi \times 10^{-6} kt(p_e - p_w)}{\phi \mu c_r f_w^2}$	$q = \frac{2\pi \times 10^{-6} kt(p_e - p_w)}{B\mu \ln(r_e/r_w)}$	$Q = \frac{0.00436332 T(h_e - h_w)}{\ln(r_e/r_w)}$			
<u>DIFFUSIVITY EQUATION</u>							
$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} =$	$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} =$	$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} =$	$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} =$	$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} =$	$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} =$	$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} =$	$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} =$
$\frac{1}{0.000263679} \frac{\phi \mu c_r}{k} \frac{\partial p}{\partial t}$	$\frac{1}{3.6 \times 10^{-6}} \frac{\phi \mu c_r}{k} \frac{\partial p}{\partial t}$	$\frac{1}{3.6 \times 10^{-6}} \frac{\phi \mu c_r}{k} \frac{\partial p}{\partial t}$	$\frac{1}{0.1336805} \frac{\phi \mu c_r}{k} \frac{\partial p}{\partial t}$	$\frac{1}{0.1336805} \frac{\phi \mu c_r}{k} \frac{\partial p}{\partial t}$	$\frac{1}{0.1336805} \frac{\phi \mu c_r}{k} \frac{\partial p}{\partial t}$	$\frac{1}{0.1336805} \frac{\phi \mu c_r}{k} \frac{\partial p}{\partial t}$	$\frac{1}{0.1336805} \frac{\phi \mu c_r}{k} \frac{\partial p}{\partial t}$
<u>GENERALIZED TRANSIENT FLOW EQUATION</u>							
$\Delta p = \frac{141.205 q B \mu \rho_D(t_D)}{kh}$	$\Delta p = \frac{q B \mu \rho_D(t_D)}{2\pi kh}$	$\Delta p = \frac{q B \mu \rho_D(t_D)}{2\pi kh}$	$\Delta p = \frac{q B \mu \rho_D(t_D)}{2\pi kh}$	$\Delta p = \frac{1}{2\pi} \frac{q B \mu}{kh} \rho_D(t_D)$	$\Delta p = \frac{1}{2\pi} \frac{q B \mu}{kh} \rho_D(t_D)$	$\Delta p = \frac{1}{2\pi} \frac{q B \mu}{kh} \rho_D(t_D)$	$\Delta p = \frac{1}{2\pi} \frac{q B \mu}{kh} \rho_D(t_D)$
<u>SLOPE OF SEMILOG STRAIGHT LINE</u>							
$m = 162.568 \frac{q B \mu}{kh}$	$m = 0.183234 \frac{q B \mu}{kh}$	$m = 1.83234 \times 10^3 \frac{q B \mu}{kh}$	$m = 0.183234 \frac{q B \mu}{kh}$	$m = 0.183234 \frac{q B \mu}{kh}$	$m = 0.183234 \frac{q B \mu}{kh}$	$m = 0.183234 \frac{q B \mu}{kh}$	$m = 0.183234 \frac{q B \mu}{kh}$
<u>GENERALIZED SKIN-FACTOR EQUATION</u>							
$S = 1.15129 \left[\frac{\rho_{hr} - \rho(\Delta t = 0)}{m} \right]$	$S = 1.15129 \left[\frac{\rho_{hr} - \rho(\Delta t = 0)}{m} \right]$	$S = 1.15129 \left[\frac{\rho_{hr} - \rho(\Delta t = 0)}{m} \right]$	$S = 1.15129 \left[\frac{\rho_{hr} - \rho(\Delta t = 0)}{m} \right]$	$\text{skin} = 1.15129 \left[\frac{s_{thr} - s(\Delta t = 0)}{M} \right]$	$\text{skin} = 1.15129 \left[\frac{s_{thr} - s(\Delta t = 0)}{M} \right]$	$\text{skin} = 1.15129 \left[\frac{s_{thr} - s(\Delta t = 0)}{M} \right]$	$\text{skin} = 1.15129 \left[\frac{s_{thr} - s(\Delta t = 0)}{M} \right]$
$- \log \left(\frac{k}{\phi \mu c_r f_w^2} \right)$	$- \log \left(\frac{k}{\phi \mu c_r f_w^2} \right)$	$- \log \left(\frac{k}{\phi \mu c_r f_w^2} \right)$	$- \log \left(\frac{k}{\phi \mu c_r f_w^2} \right)$	$- \log \left(\frac{k}{\phi \mu c_r f_w^2} \right)$	$- \log \left(\frac{k}{\phi \mu c_r f_w^2} \right)$	$- \log \left(\frac{k}{\phi \mu c_r f_w^2} \right)$	$- \log \left(\frac{k}{\phi \mu c_r f_w^2} \right)$
$+ 3.227546$	$+ 3.227546$	$+ 3.227546$	$+ 3.227546$	$+ 0.351378$	$+ 0.351378$	$+ 0.351378$	$+ 0.351378$

*The CGS system is considered to be obsolete and is replaced by SI; CGS units are included only for comparison with published material. SI is a coherent system; so equations do not contain units conversion factors.

**See Table A.8.

Appendix B

Application of Superposition To Generate Dimensionless Pressures

B.1 Introduction

As indicated in Section 2.9 and by several authors,¹⁻⁹ the principle of superposition may be used to develop dimensionless pressure data for many finite and bounded systems. This appendix shows how to use superposition to form no-flow and constant-pressure boundaries, and closed systems. A method of “desuperposition” for changing existing dimensionless-pressure solutions to solutions for different systems is explained. Finally, a general equation for calculating pressures owing to variable production rates is derived.

B.2 Dimensionless Pressure Used

When using the principle of superposition, we must choose a dimensionless pressure, p_D , that applies for the system. Normally, superposition calculations are performed using dimensionless pressures for infinite-acting systems (even when the goal is to generate a closed system^{4,6}), so the exponential-integral p_D , Eq. 2.5, is used. The exponential-integral p_D may be used when $r_D \geq 20$ and $t_D/r_D^2 \geq 0.5$ or when $t_D/r_D^2 \geq 25$. If neither of those restrictions is met, then Fig. C.1 must be used for the applicable r_D .

If superposition calculations are performed for a system that is not infinite-acting, the appropriate p_D from Appendix C must be used. In such a situation, there is no conceptual modification to the application of the superposition principle. However, p_D tables and figures for such systems often do not provide data for points other than at the well. Some useful p_D data are given in Appendix C and in Ref. 10.

B.3 Generating No-Flow and Constant-Pressure Boundaries

Fig. B.1 illustrates the method of images³⁻⁶ when used to create a no-flow boundary in an infinite system. Well 1 operates at constant flow rate q , at distance L from a single impermeable boundary, represented by the y axis in Fig. B.1. The image well, Well 2 in Fig. B.1, at a distance $-L$ from the y axis mathematically generates the boundary. By applying superposition we can calculate the pressure at any point in the x - y plane of Fig. B.1:

$$p(t, x, y) = p_i - \frac{141.2 q B \mu}{k h} [p_D(t_D, a_{D1}) + p_D(t_D, a_{D2})]. \quad (\text{B.1})$$

In Eq. B.1, the dimensionless distances a_{D1} and a_{D2} are calculated from

$$a_{D1} = \frac{a_1}{r_w} = \frac{1}{r_w} \sqrt{(x - L)^2 + y^2}, \quad (\text{B.2a})$$

$$a_{D2} = \frac{a_2}{r_w} = \frac{1}{r_w} \sqrt{(x + L)^2 + y^2}, \quad (\text{B.2b})$$

where r_w is the same for both wells. For an infinite system with $a_D > 20$, Eq. 2.5a applies.

$$p_D(t_D, a_D) = -\frac{1}{2} \operatorname{Ei}\left(-\frac{a_D^2}{4t_D}\right), \quad (\text{B.3a})$$

$$= -\frac{1}{2} \operatorname{Ei}\left(-\frac{(x \pm L)^2 + y^2}{4r_w^2 t_D}\right), \quad (\text{B.3b})$$

$$= \frac{1}{2} \int^{\infty} \frac{e^{-u}}{u} du. \quad (\text{B.3c})$$

$$\left[\frac{(x \pm L)^2 + y^2}{4r_w^2 t_D} \right]$$

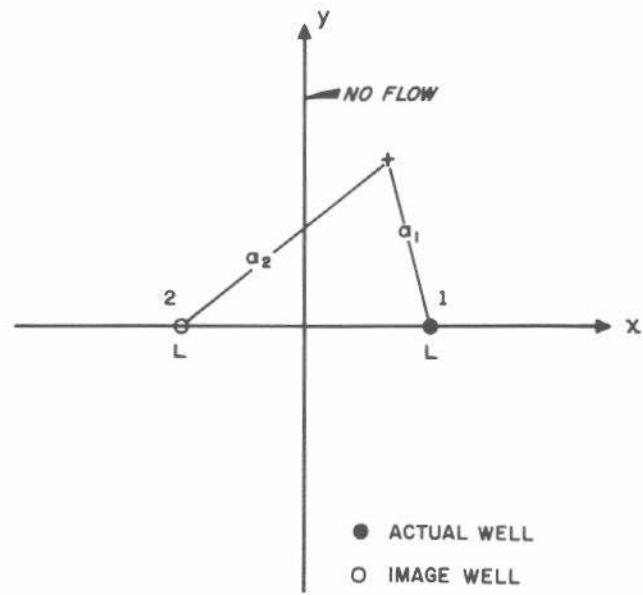


Fig. B.1 Image-well location for a no-flow boundary (sealing fault).

We wish to verify that no fluid flows across the impermeable barrier, the y axis. That is true if $(\partial p / \partial x)_{x=0} = 0$ at all points along the y axis. Differentiating Eq. B.1,

$$\frac{\partial p(t, x, y)}{\partial x} = \frac{-141.2 qB\mu}{kh} \left[\frac{\partial p_D(t_D, a_{D1})}{\partial x} + \frac{\partial p_D(t_D, a_{D2})}{\partial x} \right]. \quad (\text{B.4})$$

We wish to evaluate Eq. B.4 at $x = 0$ for arbitrary values of y . To do this we differentiate Eq. B.3c using Leibnitz's rule:¹¹

$$\frac{\partial p_D(t_D, a_D)}{\partial x} = -\frac{(x \pm L)}{(x \pm L)^2 + y^2} \exp\left(-\frac{(x \pm L)^2 + y^2}{4r_w^2 t_D}\right). \quad (\text{B.5})$$

Eq. B.5 is substituted into Eq. B.4:

$$\begin{aligned} \frac{\partial p(t, x, y)}{\partial x} &= \frac{141.2 qB\mu}{kh} \left\{ \left[\frac{x-L}{(x-L)^2 + y^2} \right. \right. \\ &\quad \left. \left. \exp\left(-\frac{(x-L)^2 + y^2}{4r_w^2 t_D}\right) \right. \right. \\ &\quad \left. \left. + \left[\frac{(x+L)}{(x+L)^2 + y^2} \right] \exp\left(-\frac{(x+L)^2 + y^2}{4r_w^2 t_D}\right) \right\}, \right. \\ &\quad \dots \quad (\text{B.6}) \end{aligned}$$

which, when evaluated at $x = 0$, becomes

$$\begin{aligned} \frac{\partial p(t, 0, y)}{\partial x} &= \frac{141.2 qB\mu}{kh} \left(\frac{1}{L^2 + y^2} \right) \left\{ -L + L \right\} \\ &\quad \exp\left(-\frac{L^2 + y^2}{4r_w^2 t_D}\right) \\ &= 0. \quad (\text{B.7}) \end{aligned}$$

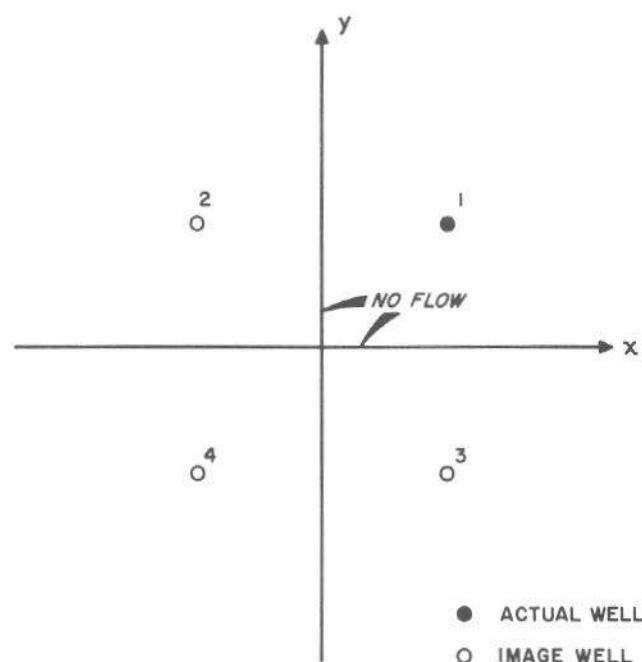


Fig. B.2 Image-well locations for two intersecting no-flow boundaries.

Since $(\partial p / \partial x)_{x=0} = 0$, we can see that an image of the operating well in the boundary creates a no-flow boundary. This is always true for straight-line boundaries no matter how many boundaries or how many wells there are. Some specific examples are given in Section B.4.

If the y axis in Fig. B.1 is to be a constant-pressure boundary, then the image well is an injection well with the same rate as the production well. In this case, superposition gives

$$p(t, x, y) = p_i - \frac{141.2 qB\mu}{kh} [p_D(t_D, a_{D1}) - p_D(t_D, a_{D2})]. \quad (\text{B.8})$$

Since

$$a_{D1} = a_{D2}, \quad (\text{B.9})$$

at $x = 0$ for all y , then

$$p_D(t_D, a_{D1}) = p_D(t_D, a_{D2}), \quad (\text{B.10})$$

and the pressure at all points along the boundary (y axis) is

$$p(t, 0, y) = p_i. \quad (\text{B.11})$$

By using the method of images to form either no-flow or constant-pressure boundaries, one can generate dimensionless pressure solutions for many important situations.

B.4 Use of Method of Images To Generate Multiple Boundary and Closed Systems

Clearly, the method of images extends to systems with more than one no-flow barrier. Fig. B.2 shows a system with two no-flow barriers intersecting at right angles. Well 1 is located near the intersection of the two barriers. Well 2, the image of Well 1 in the y axis, prevents flow across that axis resulting from Well 1. Well 3, the image of Well 1 in the x axis, prevents flow across that axis owing to Well 1. Well 4 prevents flow across the x axis resulting from Well 2 and flow across the y axis resulting from Well 3. The method of images considers barriers to be of infinite length, so flow across any barrier caused by an *image* well must be prevented by other image wells, as demonstrated in Fig. B.2.

When barriers are formed by using images, pressure may be calculated at any point by superposition. Rather than write the equation for pressure change, we may write an equation for the dimensionless pressure at any point in the two-barrier system:

$$\begin{aligned} p_D(t_D, x_D, y_D) &= p_D(t_D, a_{D1}) + p_D(t_D, a_{D2}) \\ &\quad + p_D(t_D, a_{D3}) + p_D(t_D, a_{D4}), \end{aligned} \quad (\text{B.12})$$

where a_{D1} means the dimensionless distance from the point where the pressure is being calculated to Well 1, and so on, and x_D and y_D are dimensionless Cartesian coordinates. Their precise definition varies with the application; Refs. 6 and 10 use two practical definitions of those quantities. It is easy to verify that the four wells in Fig. B.2 do create the two intersecting no-flow barriers indicated.

In general, if there are more wells or more boundaries, the dimensionless pressure may be written

$$p_D(t_D, x_D, y_D) = \sum_{i=1}^n p_D(t_D, a_{Di}), \dots \quad (\text{B.13})$$

where the number of wells, n , includes all actual wells and all image wells.

Fig. B.3 shows a single well located between two parallel no-flow barriers and the image wells required to produce those barriers. Well a is the image of the actual well in Boundary A. Well b is the image of the actual well in Boundary B. Each image well would cause flow across the other boundary so additional image wells are required. Well (a)b is the image of Well a in Boundary B and is required to prevent Well a from causing fluid to flow across Boundary B. Well (b)a is required to prevent flow across Boundary A resulting from Well b. Some of the other image wells in Fig. B.3 are marked with a similar nomenclature to indicate the reason for each well. The line of image wells goes to infinity

in both directions. The dimensionless pressure function for this system can be written

$$p_D(t_D, x_D, y_D) = \sum_{i=1}^{\infty} p_D(t_D, a_{Di}), \dots \quad (\text{B.14})$$

where a_{Di} is the dimensionless distance from Well i to x_D, y_D .

To obtain a closed system with a single well in it, add two horizontal no-flow boundaries to Fig. B.3 and image all the wells shown in Fig. B.3 in those two boundaries. Fig. B.4 shows the results — a single well in a closed rectangular drainage area. The image wells extend to infinity in all directions; the dimensionless pressure for the closed rectangular system is given by Eq. B.14. Although an infinite number of image wells is indicated, for calculation purposes it is usually only necessary to include several rows and columns of image wells before their contribution becomes negligible.

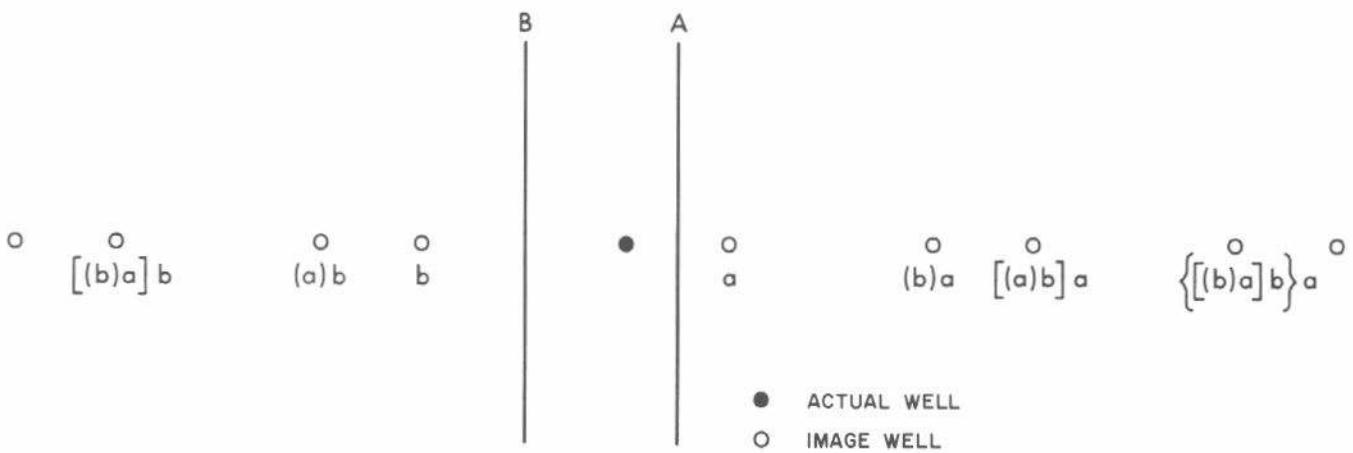


Fig. B.3 Image-well locations for two parallel no-flow boundaries.

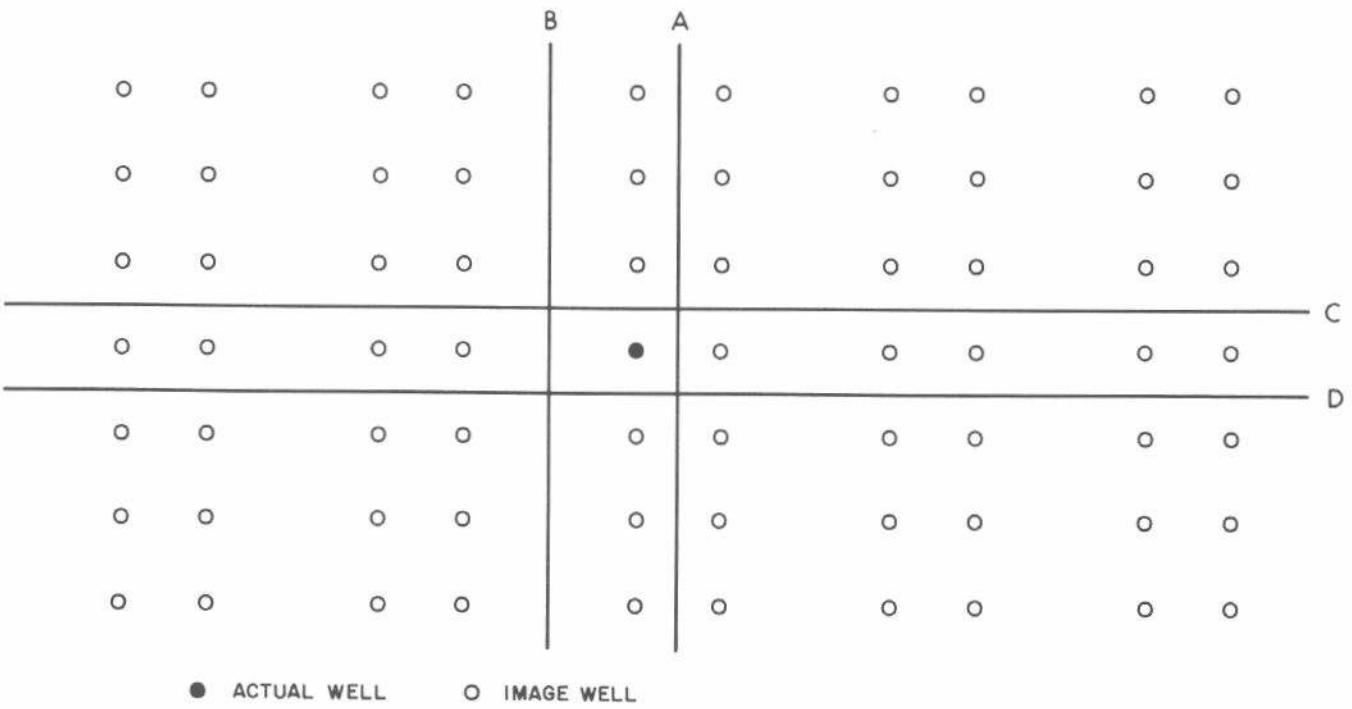


Fig. B.4 Image-well location for one well in a closed rectangle.

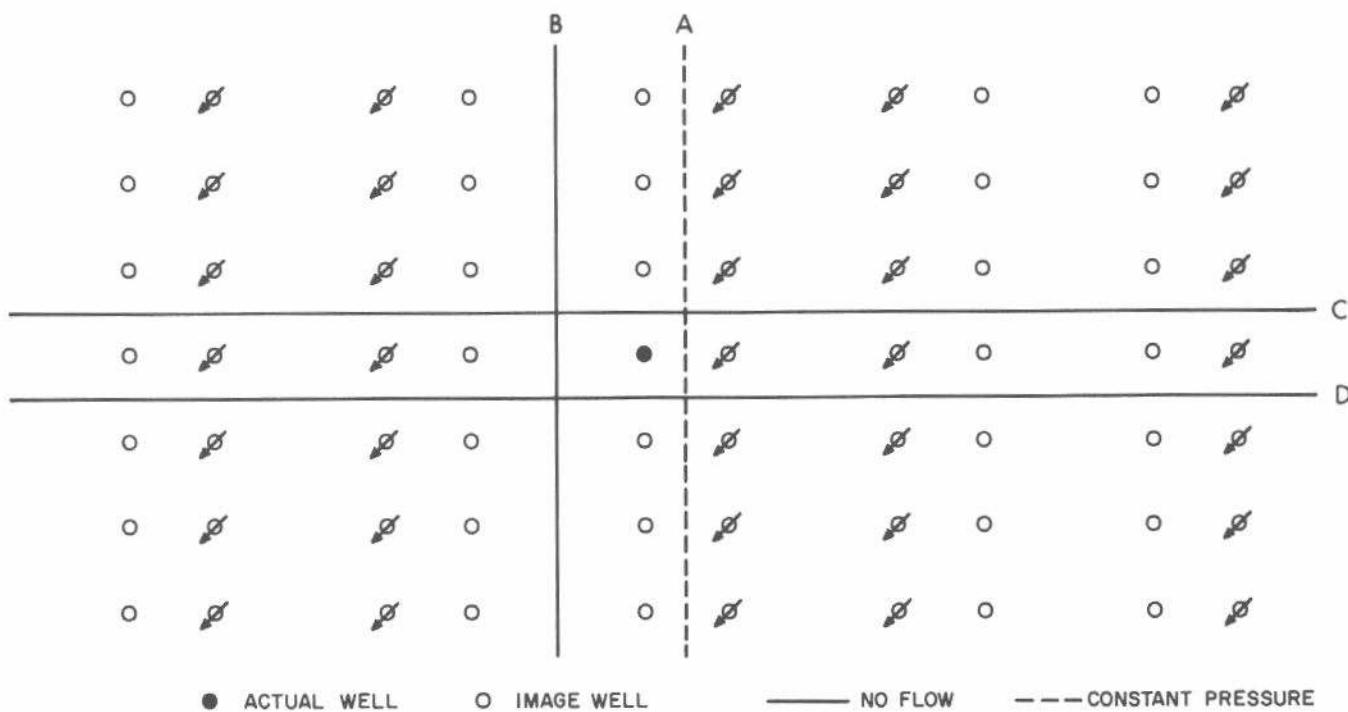


Fig. B.5 Image-well location for one well in a rectangle with three no-flow boundaries and one constant-pressure boundary.

To make one or more of the boundaries constant-pressure rather than no-flow, change some image production wells to injection wells.^{6,9} Fig. B.5 shows the image-well array for the system of Fig. B.4 but with Boundary A at constant-pressure. Whether there are one, two, three, or four constant-pressure boundaries in a rectangular system, exactly one-half the image wells will be injectors and one-half will be producers. The location of each type of well depends on which boundaries are constant-pressure and which are no-flow. Ramey, Kumar, and Gulati⁹ give several examples of systems with one or more constant-pressure boundaries and present a computer program for calculating p_D in such systems.

B.5 Superposition of Square Drainage Systems

Earlougher *et al.*⁶ showed that a useful unit of superposition is a closed-square system with a single well located at its center (Fig. B.6). The dimensionless pressures at the well and at other locations within the square may be added together to obtain dimensionless pressures for systems with different well locations and different shapes. Fig. B.7 illustrates how two square systems are added to obtain a 2:1 rectangular system with the well at the center.⁶ The open circles represent the well array for one square system; the closed circles show the well array for the second square system. The resulting well array creates a 2:1 rectangular drainage area with a well in the center as illustrated in Fig. B.7. As pointed out in Ref. 6, it is important to recognize that such superposition changes the area of the drainage system. That affects both the dimensionless time, t_{DA} , and the dimensionless pressure value *at the well* since p_D at a well in a closed drainage area depends on the value of \sqrt{A}/r_w . If p_D is desired at a well point for a system of different \sqrt{A}/r_w , it is necessary to make the correction

$$p_D(\sqrt{A}/r_w) = p_D([\sqrt{A}/r_w]_{\text{table}}) + \ln \left[\frac{(\sqrt{A}/r_w)}{(\sqrt{A}/r_w)_{\text{table}}} \right], \quad \dots \quad (\text{B.15})$$

where the p_D on the left-hand side of the equation is for the desired value of \sqrt{A}/r_w and on the right-hand side is for the value given in a table or figure (such as Table C.2 or Fig. C.12) with a different value of \sqrt{A}/r_w . Additional informa-

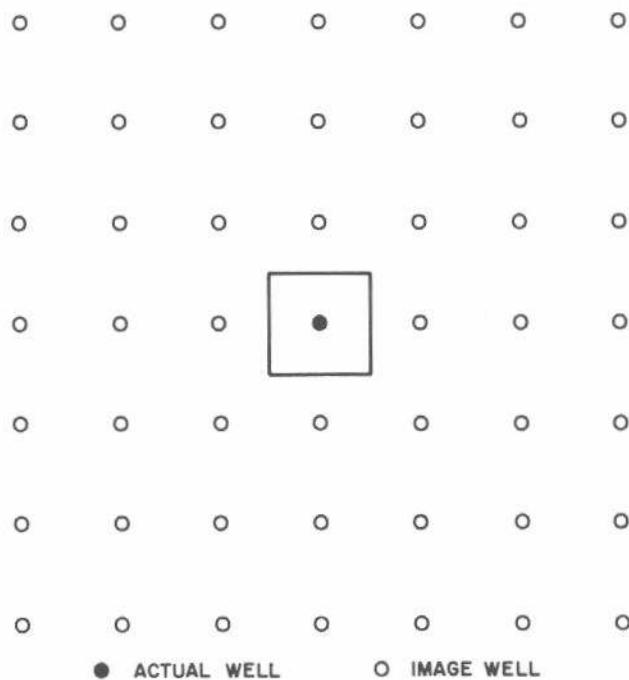


Fig. B.6 Image-well location for a single well in the center of a closed square.

tion and instructions for superposing square systems to obtain systems of other shapes are given in Ref. 6.

B.6 Desuperposition

Gringarten, Ramey, and Raghavan⁷ and Chen and Brigham⁸ have illustrated the concept of desuperposition for modifying known p_D values to p_D 's for somewhat different systems. The approach may be used for any drainage shape and well location, although we illustrate it here only for a well in the center of a closed square. Suppose that we desire to compute p_D at a well in the center of a closed square for $C_D > 0$ and $s \neq 0$. Most p_D data for a closed-square system are for $C_D = 0$, $s = 0$, so those data are not what we need. However, we may use them to get the desired results by using

$$\begin{aligned} p_D(C_D, s, \square) &= p_D(C_D = 0, s = 0, \square) \\ &\quad - p_D(C_D = 0, s = 0, \infty) + p_D(C_D, s, \infty), \\ &\quad \dots \end{aligned} \quad (\text{B.16})$$

as illustrated in Fig. B.8. We start with p_D for the closed-square system with zero skin and zero wellbore storage as indicated in Part a of Fig. B.8. From this dimensionless

pressure, the first term on the right-hand side of Eq. B.16, subtract p_D for a single well in an infinite system with zero skin and zero storage, the second term on the right-hand side of Eq. B.16. The result, shown in Part b of Fig. B.8, is an infinite array of wells with the well in the center of the square removed. Finally, add p_D for a single well in an infinite system with the desired wellbore storage coefficient and skin factor, the last term on the right-hand side of Eq. B.16, to obtain Part c of Fig. B.8. The theoretically correct dimensionless pressure is given by the right-hand side of Part d in Fig. B.8, where all the image wells have the desired skin factor and wellbore storage coefficient. However, since skin factors and wellbore storage coefficients have only a small influence⁷ at points away from the well, the approximation is a good one.⁸

Gringarten, Ramey, and Raghavan⁷ use this approach to estimate dimensionless pressure for closed fractured systems. Chen and Brigham⁸ use the approach to generate dimensionless pressures and then pressure buildup curves for a single well with wellbore storage and skin in the center of a closed square. Dimensionless pressure for many other systems can be computed with the same approach.

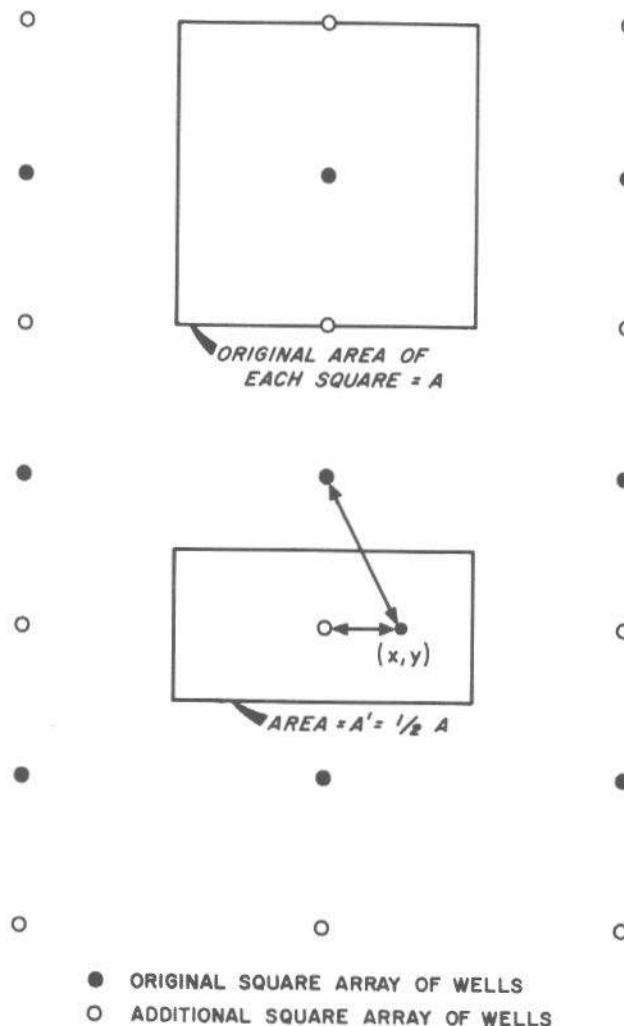


Fig. B.7 Superposition of two square arrays to form a 2:1 rectangle. After Earlougher *et al.*⁶

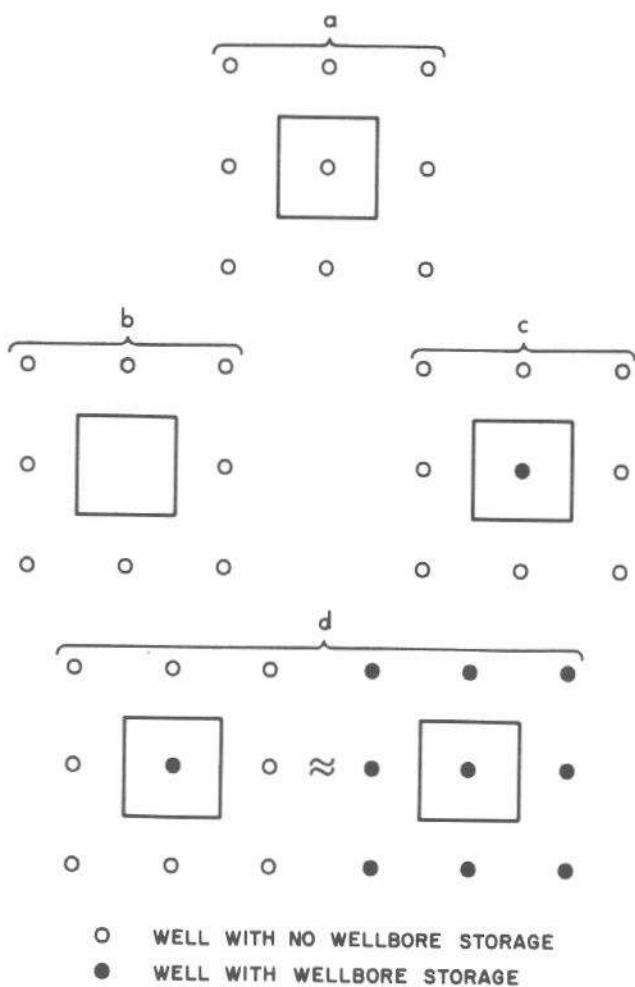


Fig. B.8 Desuperposition to approximate a single well with wellbore storage and skin in the center of a closed square. After Chen and Brigham.⁸

B.7 Superposition for Variable Rate

Fig. B.9 schematically shows a variable-rate history. In the nomenclature of that figure, production (or injection) starts at time 0; the rate remains constant at q_1 until time t_1 , then changes to q_2 until time t_2 , etc. Note that rate q_j ends at time t_j and that $t_0 = 0$. The last (and current rate) is always q_N . We may calculate the pressure at the well (or at any other point for which we know p_D) at any time during rate q_N by using the principle of superposition as indicated by Eq. 2.31. The pressure at the well is

$$\begin{aligned} p_{wf}(t) = p_i - \frac{141.2 B \mu}{kh} & \left\{ q_1 [p_D(t_D) + s] \right. \\ & + (q_2 - q_1)[p_D([t-t_1]_D) + s] \\ & + (q_3 - q_2)[p_D([t-t_2]_D) + s] + \dots \\ & \left. + (q_N - q_{N-1})[p_D([t-t_{N-1}]_D) + s] \right\}. \quad \dots \text{ (B.17)} \end{aligned}$$

This may be rearranged to

$$\begin{aligned} p_{wf}(t) = p_i - \frac{141.2 B \mu}{kh} & \left\{ q_1 [p_D(t_D) - p_D([t-t_1]_D)] \right. \\ & + q_2 [p_D([t-t_1]_D) - p_D([t-t_2]_D)] + \dots \\ & + q_{N-1} [p_D([t-t_{N-2}]_D) - p_D([t-t_{N-1}]_D)] \\ & \left. + q_N [p_D([t-t_{N-1}]_D) + s] \right\}. \quad \dots \text{ (B.18)} \end{aligned}$$

If the system is infinite-acting and if the logarithmic approximation of the exponential integral, Eq. 2.5b, applies, Eq. B.18 may be written

$$\begin{aligned} p_{wf}(t) = p_i - \frac{70.60 B \mu}{kh} & \left\{ q_1 \ln \left(\frac{t}{t-t_1} \right) \right. \\ & + q_2 \ln \left(\frac{t-t_1}{t-t_2} \right) + \dots \\ & + q_{N-1} \ln \left(\frac{t-t_{N-2}}{t-t_{N-1}} \right) + q_N \left[\ln(t-t_{N-1}) \right. \\ & \left. + \ln \left(\frac{k}{\phi \mu c_t r_w^2} \right) - 7.4316 + 2s \right], \quad \dots \text{ (B.19)} \end{aligned}$$

or

$$\begin{aligned} p_{wf}(t) = p_i - \frac{162.6 B \mu}{kh} & \left\{ \sum_{j=1}^{N-1} q_j \log \left(\frac{t-t_{j-1}}{t-t_j} \right) \right. \\ & + q_N \left[\log(t-t_{N-1}) + \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) \right. \\ & \left. \left. - 3.2275 + 0.86859s \right] \right\}. \quad \dots \text{ (B.20)} \end{aligned}$$

Eqs. B.17 through B.20 are convenient for estimating pressures resulting from multiple-rate histories. However, the form

$$\begin{aligned} \frac{p_i - p_{wf}(t)}{q_N} = \frac{162.6 B \mu}{kh} & \left\{ \sum_{j=1}^N \left[\left(\frac{q_j - q_{j-1}}{q_N} \right) \right. \right. \\ & \times \log(t-t_{j-1}) \left. \right] + \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) \\ & \left. - 3.2275 + 0.86859s \right\}, \quad \dots \text{ (B.21)} \end{aligned}$$

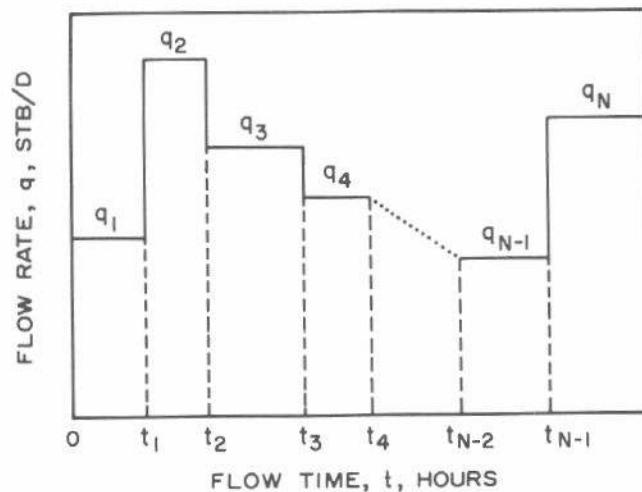


Fig. B.9 Schematic representation of a variable production-rate schedule.

which results from combining Eqs. B.17 and 2.5b, is more convenient for analyzing multiple-rate test data, as indicated in Section 4.2.

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Appendix C

Dimensionless Pressure Solutions

C.1 Introduction

This appendix presents correlations of dimensionless pressure with dimensionless time for single-well systems producing at constant rate. Some data from the literature have been modified to be consistent with the nomenclature used in this monograph. We retain the definition of dimensionless pressure corresponding to

$$p_i - p = \Delta p = \frac{141.2 q B \mu}{kh} p_D(t_D, \dots) . \quad \dots \quad (C.1)$$

Dimensionless time is usually based on wellbore radius,

$$t_D = \frac{0.0002637 kt}{\phi \mu c_r r_w^2} , \quad \dots \quad (C.2a)$$

or based on drainage area,

$$t_{DA} = \frac{0.0002637 kt}{\phi \mu c_r A} = t_D \left(\frac{r_w^2}{A} \right) . \quad \dots \quad (C.2b)$$

We clearly indicate when it is convenient to base the dimensionless time on some other characteristic dimension.

C.2 Infinite Systems

No Wellbore Storage, No Skin

After wellbore storage effects are no longer important, the dimensionless pressure for infinite and infinite-acting systems is given by¹⁻⁴

$$p_D = -\frac{1}{2} \operatorname{Ei}\left(-\frac{r_D^2}{4t_D}\right) , \quad \dots \quad (C.3)$$

when $t_D \geq 20$ and $t_D/r_D^2 \geq 0.5$ or when $t_D/r_D^2 \geq 25$. Dimensionless pressure values for smaller t_D and r_D are given in Fig. C.1 for a range of r_D and t_D ; tabulated values for $r_D = 1$ are given by van Everdingen and Hurst.⁵ The lowermost curve in Fig. C.1 ($r_D > 20$), which is the exponential-integral solution (Eq. C.3), is shown on an expanded scale in Fig. C.2.*

A simplification of Eq. C.3 and Figs. C.1 and C.2 applies when $t_D/r_D^2 > 100$ (or with less than 1-percent error when $t_D/r_D^2 > 10$):

$$p_D = \frac{1}{2} [\ln(t_D/r_D^2) + 0.80907] . \quad \dots \quad (C.4)$$

*See footnote on Page 24.

These dimensionless pressure solutions apply for a single undamaged well in an infinite-acting system with no wellbore storage. Damage or improvement may be included as indicated in Eq. 2.2.

Single Vertical Fracture, No Wellbore Storage

Figs. C.3* and C.4 give dimensionless pressure data for a vertically fractured well in an infinite-acting system. A single fracture of half-length x_f intersects the well. Two situations are shown:

1. The *uniform-flux fracture* is a first approximation to the behavior of a vertically fractured well.^{6,7} Fluid enters the fracture at a uniform flow rate per unit area of fracture face so that there is a pressure drop in the fracture. The dimensionless pressure at the well for the uniform-flux fracture case is computed from^{6,7}

$$p_D = \sqrt{\pi t_{Dxf}} \operatorname{erf}\left(\frac{1}{2\sqrt{t_{Dxf}}}\right) - \frac{1}{2} \operatorname{Ei}\left(\frac{-1}{4t_{Dxf}}\right) , \quad \dots \quad (C.5)$$

where dimensionless time based on the half-fracture length is defined as

$$t_{Dxf} = t_D(r_w/x_f)^2 . \quad \dots \quad (C.6)$$

When $t_{Dxf} > 10$, Eq. C.5 becomes^{6,7}

$$p_D = \frac{1}{2} [\ln t_{Dxf} + 2.80907] , \quad \dots \quad (C.7)$$

with less than 1-percent error. For $t_{Dxf} < 0.1$, Eq. C.5 becomes^{6,7}

$$p_D = \sqrt{\pi t_{Dxf}} , \quad \dots \quad (C.8)$$

indicating that at short times flow into the fracture is linear.

2. The *infinite-conductivity fracture* has infinite permeability and, therefore, uniform pressure throughout. The dimensionless well pressure for that case is given in Figs. C.3 and C.4, and may be computed from^{6,7}

$$\begin{aligned} p_D = & \frac{1}{2} \sqrt{\pi t_{Dxf}} \left[\operatorname{erf}\left(\frac{0.134}{\sqrt{t_{Dxf}}}\right) + \operatorname{erf}\left(\frac{0.866}{\sqrt{t_{Dxf}}}\right) \right] \\ & - 0.067 \operatorname{Ei}\left(-\frac{0.018}{t_{Dxf}}\right) - 0.433 \operatorname{Ei}\left(-\frac{0.750}{t_{Dxf}}\right) . \end{aligned} \quad \dots \quad (C.9)$$

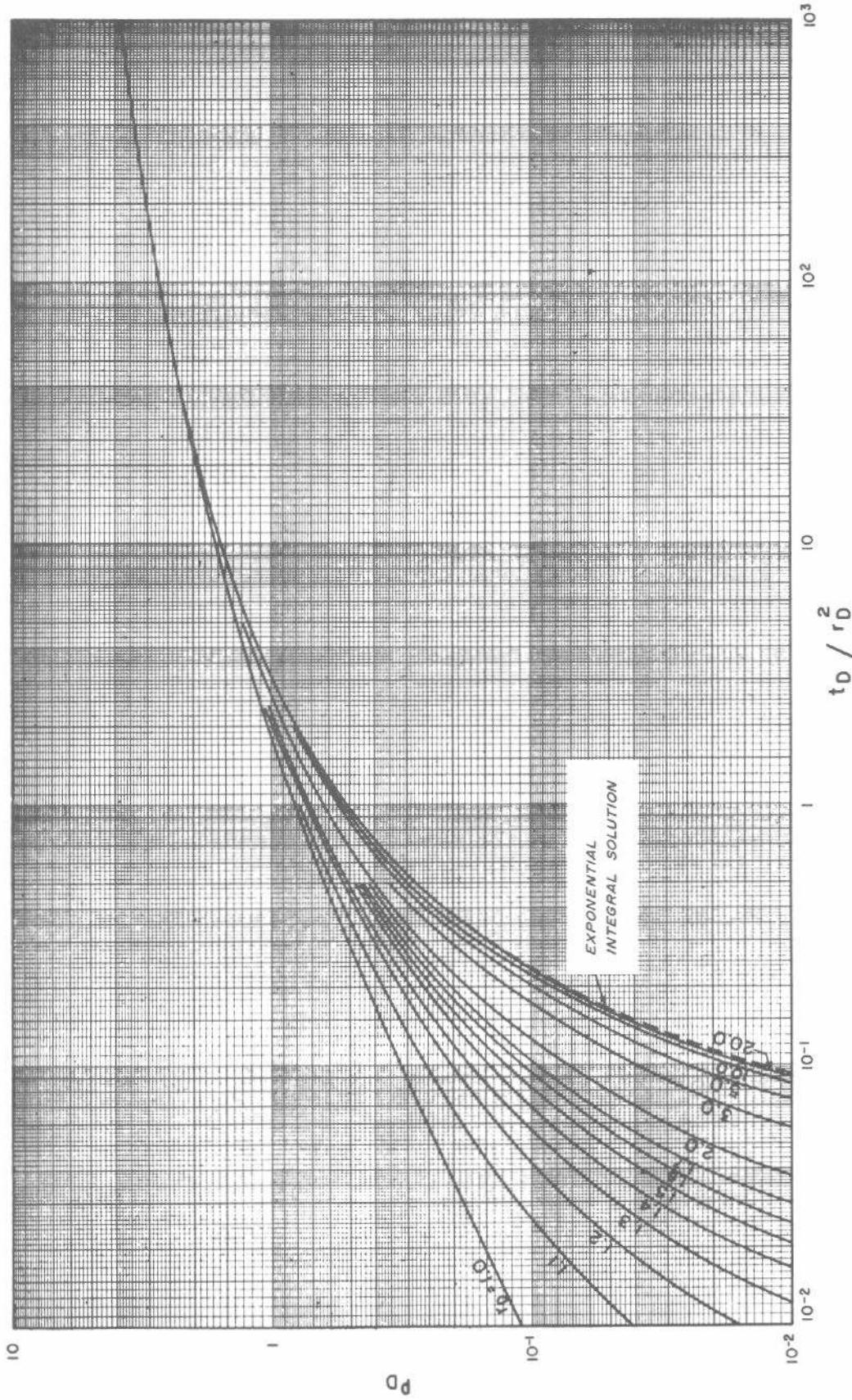


Fig. C.1 Dimensionless pressure for single well in an infinite system, small r_D , short time, no wellbore storage, no skin.
After Mueller and Witherspoon.⁴

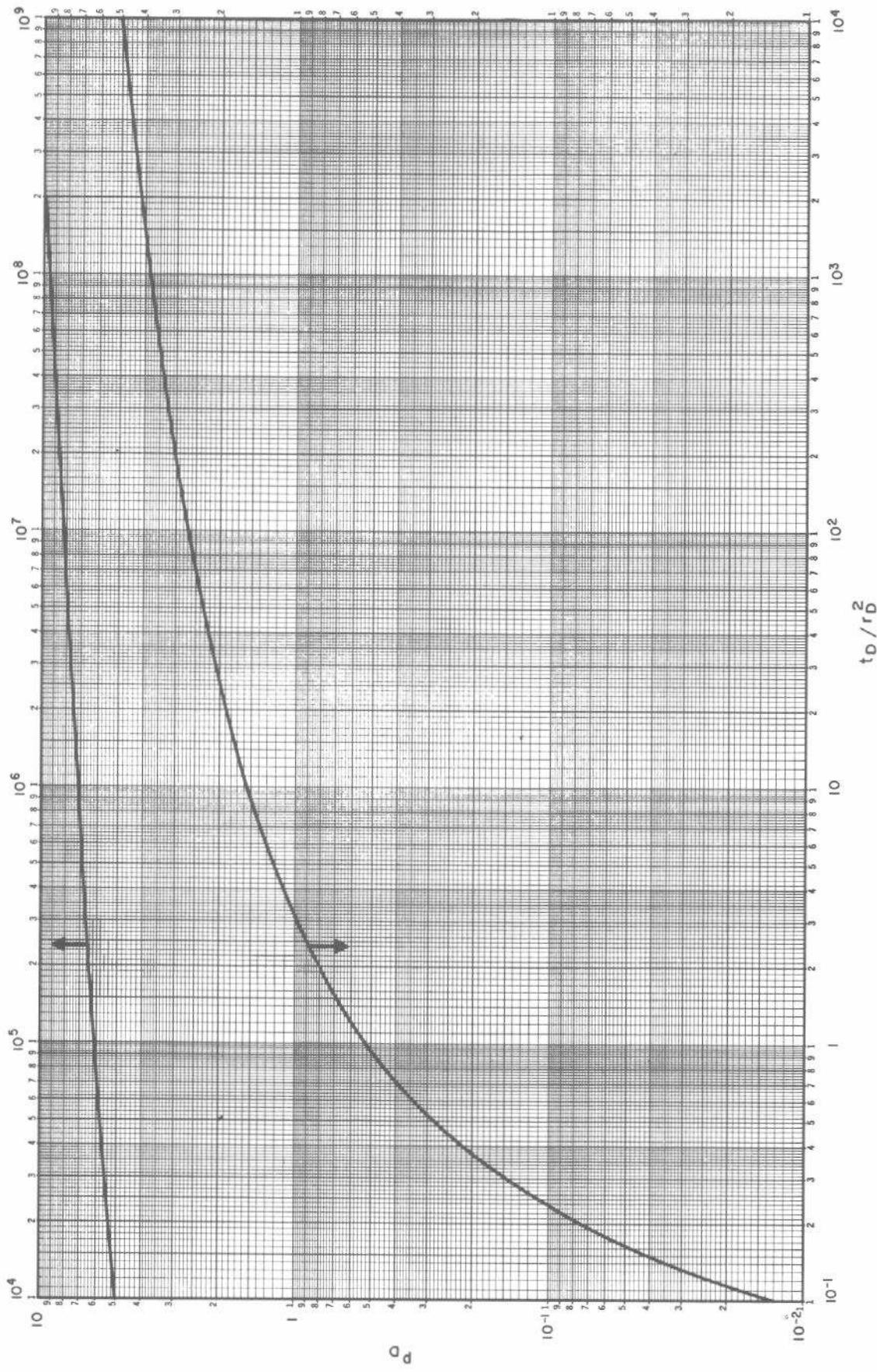


Fig. C.2 Dimensionless pressure for a single well in an infinite system, no wellbore storage, no skin. Exponential-integral solution.

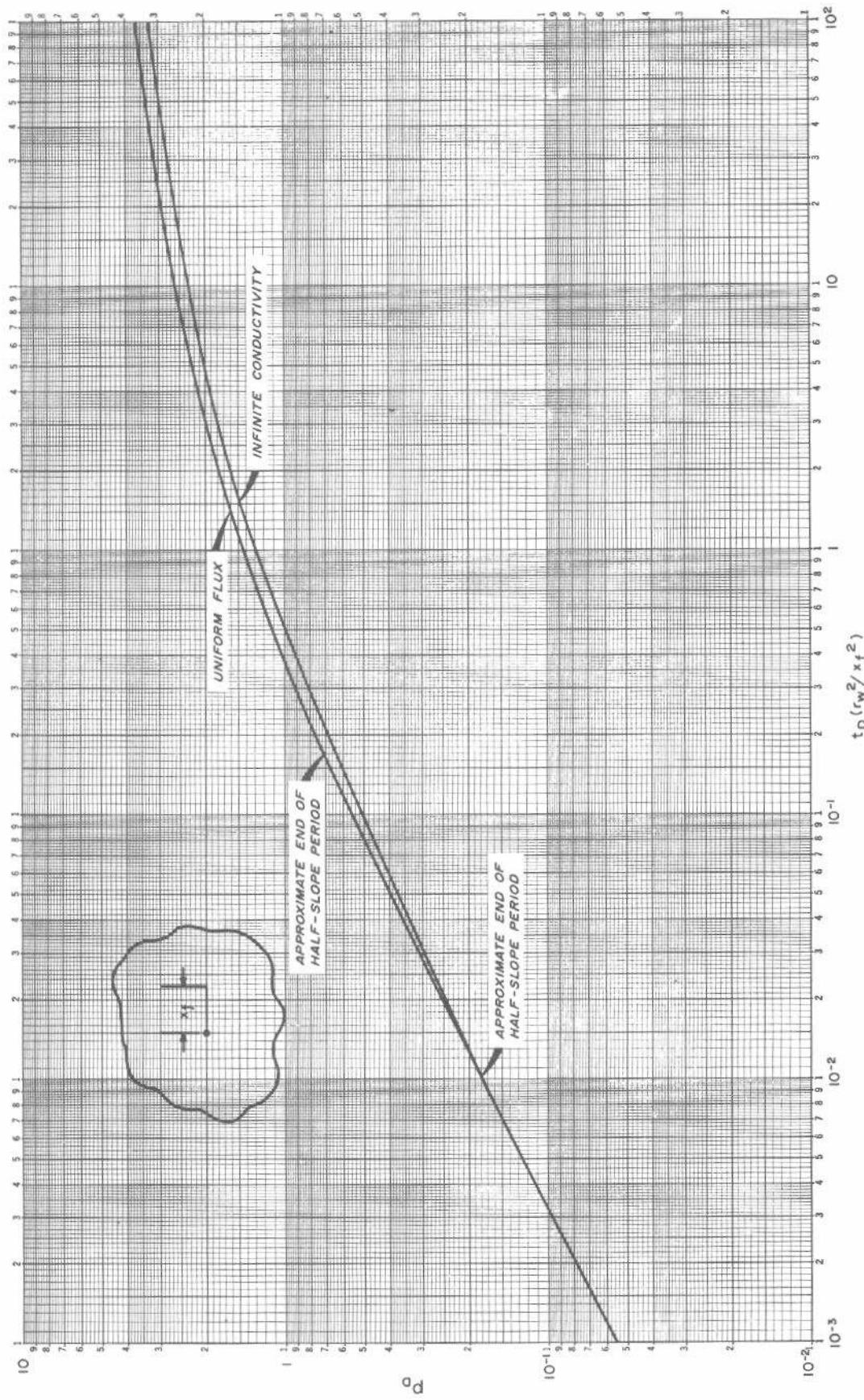


Fig. C.3 Dimensionless pressure for single, vertically fractured well in an infinite system, no wellbore storage. Log-log plot.
Data of Gringarten, Ramey, and Raghavan.^{6,7}

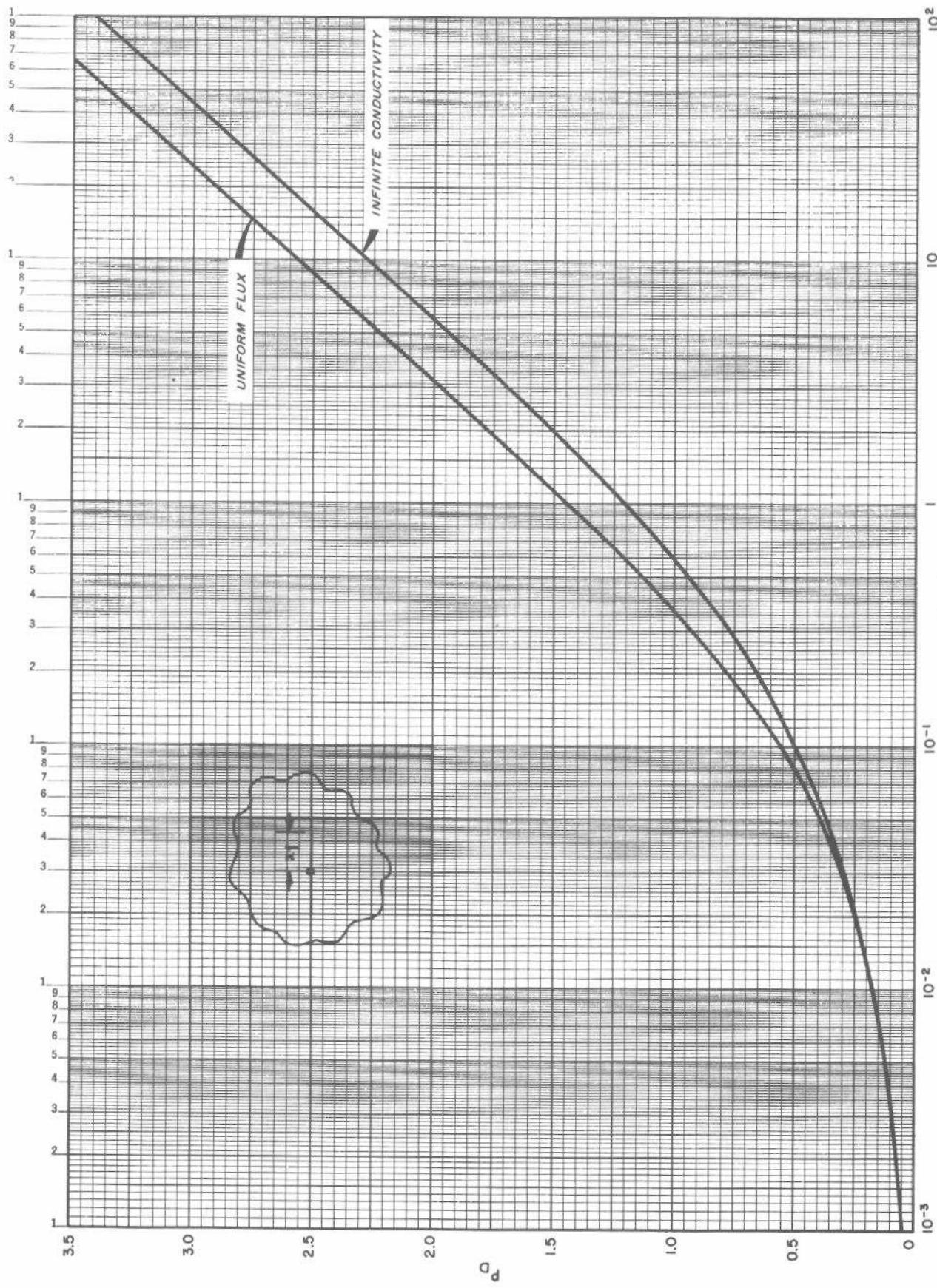


Fig. C.4 Dimensionless pressure for single, vertically fractured well in an infinite system, no wellbore storage. Semilog plot.
Data of Gringarten, Ramey, and Raghavan.^{6,7}

When $t_{Drf} > 10$, Eq. C.9 becomes^{6,7}

$$p_D = \frac{1}{2} [\ln t_{Drf} + 2.2000] , \quad \dots \dots \dots \quad (\text{C.10})$$

with less than 1-percent error. When t_{Drf} is less than 0.01, Eq. C.8 applies.

An important feature of Fig. C.3 is the early-time slope of one-half cycle in pressure per cycle in time. Such a half-slope straight line on the log-log plot is often diagnostic of a vertically fractured well.

Gringarten, Ramey, and Raghavan^{6,7} tabulate p_D values for the two types of vertical fractures.

Single Horizontal Fracture, No Wellbore Storage

Fig. C.5* shows p_D data at the well for an infinite system with a single horizontal fracture located at the formation midpoint. The dimensionless time used,

$$t_{Drf} = \frac{0.0002637 kt}{\phi \mu c_i r_f^2} = t_D \left(\frac{r_w^2}{r_f^2} \right) , \quad \dots \dots \quad (\text{C.11})$$

is based on the horizontal fracture radius, r_f . In Fig. C.5, dimensionless pressure is normalized by the parameter on the curves,

$$h_p = \frac{h}{r_f} \sqrt{\frac{k_r}{k_z}} , \quad \dots \dots \quad (\text{C.12})$$

where k_r and k_z are radial and vertical permeabilities, respectively. At short times and for large h_p , the curves in Fig. C.5 have a half-slope portion. It is apparent, however, that many horizontally fractured systems would not exhibit a half-slope straight line on the log-log plot. At low values of h_p , the curves in Fig. C.5 have a unit slope, like the unit slope caused by wellbore storage effects. There is *no wellbore storage effect* included in Fig. C.5, so the unit slope there is a result of the *fracture*, not the wellbore. Gringarten⁸ and Ramey⁹ tabulate p_D values for the horizontal-fracture case.

Wellbore Storage and Thin Skin Included

Fig. C.6* shows dimensionless pressure data for a single well in an infinite system with wellbore storage and skin effect included.¹⁰ The dimensionless wellbore storage coefficient is

$$C_D = \frac{5.6146 C}{2\pi\phi c_i h r_w^2} . \quad \dots \dots \quad (\text{C.13})$$

When $C_D > 0$, Fig. C.6 shows that the log-log plot has an early-time unit slope. At later times, the curves approach those for zero wellbore storage. Tabulated dimensionless pressure data are given by Agarwal, Al-Hussainy, and Ramey.¹⁰ Although t_D in Fig. C.6 is based on r_w , generation of the negative skin solutions involved use of an apparent larger wellbore radius as defined by Eq. 2.11.

Wellbore Storage and Finite Skin Included

Fig. C.7* gives dimensionless pressure data for a single

well in an infinite reservoir with wellbore storage and a finite skin effect.¹¹ Fig. 2.6 schematically illustrates the finite skin. The skin factor is calculated from

$$s = \left(\frac{k}{k_s} - 1 \right) \ln(r_{SD}) , \quad \dots \dots \quad (\text{C.14})$$

where

$$r_{SD} = r_s/r_w . \quad \dots \dots \quad (\text{C.15})$$

Wattenbarger and Ramey¹¹ provide tables of p_D vs t_D for the conditions of Fig. C.7.

Other Useful Type Curves

Fig. C.8* shows another relation between pressure and time for a single well with wellbore storage and skin effect in an infinite system.¹² The graph can be changed to a dimensionless pressure-dimensionless time basis by using equations given in Ref. 12. This type curve is particularly useful for curve matching and is not recommended for calculating pressure response. Its use is illustrated in Section 3.3.

Fig. C.9* is a type curve presented by McKinley¹³ for a single well with wellbore storage, but no skin factor, in an infinite system. Fig. C.9 assumes

$$\frac{k}{\phi \mu c_i r_w^2} = 9.728 \times 10^6 . \quad \dots \dots \quad (\text{C.16})$$

Although the figure is plotted on the basis of actual variables, it may be reduced to a dimensionless graph by using the definitions of C_D , t_D , and p_D with Eq. C.16. The main utility of Fig. C.9 is for type-curve matching of test data, not for calculating pressure response.

C.3 Closed Systems

All closed reservoir systems (that is, those with no-flow outer boundaries) have the transient behavior illustrated in Fig. 2.1. Within 1 percent,

$$p_D = \frac{1}{2} [\ln(t_{DA}) + \ln\left(\frac{A}{r_w^2}\right) + 0.80907] , \quad \dots \dots \quad (\text{C.17})$$

if $0.000025 < t_{DA}$ and t_{DA} is less than the value in the "Use Infinite System Solution With Less Than 1% Error for $t_{DA} <$ " column of Table C.1. At long times the system reaches pseudosteady state and¹⁴

$$p_D = 2\pi t_{DA} + \frac{1}{2} \ln\left(\frac{A}{r_w^2}\right) + \frac{1}{2} \ln\left(\frac{2.2458}{C_A}\right) . \quad \dots \dots \quad (\text{C.18})$$

Eq. C.18 applies when t_{DA} exceeds the value in the "Less Than 1% Error for $t_{DA} >$ " column of Table C.1. Values of C_A and of the last term on the right-hand side of Eq. C.18 are given in Table C.1 for many closed drainage shapes. Values of C_A are also given in Refs. 15 through 18 and in Table C.2.

Dimensionless pressure data *at the well* in closed reservoir systems are always given for a specific \sqrt{A}/r_w . If p_D is desired for a system of similar shape and geometry but with a different value of this parameter, it may be computed from¹⁷

$$(p_D)_{\text{desired}} = (p_D)_{\text{table}} + \ln[(\sqrt{A}/r_w)_{\text{desired}}/(\sqrt{A}/r_w)_{\text{table}}] , \quad \dots \dots \quad (\text{C.19})$$

*See footnote on Page 24.

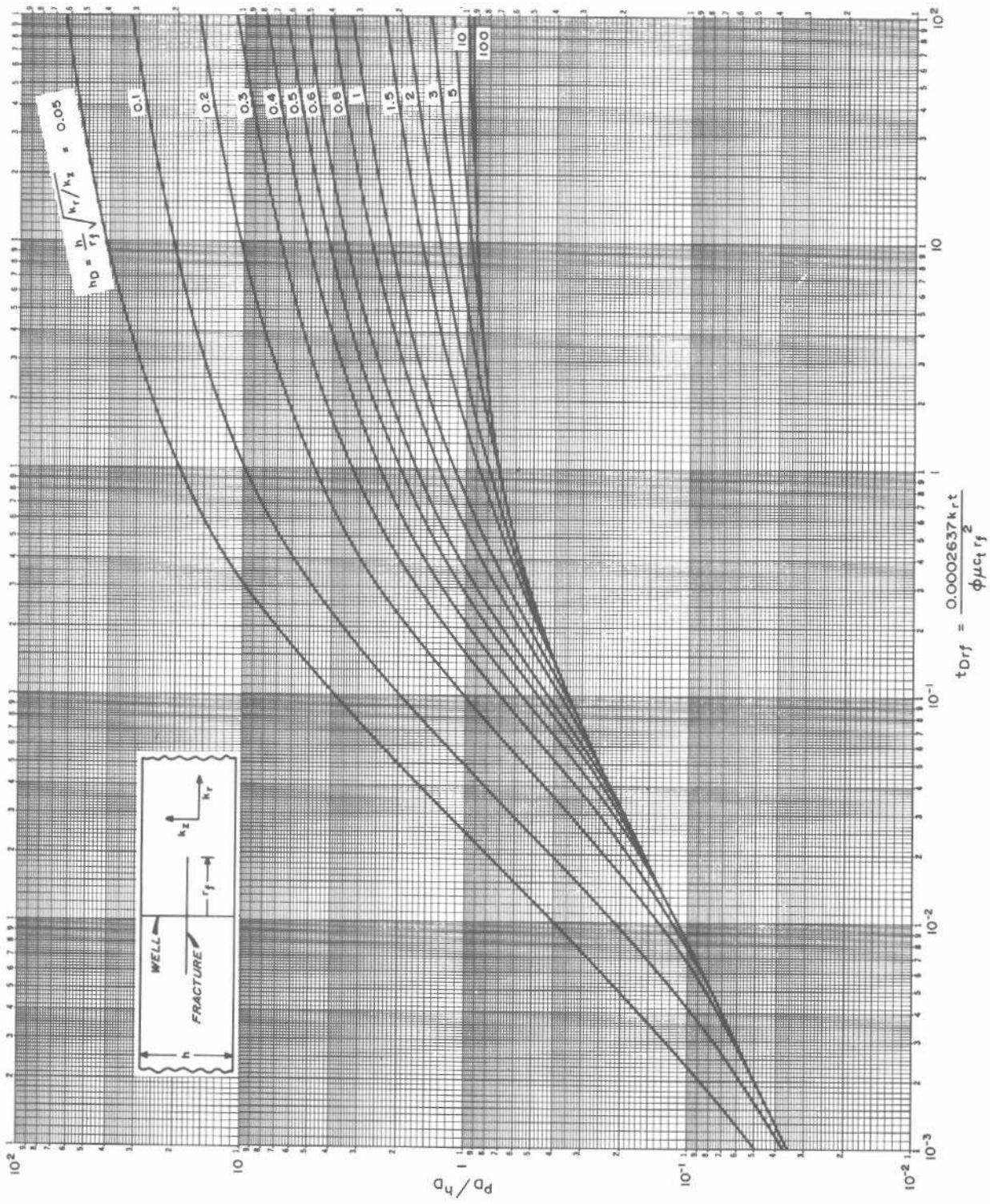


Fig. C.5 Dimensionless pressure for a single, horizontally fractured (uniform-flux) well in an infinite system, no wellbore storage.
Fracture located in the center of the interval. After Gringarten, Ramey, and Raghavan.⁶

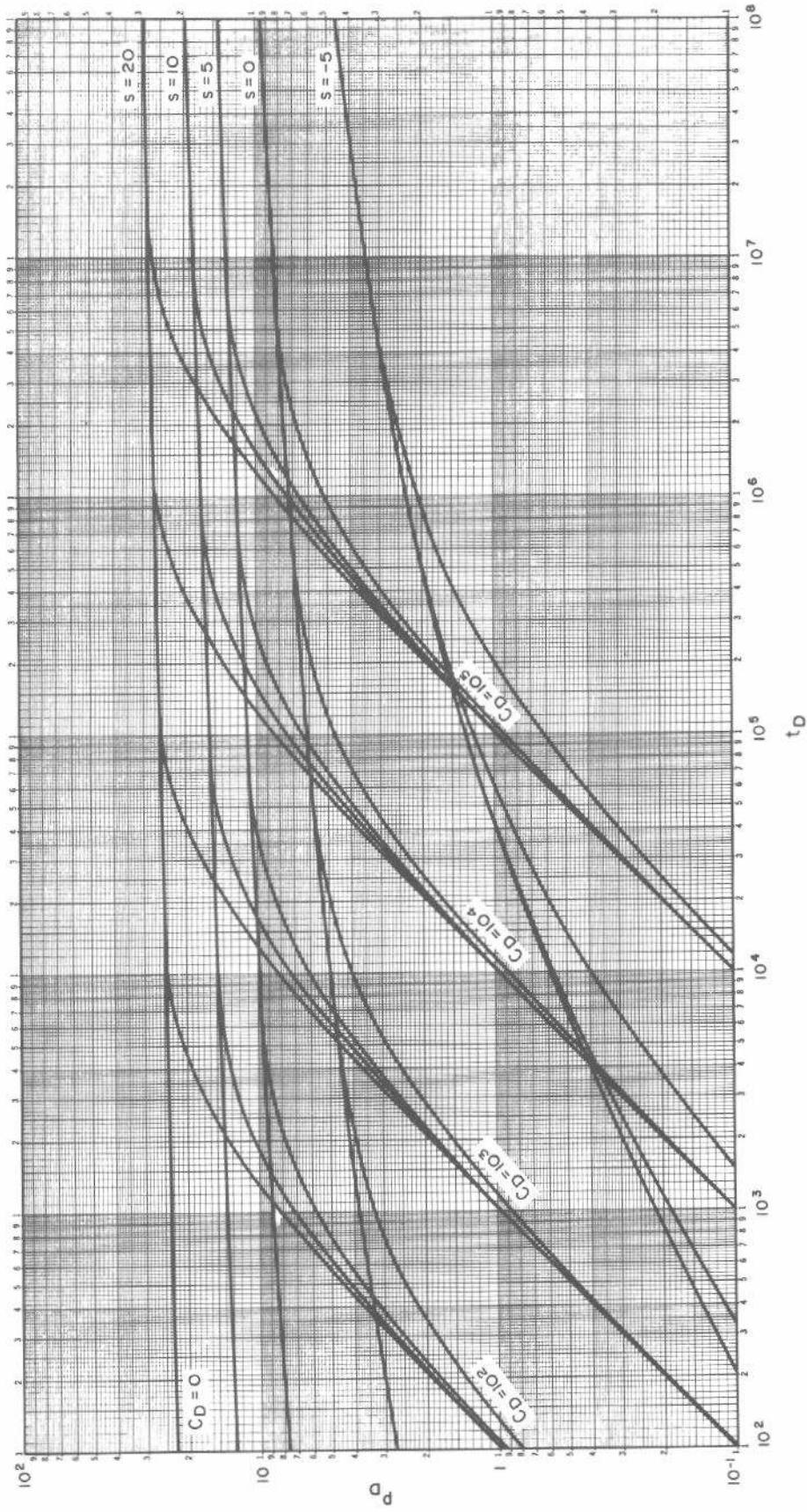


Fig. C.6 Dimensionless pressure for a single well in an infinite system, wellbore storage and skin included. After Agarwal, Al-Hussainy, and Ramey.¹⁰ Graph courtesy H. J. Ramey, Jr.

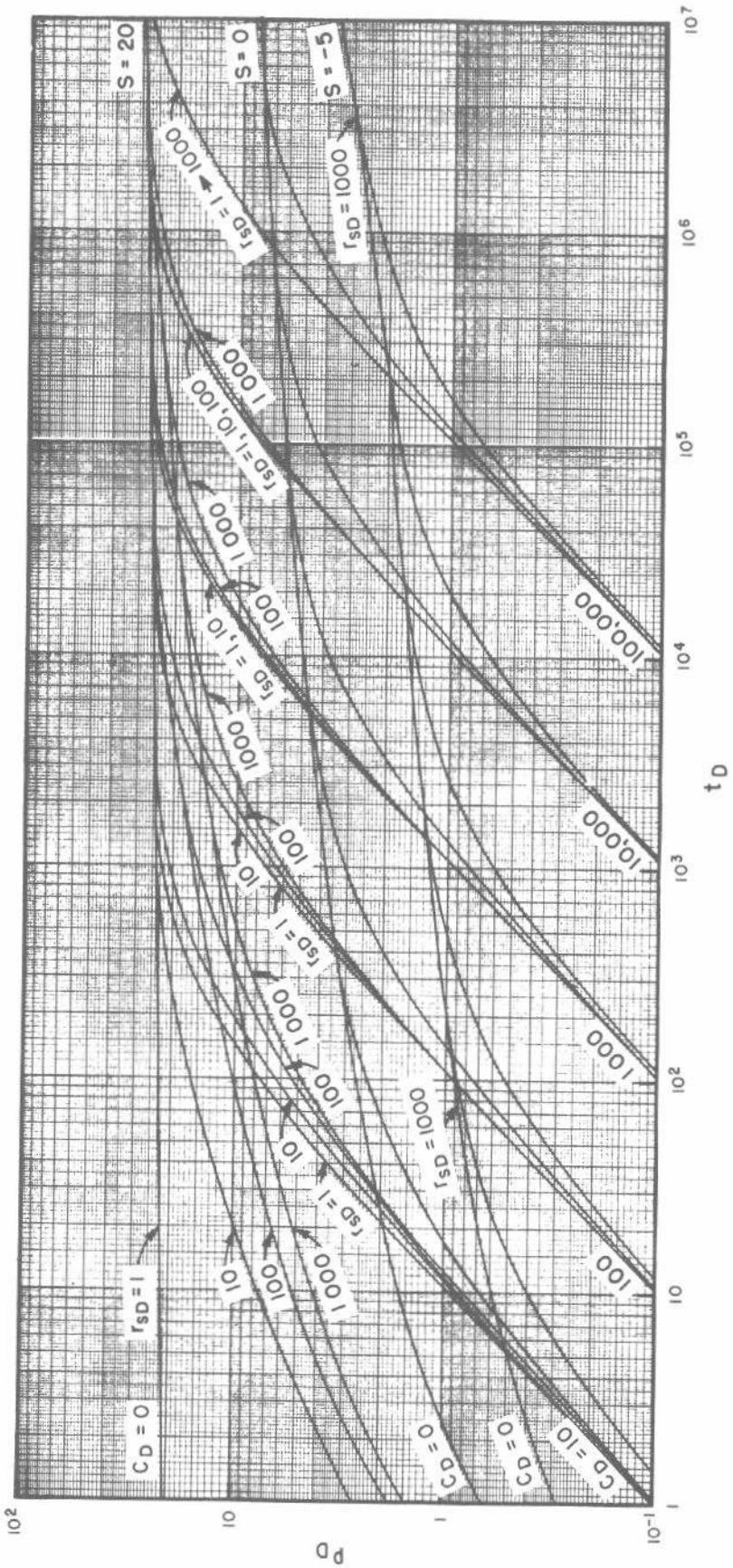


Fig. C.7 Dimensionless pressure for a single well in an infinite reservoir including wellbore storage and a finite skin (composite reservoir).
After Wattenbarger and Ramey.¹¹ Graph courtesy H. J. Ramey, Jr.

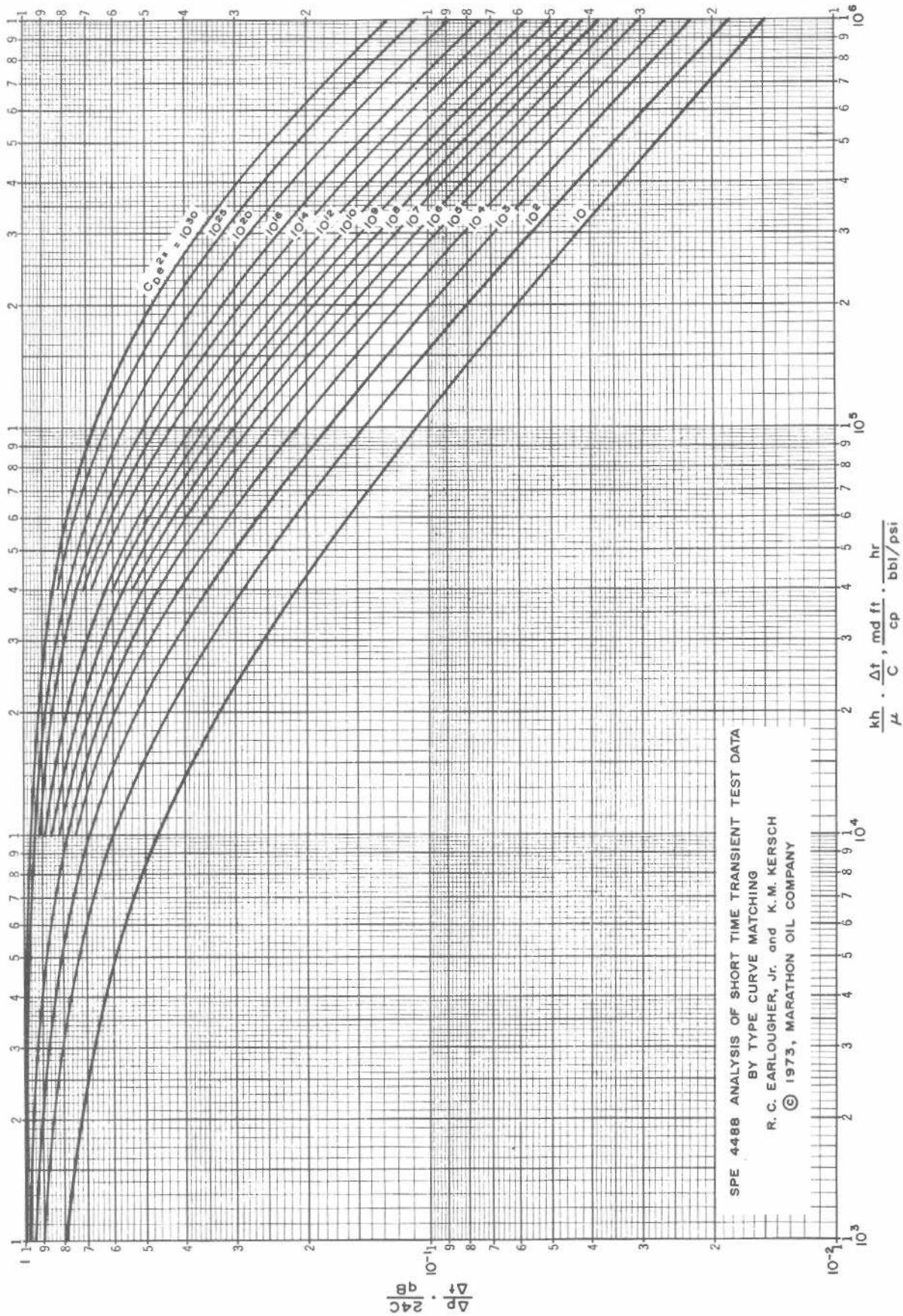


Fig. C.8 Type curve for a single well in an infinite system, wellbore storage and skin effects included. After Earlougher and Kersch.¹²
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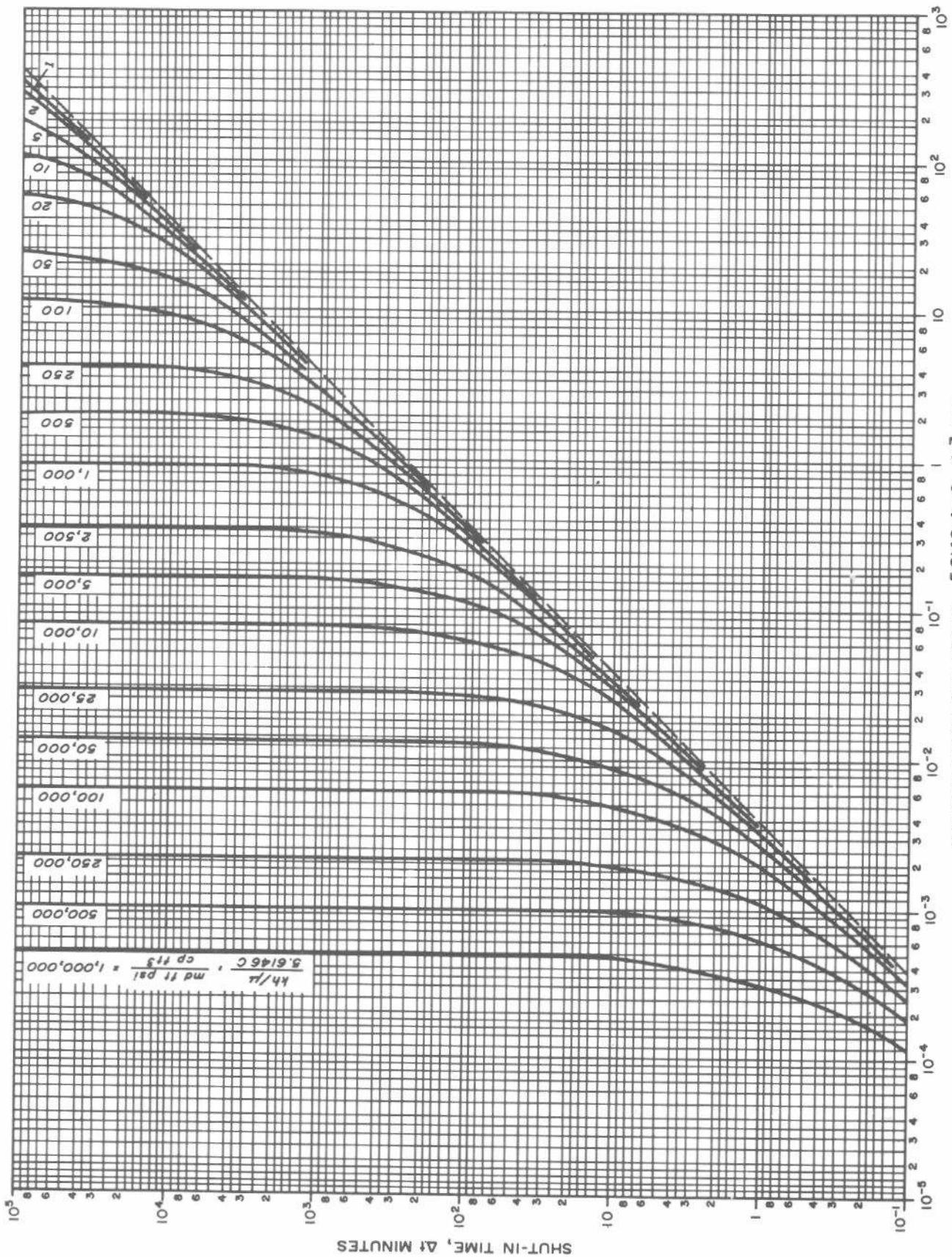
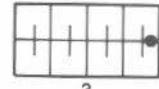
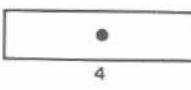
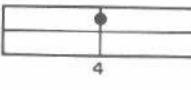
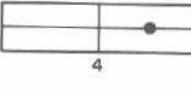
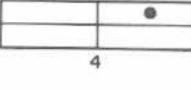
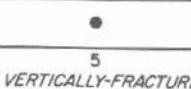


Fig. C.9 Type curve for a single well in an infinite system, wellbore storage included, no skin. After McKinley.¹³

TABLE C.1—SHAPE FACTORS FOR VARIOUS CLOSED SINGLE-WELL DRAINAGE AREAS.

IN BOUNDED RESERVOIRS	C_A	$\ln C_A$	$1/2 \ln \left(\frac{2.2458}{C_A} \right)$	EXACT FOR $t_{DA} >$	LESS THAN 1% ERROR FOR $t_{DA} >$	USE INFINITE SYSTEM SOLUTION WITH LESS THAN 1% ERROR FOR $t_{DA} <$
	31.62	3.4538	-1.3224	0.1	0.06	0.10
	31.6	3.4532	-1.3220	0.1	0.06	0.10
	27.6	3.3178	-1.2544	0.2	0.07	0.09
	27.1	3.2995	-1.2452	0.2	0.07	0.09
	21.9	3.0865	-1.1387	0.4	0.12	0.08
	0.098	-2.3227	+1.5659	0.9	0.60	0.015
	30.8828	3.4302	-1.3106	0.1	0.05	0.09
	12.9851	2.5638	-0.8774	0.7	0.25	0.03
	4.5132	1.5070	-0.3490	0.6	0.30	0.025
	3.3351	1.2045	-0.1977	0.7	0.25	0.01
	21.8369	3.0836	-1.1373	0.3	0.15	0.025
	10.8374	2.3830	-0.7870	0.4	0.15	0.025
	4.5141	1.5072	-0.3491	1.5	0.50	0.06
	2.0769	0.7309	+0.0391	1.7	0.50	0.02
	3.1573	1.1497	-0.1703	0.4	0.15	0.005

TABLE C.1—CONT'D.

	C_A	$\ln C_A$	$1/2 \ln \left(\frac{2.2458}{C_A} \right)$	EXACT FOR $t_{DA} >$	LESS THAN 1% ERROR FOR $t_{DA} >$	USE INFINITE SYSTEM SOLUTION WITH LESS THAN 1% ERROR FOR $t_{DA} <$
 1 2	0.5813	-0.5425	+0.6758	2.0	0.60	0.02
 1 2	0.1109	-2.1991	+1.5041	3.0	0.60	0.005
 1 4	5.3790	1.6825	-0.4367	0.8	0.30	0.01
 1 4	2.6896	0.9894	-0.0902	0.8	0.30	0.01
 1 4	0.2318	-1.4619	+1.1355	4.0	2.00	0.03
 1 4	0.1155	-2.1585	+1.4838	4.0	2.00	0.01
 1 5	2.3606	0.8589	-0.0249	1.0	0.40	0.025
<i>IN VERTICALLY-FRACTURED RESERVOIRS</i>						
USE $(x_e/x_f)^2$ IN PLACE OF A/r_w^2 FOR FRACTURED SYSTEMS						
 1 1	2.6541	0.9761	-0.0835	0.175	0.08	CANNOT USE
 1 1	2.0348	0.7104	+0.0493	0.175	0.09	CANNOT USE
 1 1	1.9986	0.6924	+0.0583	0.175	0.09	CANNOT USE
 1 1	1.6620	0.5080	+0.1505	0.175	0.09	CANNOT USE
 1 1	1.3127	0.2721	+0.2685	0.175	0.09	CANNOT USE
 1 1	0.7887	-0.2374	+0.5232	0.175	0.09	CANNOT USE
<i>IN WATER-DRIVE RESERVOIRS</i>						
 1	19.1	2.95	-1.07	—	—	—
<i>IN RESERVOIRS OF UNKNOWN CHARACTER</i>						
 1	25.0	3.22	-1.20	—	—	—

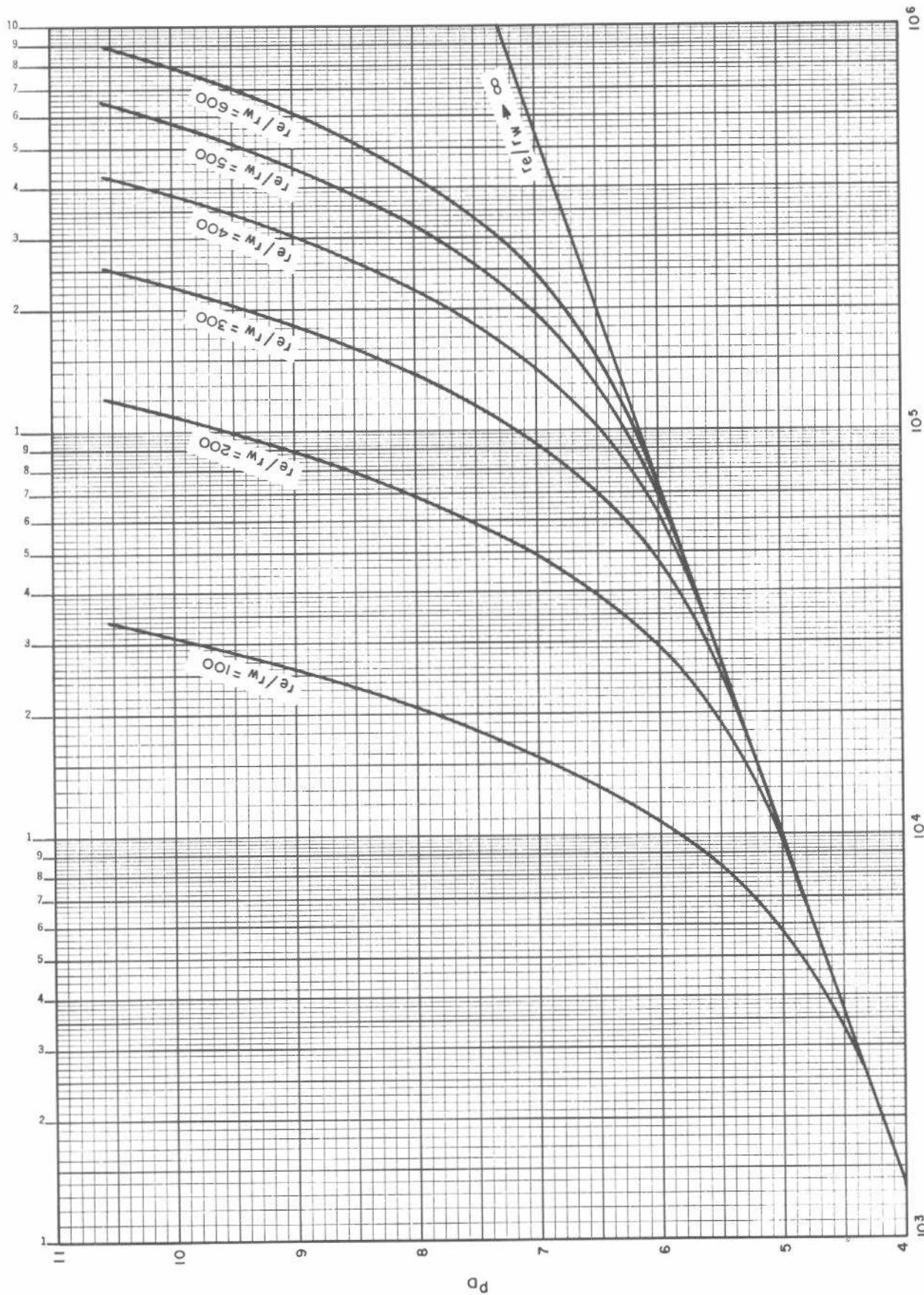


Fig. C.10A Dimensionless pressure for a well in the center of a closed circular reservoir, no wellbore storage, no skin. Calculated from Eq. C.20.

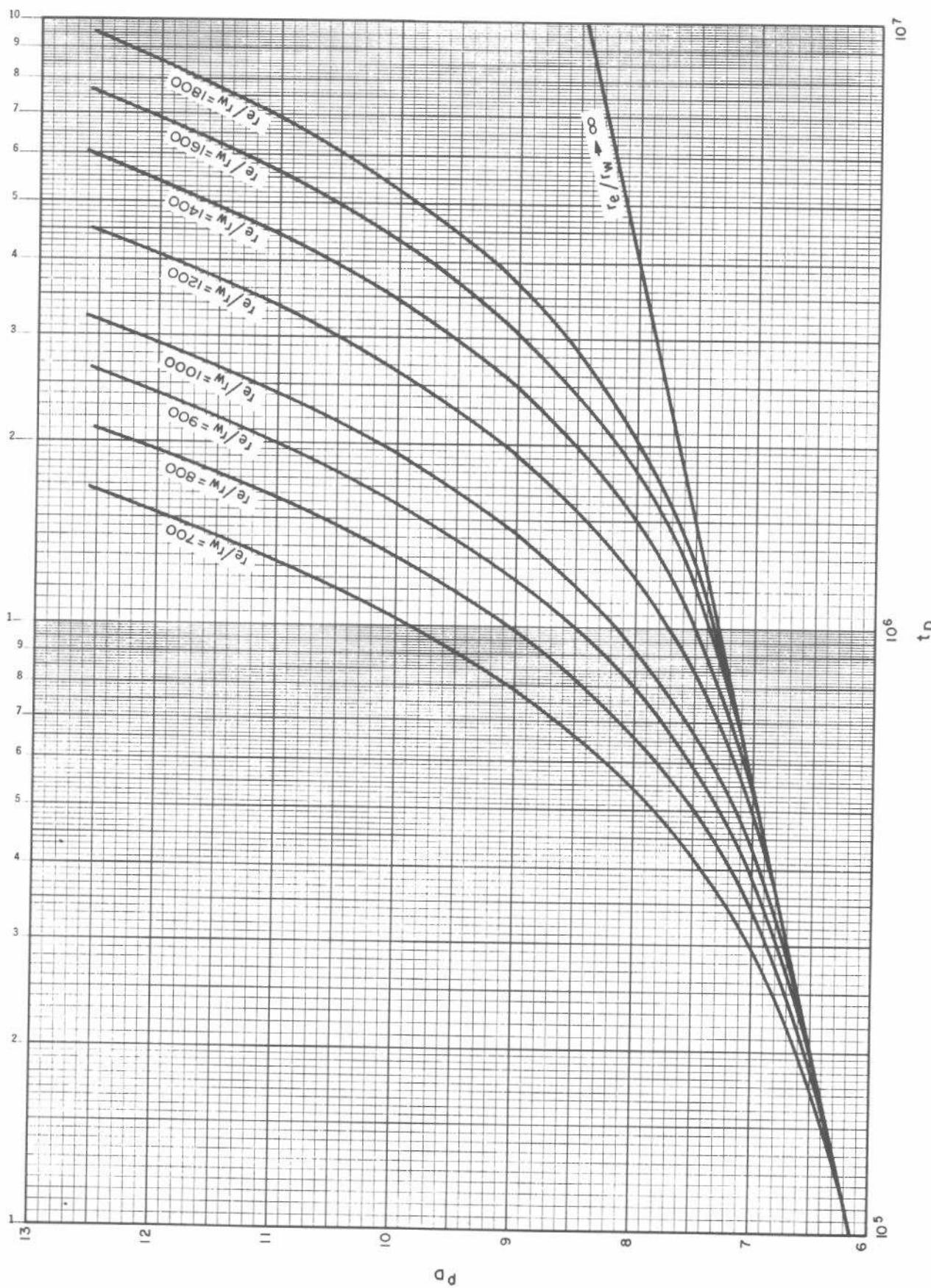


Fig. C.10B Dimensionless pressure for a well in the center of a closed circular reservoir, no wellbore storage, no skin. Calculated from Eq. C.20.

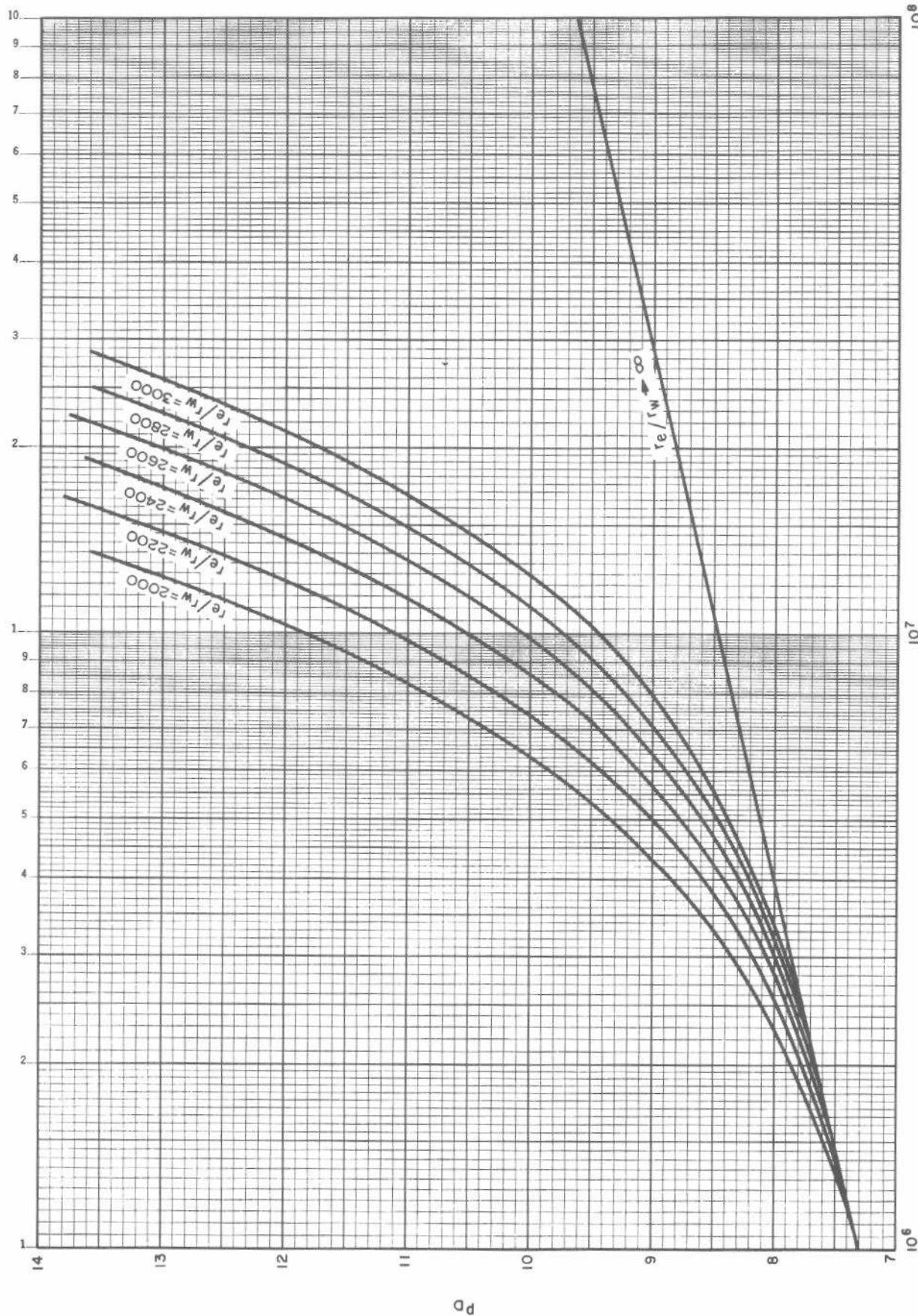


Fig. C.10C Dimensionless pressure for a well in the center of a closed circular reservoir, no wellbore storage, no skin. Calculated from Eq. C.20.

where

$$(p_D)_{\text{desired}} = \text{value of } p_D \text{ at desired value of } \sqrt{A/r_w}, \text{ and}$$

$$(p_D)_{\text{table}} = \text{tabulated or plotted value of } p_D \text{ using table or graph value of } \sqrt{A/r_w}.$$

The quantity $\sqrt{A/r_w}$ affects the dimensionless pressure only at the well; values given at points away from the well are independent of this parameter.

Closed Circular Reservoir, No Wellbore Storage, No Skin

Figs. C.10A, C.10B, and C.10C show dimensionless pressure data for a well in the center of a closed circular reservoir with no wellbore storage and no skin. Skin effect may be included by using Eq. 2.2. The data in Fig. C.10 have been computed from³

$$p_D = -\frac{1}{2} \left\{ \text{Ei}\left(-\frac{1}{4t_D}\right) - \text{Ei}\left(-\frac{1}{4t_{De}}\right) - 4t_{De} \exp(-1/4t_{De}) \right\}, \quad (\text{C.20})$$

where the dimensionless time based on the external radius of the system is

$$t_{De} = \frac{0.0002637 kt}{\phi \mu c r_e^2} = t_D \left(\frac{r_w^2}{r_e^2} \right). \quad (\text{C.21})$$

Hornor³ points out that Eq. C.20 is "not even a mathematical solution of the basic flow equation," Eq. 2.1. However, it is an excellent approximation to the exact solution³ (Ref. 3, Eq. XII, and Ref. 1, Eq. 2.36).

Closed-Square Reservoir, No Wellbore Storage, No Skin

Earlougher, Ramey, Miller, and Mueller¹⁷ give dimensionless pressure data at several points in a closed-square drainage area with the well at the center of the system. Wellbore storage and skin effect are not included. Fig. C.11 schematically illustrates system geometry and the points for which p_D data are given. Figs. C.12A and C.12B show p_D at several points in the system. Table C.2 presents the data for this system.

Single-Well Rectangular Systems, No Wellbore Storage, No Skin

Figs. C.13 through C.16 present dimensionless pressure data at the well for a single well at various locations in various closed rectangular systems. Wellbore storage and skin factors are not included. Earlougher and Ramey¹⁸ give tabular data for these figures; they also present p_D data for points away from the well.

The data in Figs. C.13 through C.16 are related to the data presented by Matthews, Brons, and Hazebroek¹⁹ (Figs. 6.2 through 6.5) by¹⁷

$$p_D(t_{DA}) = 2\pi t_{DA} + \frac{1}{2} \left[\ln \left(t_{DA} \frac{A}{r_w^2} \right) + 0.80907 - p_{D\text{MBH}}(t_{DA}) \right], \quad (\text{C.22})$$

where

$$p_{D\text{MBH}} = \frac{kh(p^* - \bar{p})}{70.6 qB\mu}. \quad (\text{C.23})$$

Closed-Square Reservoir, Vertically Fractured Well, No Wellbore Storage

Fig. C.17* gives dimensionless pressure data for a single vertically fractured well (infinite-conductivity fracture case) in the center of a closed-square drainage region. Wellbore storage effects are not included. The data in Fig. C.17 are from Gringarten, Ramey, and Raghavan^{6,7} and are considered to be slightly more accurate than other similar data.^{20,21} As for the infinite, vertically fractured system, there is an initial half-slope straight line on the log-log plot; the duration of this line depends on the fracture length.

Fig. C.18* shows additional p_D data for a single, verti-

*See footnote on Page 24.

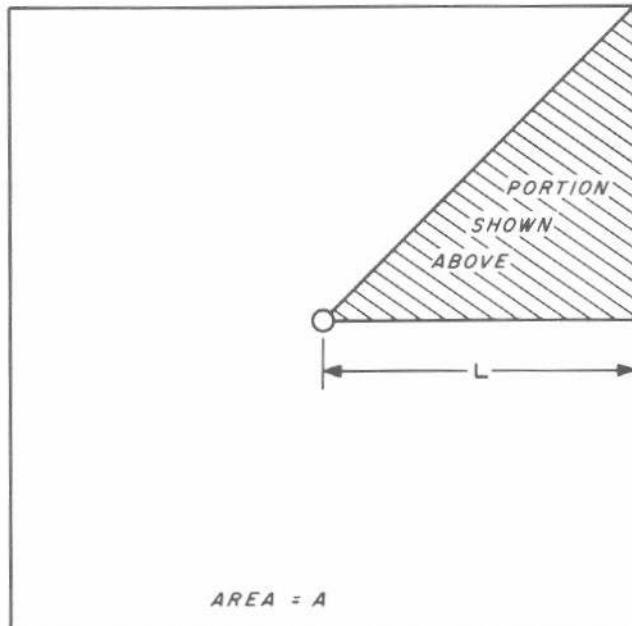
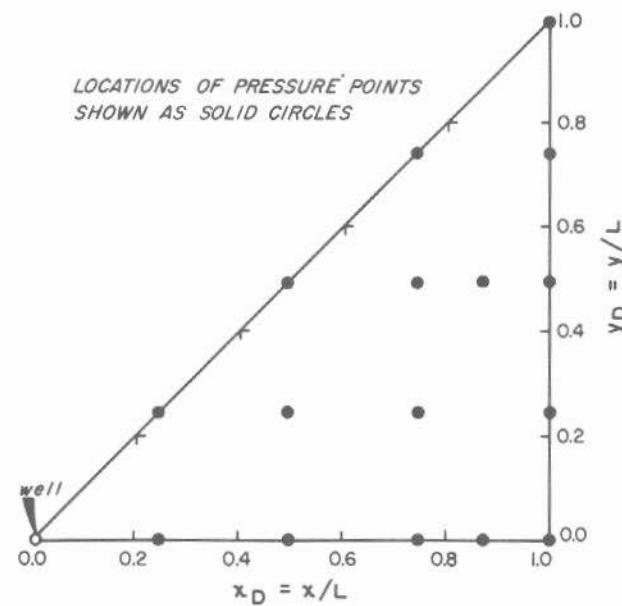


Fig. C.11 Well in the center of a closed-square system, well and pressure point location. After Earlougher, Ramey, Miller, and Mueller.¹⁷

cally fractured (infinite-conductivity) well in a closed-square drainage area. Wellbore storage is not included. Fig. C.18 may be more useful for type-curve matching than Fig. C.17 under some circumstances. Gringarten, Ramey, and Raghavan⁶ give tabular p_D data for this case.

Fig. C.19* gives dimensionless pressure data for the systems of Figs. C.17 and C.18 but for a *uniform-flux* vertical fracture. Tabular data appear in Ref. 6. It is generally believed^{6,7} that the uniform-flux fracture solution more closely approximates actual fractured systems than does the infinite-conductivity fracture solution.

*See footnote on Page 24.

A vertically fractured well in a closed system has the same general transient behavior as an unfractured well in a closed system. For the fractured well, the dimensionless pressure during the infinite-acting period is given by Eqs. C.5 and C.7 through C.10, depending on the fracture solution (infinite-conductivity or uniform-flux) and the time. Vertically fractured systems also exhibit pseudosteady-state behavior:

$$p_D = 2\pi t_{DA} + \frac{1}{2} \ln \left[\left(\frac{x_e}{x_f} \right)^2 \right] + \frac{1}{2} \ln \left(\frac{2.2458}{C_A} \right). \quad \dots \quad (C.24)$$

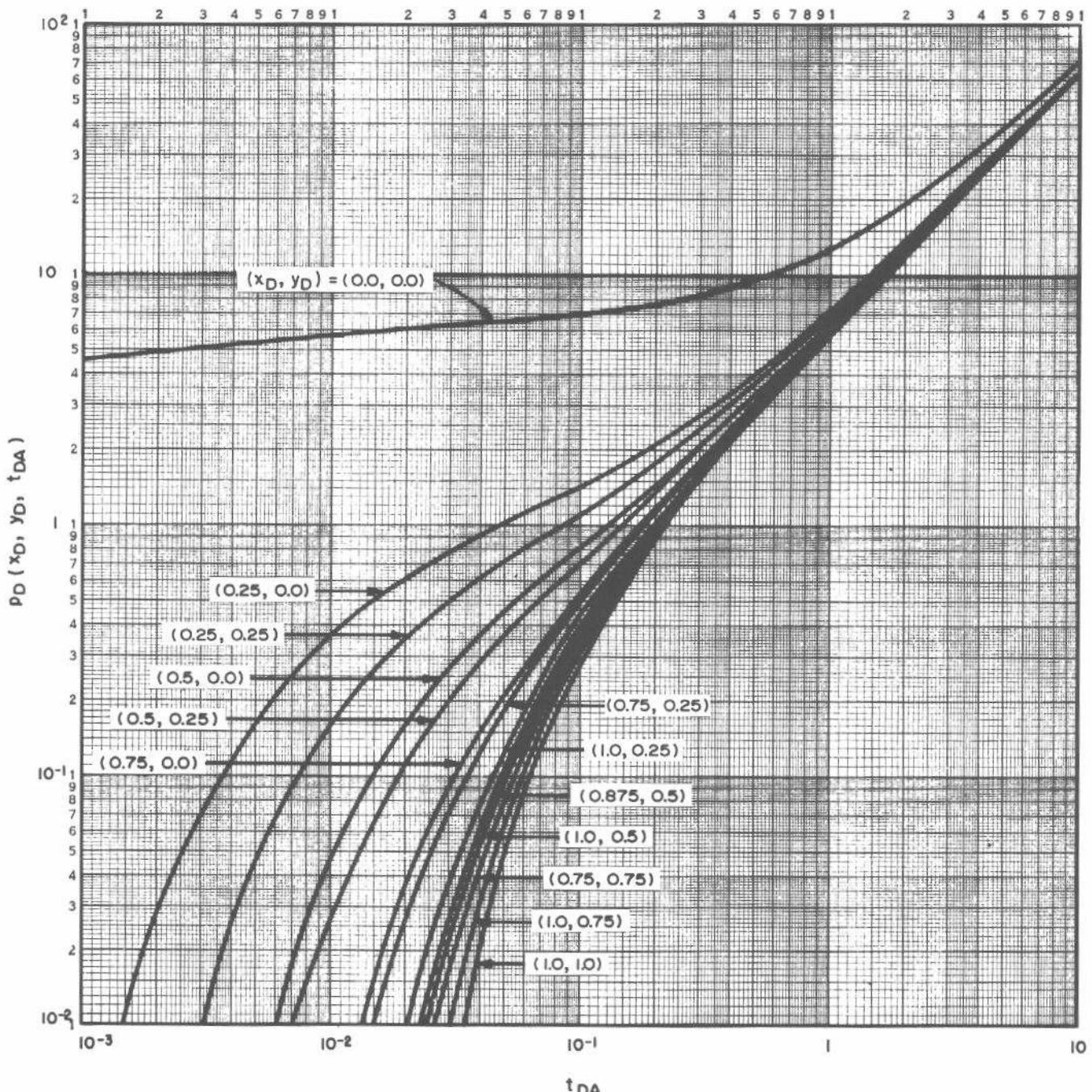


Fig. C.12A Dimensionless pressure at various points in a closed-square system with the well at the center, no wellbore storage, no skin, $\sqrt{A/r_w} = 2.000$. Log-log plot. See Fig. C.11 for point locations. After Earlougher, Ramey, Miller, and Mueller.¹⁷

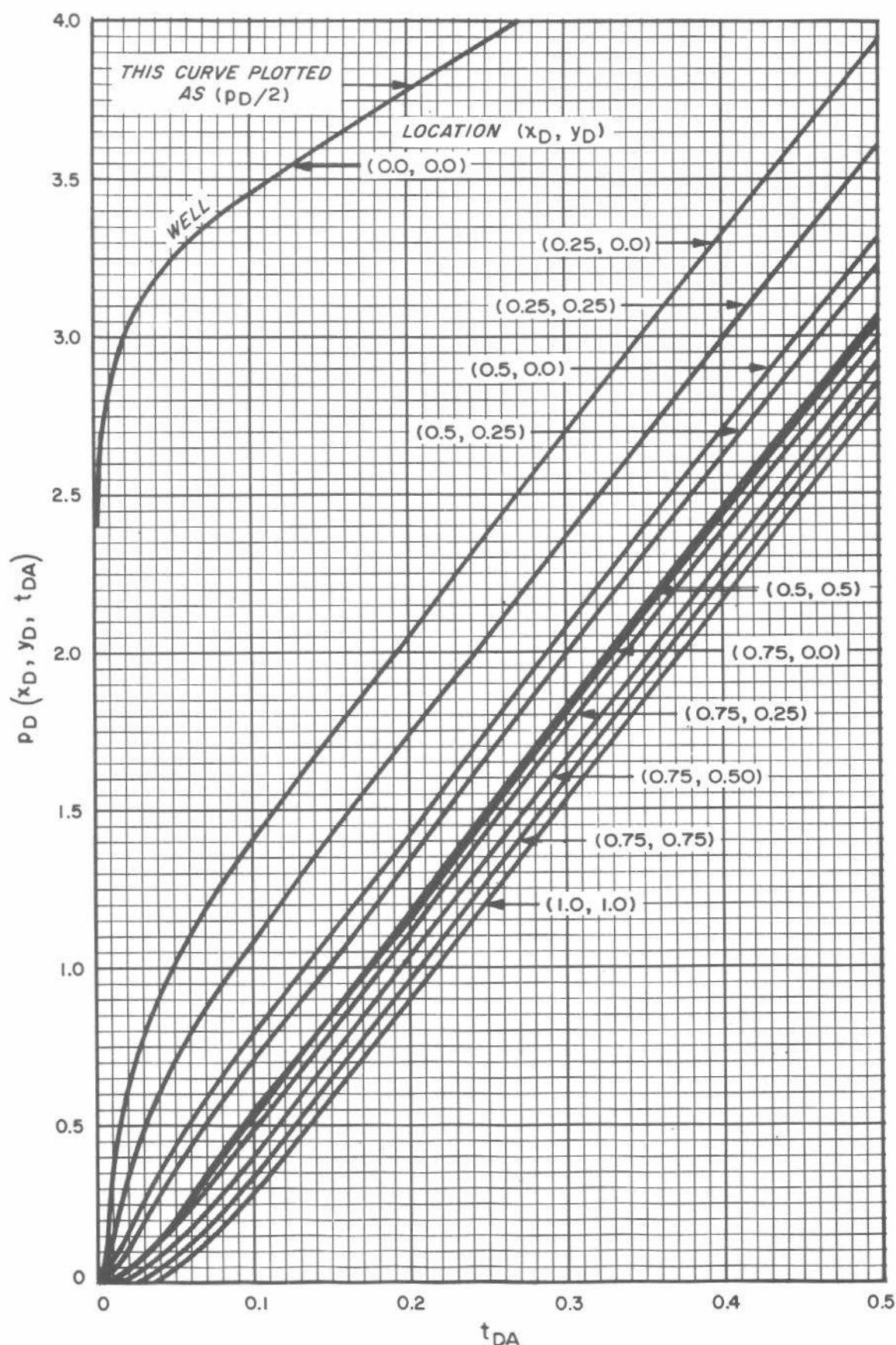


Fig. C.12B Dimensionless pressure at various points in a closed-square system with the well at the center, no wellbore storage, no skin, $\sqrt{A/r_w} = 2,000$. Coordinate plot. See Fig. C.11 for point locations. After Earlougher, Ramey, Miller, and Mueller.¹⁷

TABLE C.2—DIMENSIONLESS PRESSURE AT VARIOUS POINTS IN A CLOSED SQUARE WITH A WELL AT THE CENTER,
NO WELBORE STORAGE, NO SKIN. SEE FIG. C.11 FOR DEFINITION OF POINTS. $\sqrt{A}/r_w = 2,000$.
After Earlougher, Ramey, Miller, and Mueller.¹⁷

P ₀									
t _{DA}	x _D =0.000	x _D =0.250	x _D =0.250	x _D =0.500	x _D =0.500	x _D =0.500	x _D =0.750	x _D =0.750	x _D =0.750
0.0010	4.5516	0.0021	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0015	4.7543	0.0109	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0020	4.9981	0.0261	0.0021	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0025	5.0097	0.0456	0.0056	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
0.0030	5.1109	0.0675	0.0109	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000
0.0040	5.2447	0.1141	0.0261	0.0021	0.0007	0.0000	0.0000	0.0000	0.0000
0.0050	5.3563	0.1607	0.0456	0.0056	0.0021	0.0001	0.0001	0.0001	0.0000
0.0060	5.4474	0.2053	0.0675	0.0109	0.0047	0.0004	0.0002	0.0001	0.0001
0.0070	5.5245	0.2475	0.0906	0.0178	0.0085	0.0011	0.0006	0.0003	0.0003
0.0080	5.5913	0.2871	0.1141	0.0261	0.0135	0.0021	0.0012	0.0007	0.0007
0.0090	5.6502	0.3243	0.1376	0.0354	0.0194	0.0036	0.0021	0.0013	0.0013
0.0100	5.7029	0.3592	0.1607	0.0456	0.0261	0.0056	0.0034	0.0021	0.0021
0.0150	5.9056	0.5063	0.2676	0.1023	0.0675	0.0218	0.0154	0.0109	0.0109
0.0200	6.0494	0.6211	0.3592	0.1607	0.1142	0.0456	0.0350	0.0266	0.0266
0.0250	6.1610	0.7147	0.4379	0.2164	0.1609	0.0735	0.0597	0.0474	0.0474
0.0300	6.2522	0.7939	0.5065	0.2685	0.2061	0.1032	0.0876	0.0716	0.0716
0.0400	6.3965	0.9232	0.6224	0.3628	0.2906	0.1650	0.1485	0.1263	0.1263
0.0500	6.5099	1.0279	0.7192	0.4470	0.3685	0.2276	0.2125	0.1854	0.1854
0.0600	6.6050	1.1178	0.8041	0.5242	0.4415	0.2904	0.2772	0.2466	0.2466
0.0700	6.6888	1.1983	0.8815	0.5968	0.5112	0.3532	0.3418	0.3086	0.3086
0.0800	6.7654	1.2728	0.9539	0.6661	0.5786	0.4160	0.4061	0.3711	0.3711
0.0900	6.8374	1.3434	1.0231	0.7334	0.6446	0.4788	0.4700	0.4338	0.4338
0.1000	6.9063	1.4114	1.0902	0.7992	0.7095	0.5417	0.5336	0.4965	0.4965
0.1500	7.2311	1.7347	1.4119	1.1186	1.0274	0.8558	0.8492	0.8106	0.8106
0.2000	7.5468	2.0501	1.7271	1.4335	1.3421	1.1700	1.1636	1.1248	1.1248
0.2500	7.8611	2.3644	2.0414	1.7476	1.6563	1.4841	1.4778	1.4390	1.4390
0.3000	8.1753	2.6786	2.3556	2.0620	1.9705	1.7933	1.7919	1.7531	1.7531
0.4000	8.8036	3.0369	2.9839	2.6903	2.5988	2.4266	2.4202	2.3814	2.3814
0.5000	9.4320	3.9352	3.6122	3.3186	3.2271	3.0549	3.0486	3.0098	3.0098
0.6000	10.0603	4.5636	4.2406	3.9469	3.8555	3.6833	3.6769	3.6381	3.6381
0.7000	10.6886	5.1919	4.8689	4.5752	4.4838	4.3116	4.3052	4.2664	4.2664
0.8000	11.3169	5.8202	5.4972	5.2036	5.1121	4.9399	4.9335	4.8847	4.8847
0.9000	11.9452	6.4485	6.1255	5.8319	5.7404	5.5682	5.5618	5.5230	5.5230
1.0000	12.5735	7.0768	6.7538	6.4602	6.3687	6.1965	6.1902	6.1513	6.1513
2.0000	18.8567	13.3600	13.0370	12.7433	12.6519	12.4797	12.4733	12.4345	12.4345
4.0000	31.4230	25.9263	25.6033	25.3097	25.2182	25.0460	25.0397	25.0009	25.0009
8.0000	46.5557	51.0590	50.7360	50.4423	50.3509	50.1787	50.1723	50.1335	50.1335
10.0000	49.1220	63.6253	63.3023	63.0087	62.9172	62.7450	62.7386	62.6999	62.6999
C _A	30.8828	1.837 x 10 ⁶	3.504 x 10 ⁶	6.303 x 10 ⁶	7.570 x 10 ⁶	10.68 x 10 ⁶	10.82 x 10 ⁶	11.69 x 10 ⁶	11.69 x 10 ⁶
1/2 ln (C _A / C _A)	-1.3106	-6.807	-7.130	-7.424	-7.515	-7.688	-7.694	-7.733	-7.733

t_{DA}	$x=0.750$	$x=0.750$	$x=0.875$	$x=0.875$	$x=1.000$	$x=1.000$	$x=1.000$	$x=1.000$	$x=1.000$
	$y=0.500$	$y=0.750$	$y=0.000$	$y=0.500$	$y=0.000$	$y=0.250$	$y=0.500$	$y=0.750$	$y=1.000$
0.0010	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0015	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0020	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0025	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0030	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0040	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0050	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0060	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0070	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0080	0.0001	0.0000	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0090	0.0003	0.0000	0.0004	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
0.0100	0.0005	0.0001	0.0008	0.0001	0.0003	0.0002	0.0000	0.0000	0.0000
0.0150	0.0040	0.0008	0.0055	0.0016	0.0031	0.0023	0.0009	0.0002	0.0001
0.0200	0.0121	0.0036	0.0164	0.0060	0.0111	0.0087	0.0042	0.0014	0.0005
0.0250	0.0245	0.0091	0.0329	0.0143	0.0249	0.0203	0.0112	0.0045	0.0023
0.0300	0.0404	0.0177	0.0539	0.0264	0.0436	0.0365	0.0219	0.0102	0.0062
0.0400	0.0805	0.0437	0.1050	0.0600	0.0913	0.0793	0.0532	0.0307	0.0223
0.0500	0.1281	0.0800	0.1628	0.1030	0.1469	0.1308	0.0947	0.0623	0.0498
0.0600	0.1807	0.1241	0.2237	0.1525	0.2065	0.1871	0.1431	0.1029	0.0872
0.0700	0.2366	0.1740	0.2859	0.2064	0.2678	0.2460	0.1962	0.1502	0.1321
0.0800	0.2948	0.2279	0.3486	0.2632	0.3299	0.3064	0.2525	0.2023	0.1826
0.0900	0.3546	0.2846	0.4114	0.3219	0.3925	0.3677	0.3109	0.2579	0.2369
0.1000	0.4153	0.3433	0.4744	0.3820	0.4551	0.4296	0.3708	0.3157	0.2939
0.1500	0.7257	0.6500	0.7888	0.6913	0.7692	0.7421	0.6797	0.6209	0.5976
0.2000	1.0393	0.9632	1.1030	1.0047	1.0834	1.0561	0.9931	0.9338	0.9103
0.2500	1.3534	1.2772	1.4172	1.3188	1.3975	1.3702	1.3071	1.2478	1.2243
0.3000	1.6676	1.5913	1.7313	1.6330	1.7117	1.6843	1.6213	1.5620	1.5384
0.4000	2.2959	2.2196	2.3597	2.2613	2.3400	2.3127	2.2496	2.1903	2.1667
0.5000	2.9242	2.8479	2.9880	2.8896	2.9683	2.9410	2.8779	2.8186	2.7950
0.6000	3.5525	3.4763	3.6163	3.5179	3.5966	3.5693	3.5062	3.4469	3.4233
0.7000	4.1808	4.1046	4.2446	4.1462	4.2249	4.1976	4.1346	4.0752	4.0517
0.8000	4.8092	4.7329	4.8729	4.7745	4.8533	4.8259	4.7629	4.7036	4.6680
0.9000	5.4375	5.3612	5.5012	5.4029	5.4816	5.4542	5.3912	5.3319	5.3083
1.0000	6.0658	5.9895	6.1296	6.0312	6.1099	6.0826	6.0195	5.9602	5.9366
2.0000	12.3490	12.2727	12.4127	12.3144	12.3930	12.3657	12.3027	12.2434	12.2198
4.0000	24.9153	24.8391	24.9791	24.8807	24.9594	24.9321	24.8690	24.8097	24.7861
8.0000	50.0480	49.9717	50.1117	50.0134	50.0921	50.0647	50.0017	49.9424	49.9188
10.0000	62.6143	62.5381	62.6701	62.5979	62.6584	62.6311	62.5680	62.5087	62.4851
C_A	13.87×10^6	16.16×10^6	12.21×10^6	14.87×10^6	12.70×10^6	13.41×10^6	15.22×10^6	17.14×10^6	18.96×10^6
$\frac{1}{2} f_1 \left(\frac{P_0}{C_A} \right)$	-7.818	-7.894	-7.754	-7.853	-7.774	-7.801	-7.865	-7.924	-7.974

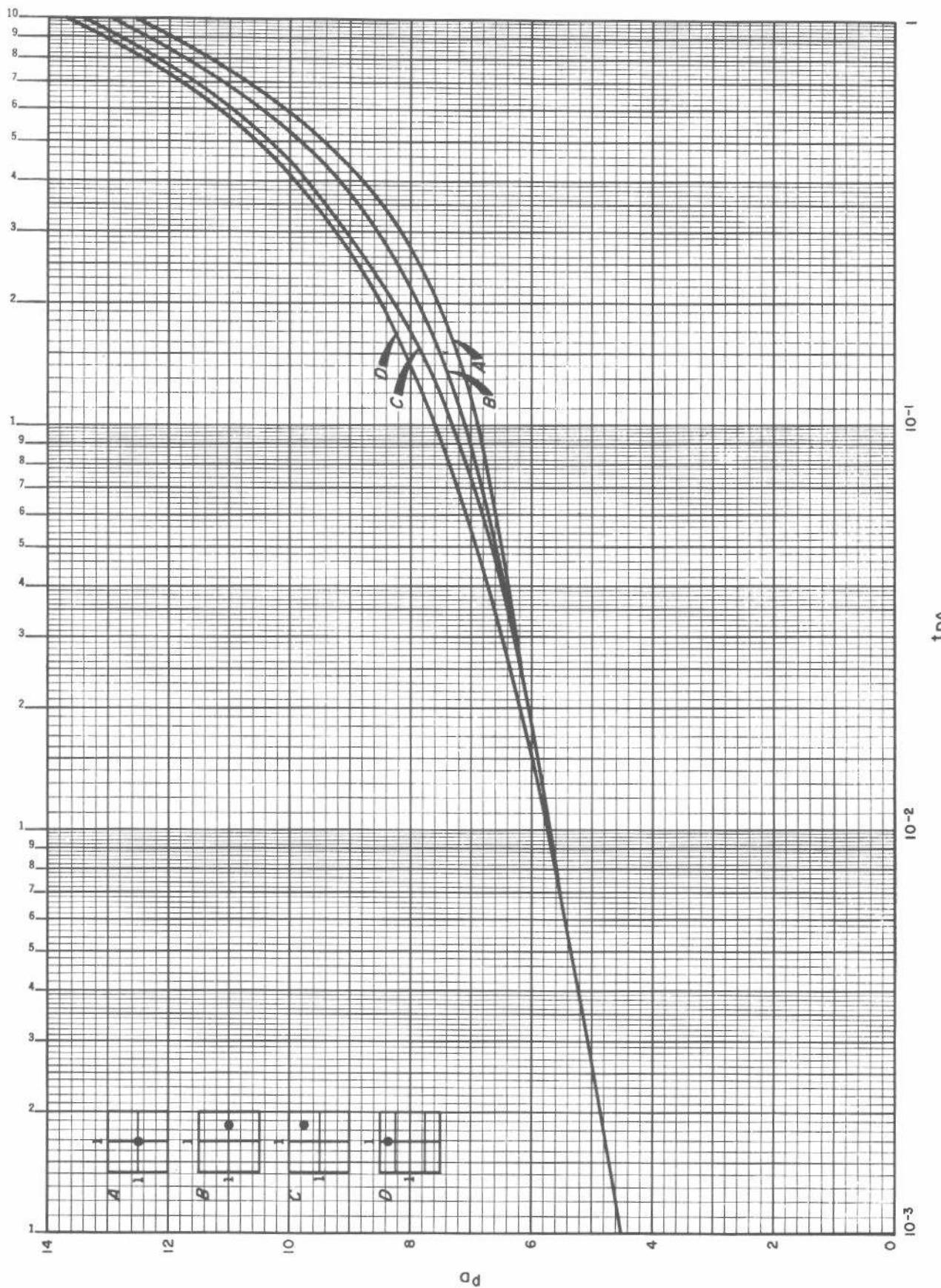


Fig. C.13 Dimensionless pressure for a single well in various closed rectangular systems, no skin, $\sqrt{A/r_w} = 2,000$. Data of Earlougher and Ramey.¹⁸

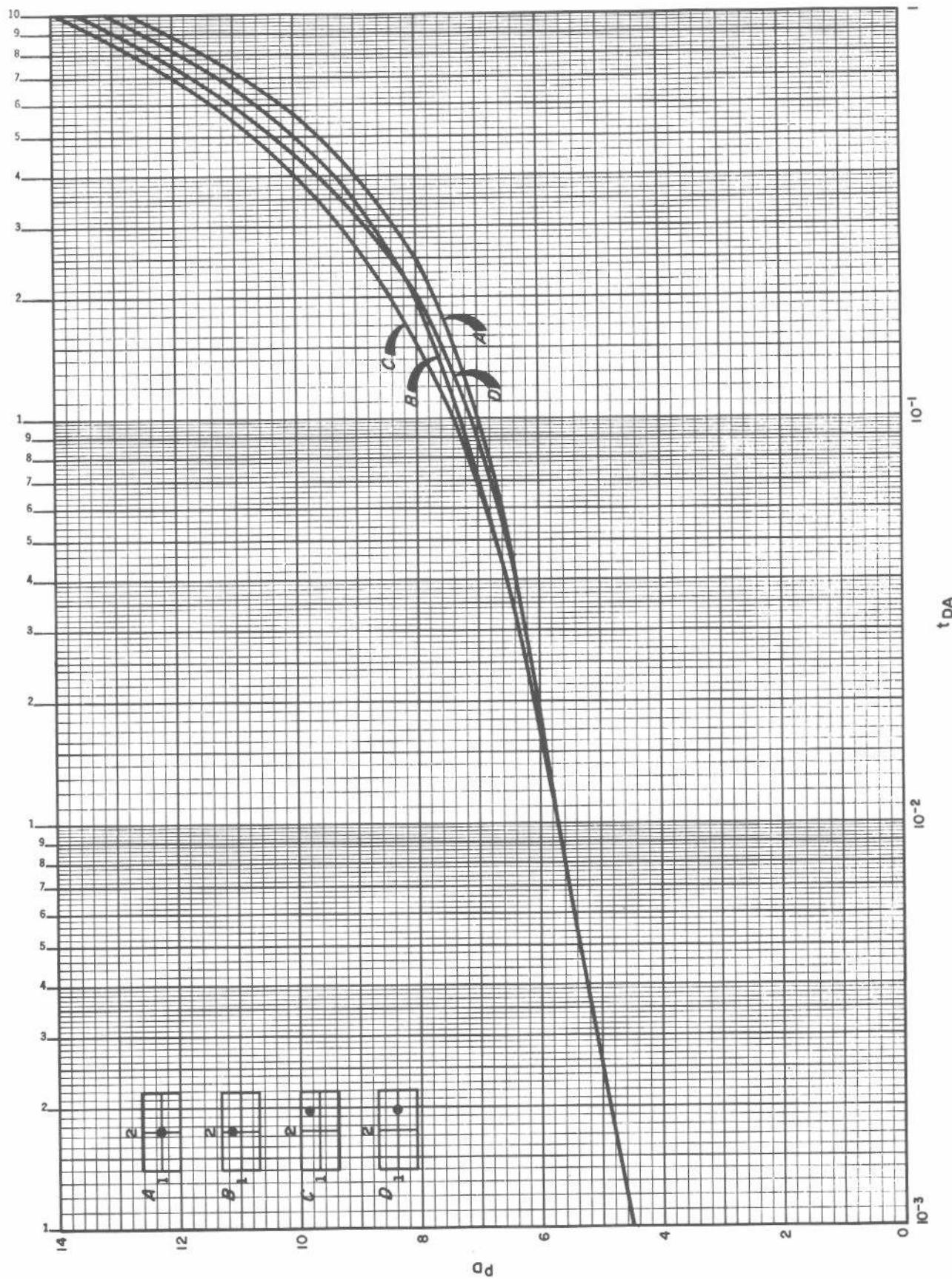


Fig. C.14 Dimensionless pressure for a single well in various closed rectangular systems, no skin, no wellbore storage, no skin, $\sqrt{A/r_w} = 2,000$. Data of Earlougher and Ramey.¹⁸

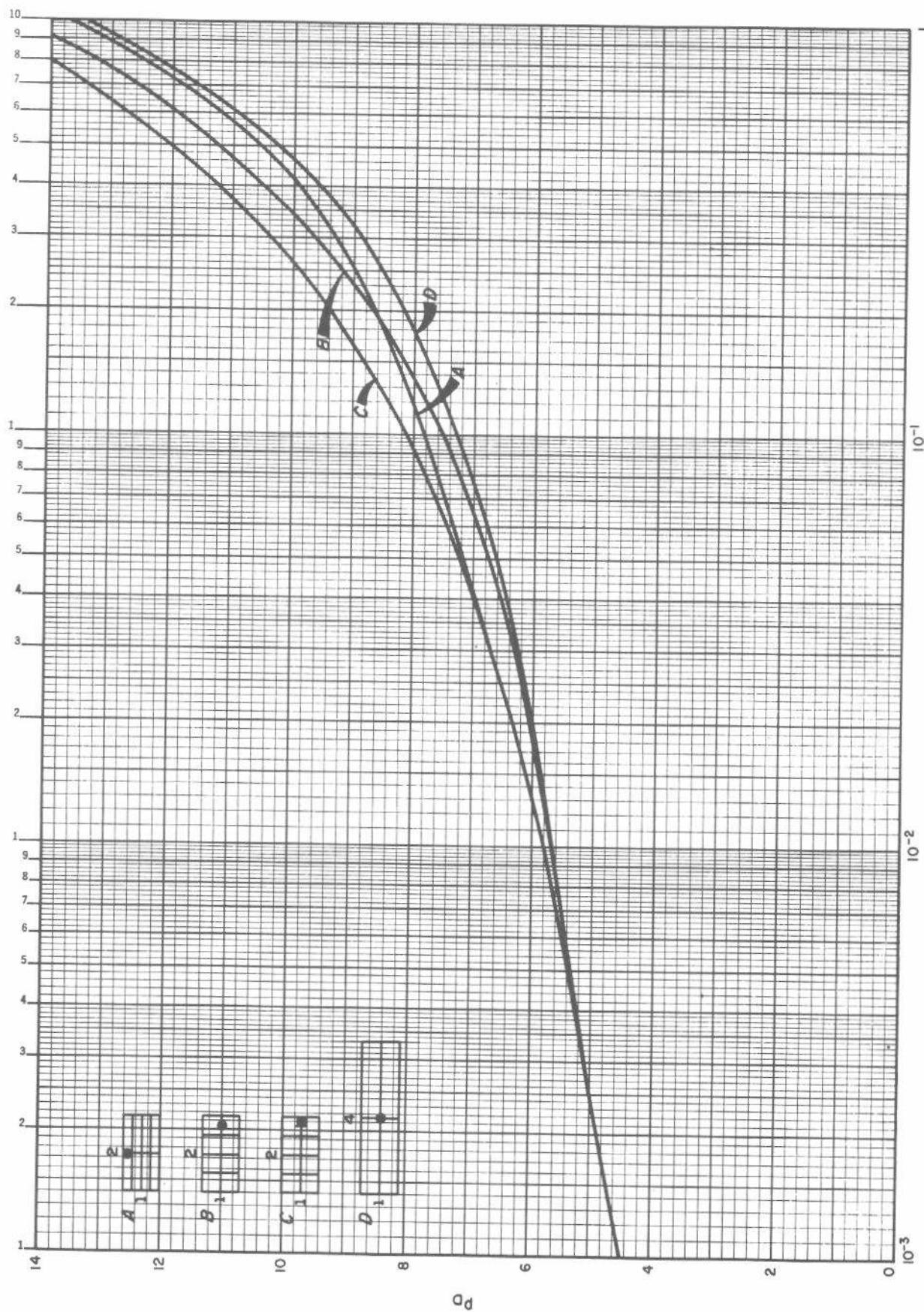


Fig. C.15 Dimensionless pressure for a single well in various closed rectangular systems, no skin, $\sqrt{A/r_w} = 2,000$. Data of Earlougher and Ramey.¹⁸

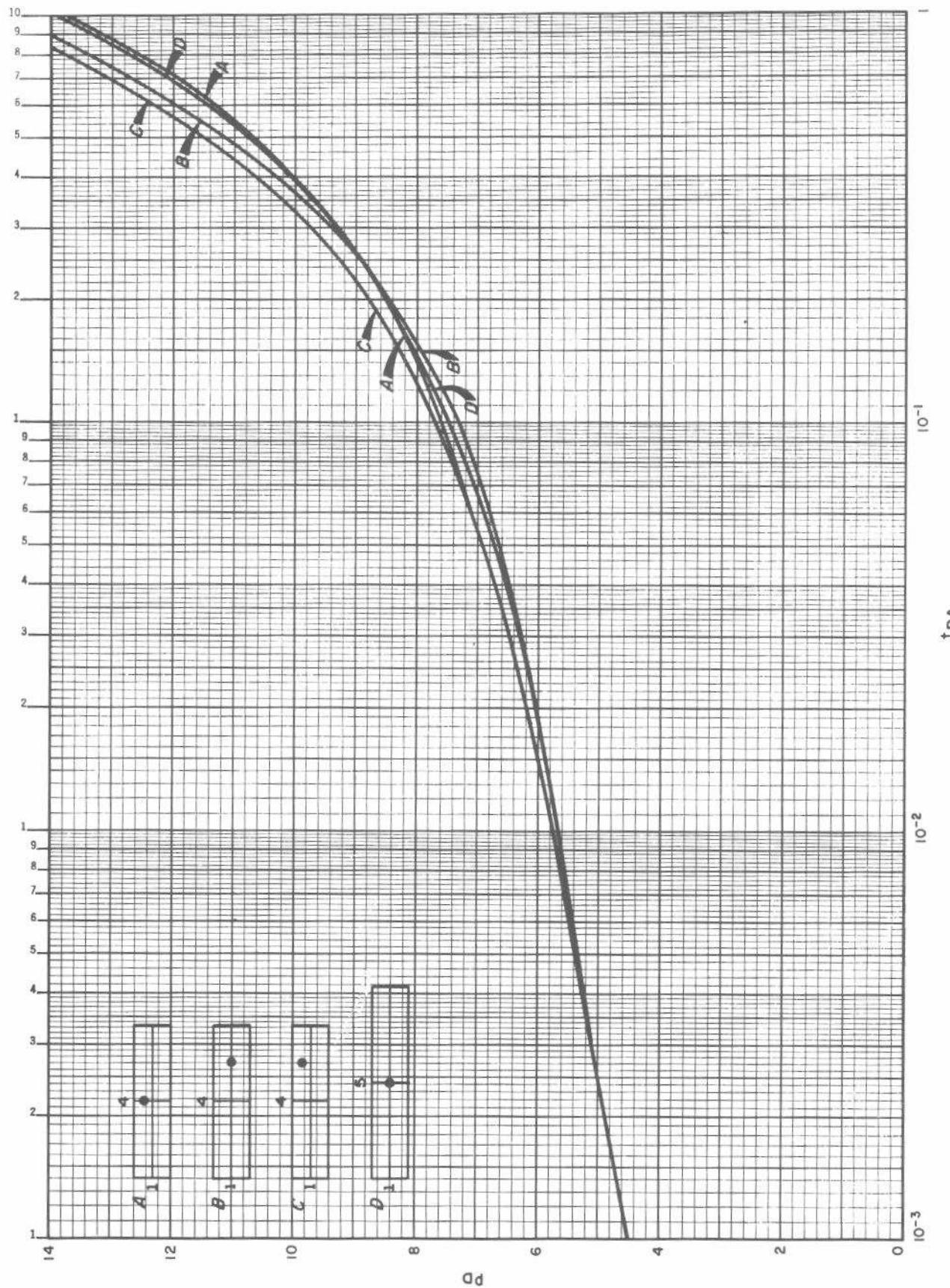


Fig. C.16 Dimensionless pressure for a single well in various closed rectangular systems, no skin, no wellbore storage, no skin, $\sqrt{A/r_w} = 2,000$. Data of Earlougher and Ramey.¹⁸

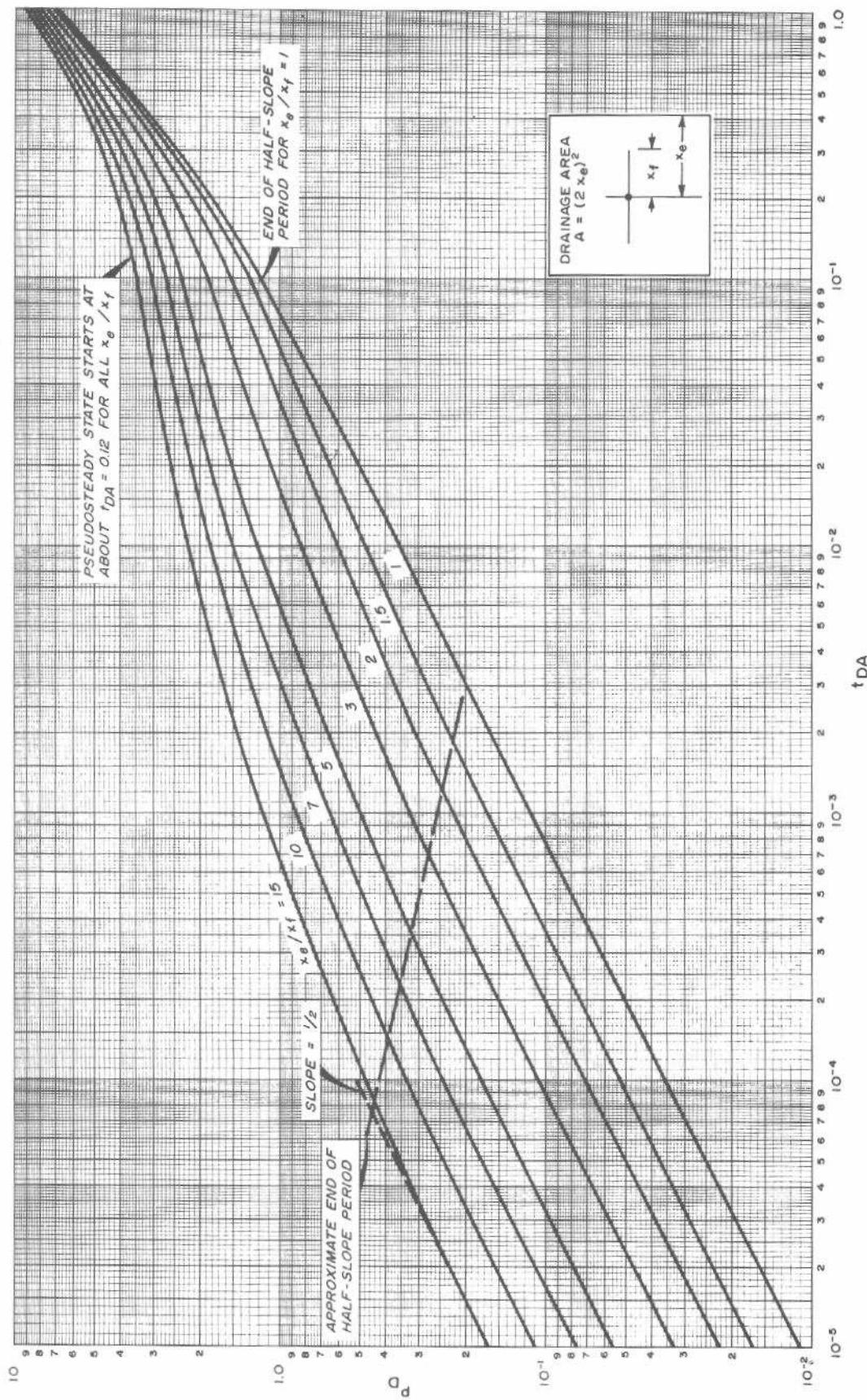


Fig. C.17 Dimensionless pressure for vertically fractured well in the center of a closed system, no wellbore storage, infinite-conductivity fracture. Data of Gringarten, Ramey, and Raghavan.^{6,7} Graph courtesy H. J. Ramey, Jr.

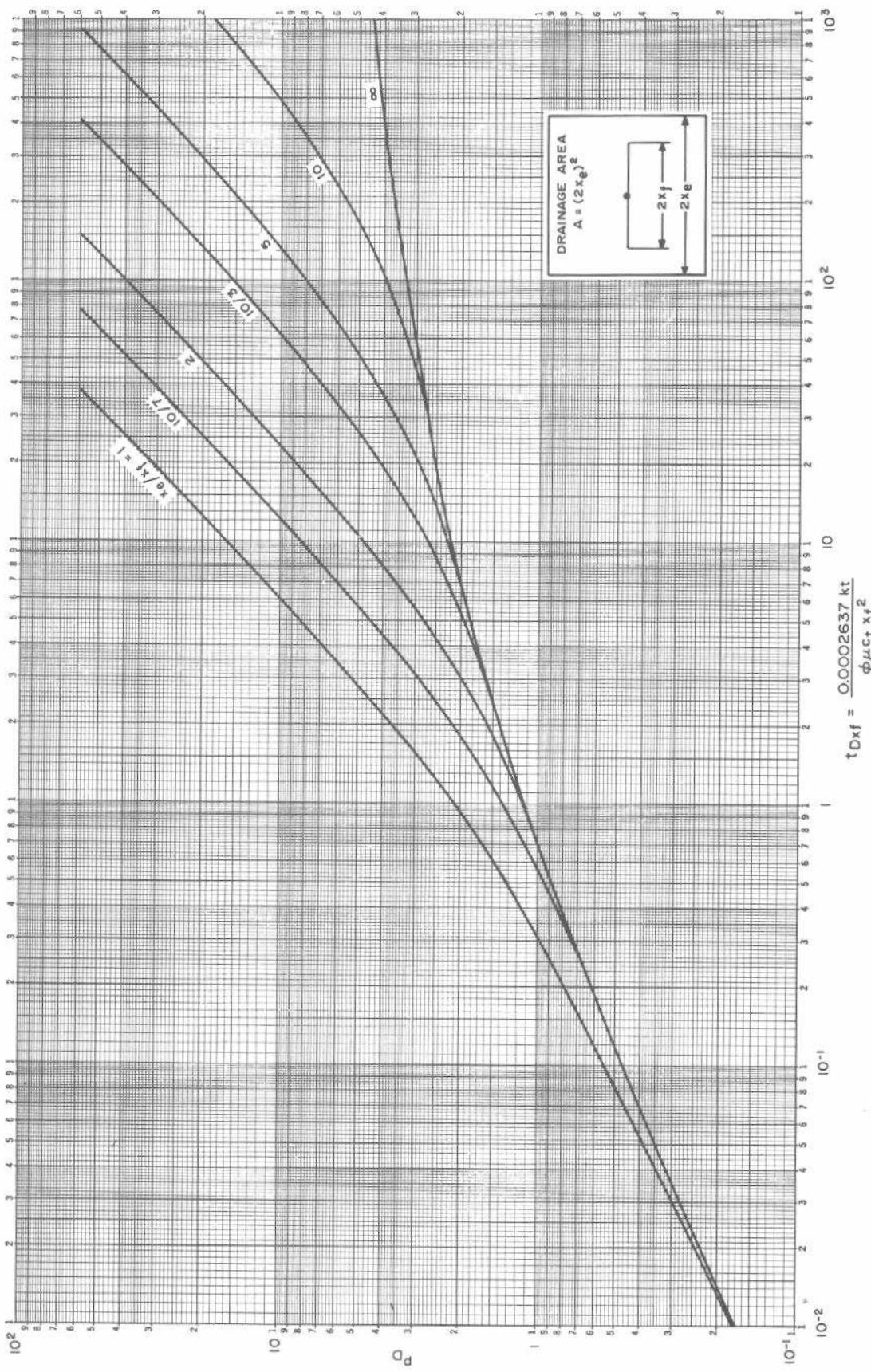


Fig. C.18 Dimensionless pressure for vertically fractured well in the center of a closed square, no wellbore storage, infinite-conductivity fracture. After Gringarten, Ramey, and Raghavan.^{6,7}

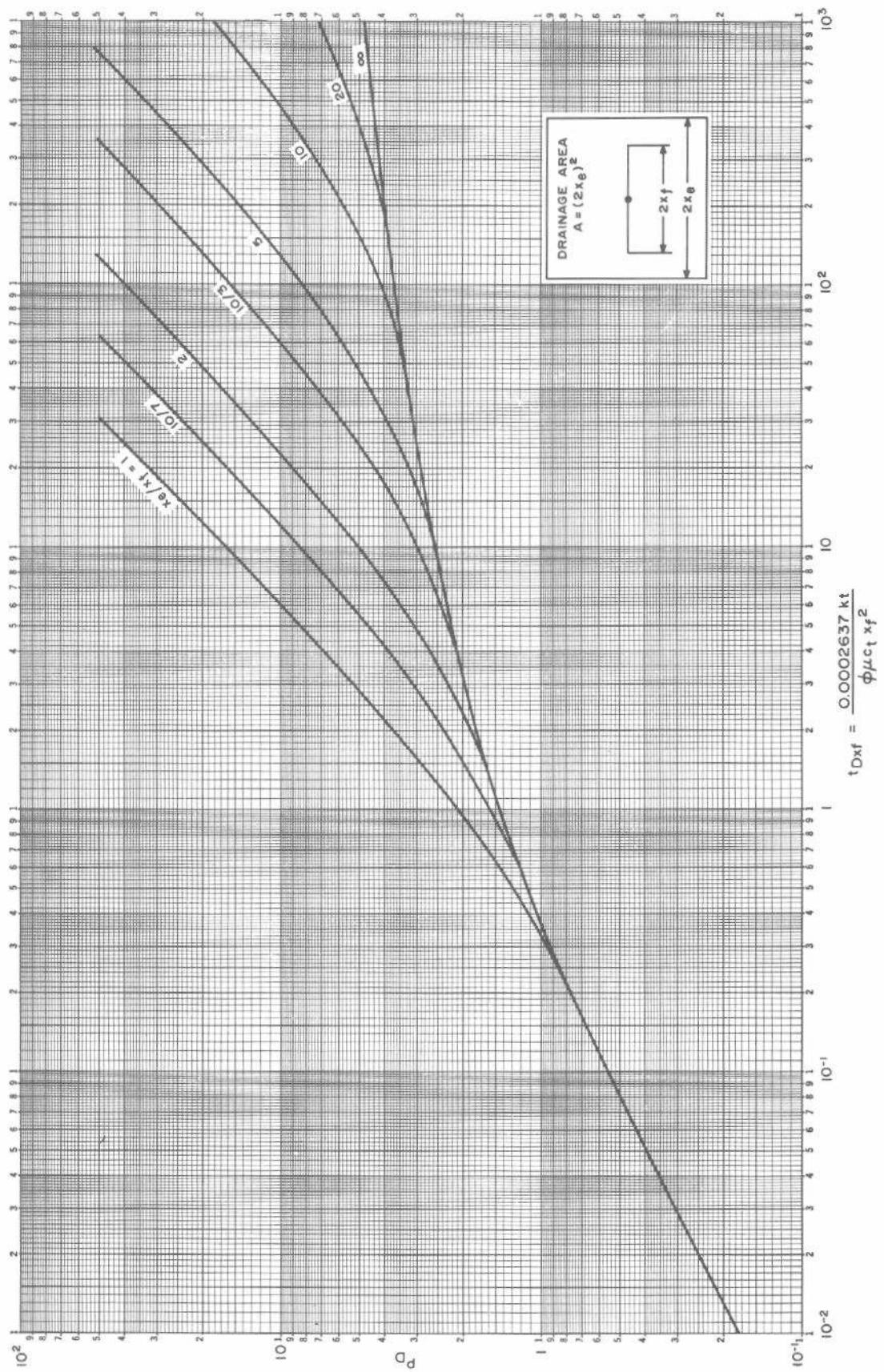


Fig. C.19 Dimensionless pressure for vertically fractured well in the center of a closed square, no wellbore storage, uniform-flux fracture.
 After Gringarten, Ramey and Raghavan.^{6,7}

Eq. C.24 is Eq. C.18 with x_e/x_f substituted for $\sqrt{A/r_w}$. Shape factors for fractured systems based on this dimensional parameter are given in Table C.1.

C.4 Constant-Pressure Systems

Circular Reservoir, No Wellbore Storage, No Skin

Fig. C.20 shows p_D for a single well in the center of a circular reservoir with constant external pressure, no wellbore storage, and no skin. The system reaches true steady state when

$$t_D > 1.25 \left(\frac{r_e^2}{r_w^2} \right), \quad \dots \quad (\text{C.25a})$$

or

$$t_{DA} > 0.40 \dots \quad (\text{C.25b})$$

After that time the dimensionless pressure is given by Eq. 2.26a. Tabular data are presented by van Everdingen and Hurst.⁵

Rectangular Reservoirs, No Wellbore Storage, No Skin

Fig. C.21 gives dimensionless pressures for several square and rectangular systems with a single well and one or more boundaries at constant pressure. No wellbore storage or skin effects are included in Fig. C.21. Each system reaches steady state at some time. Ramey, Kumar, and Gulati²² give additional information about steady state and

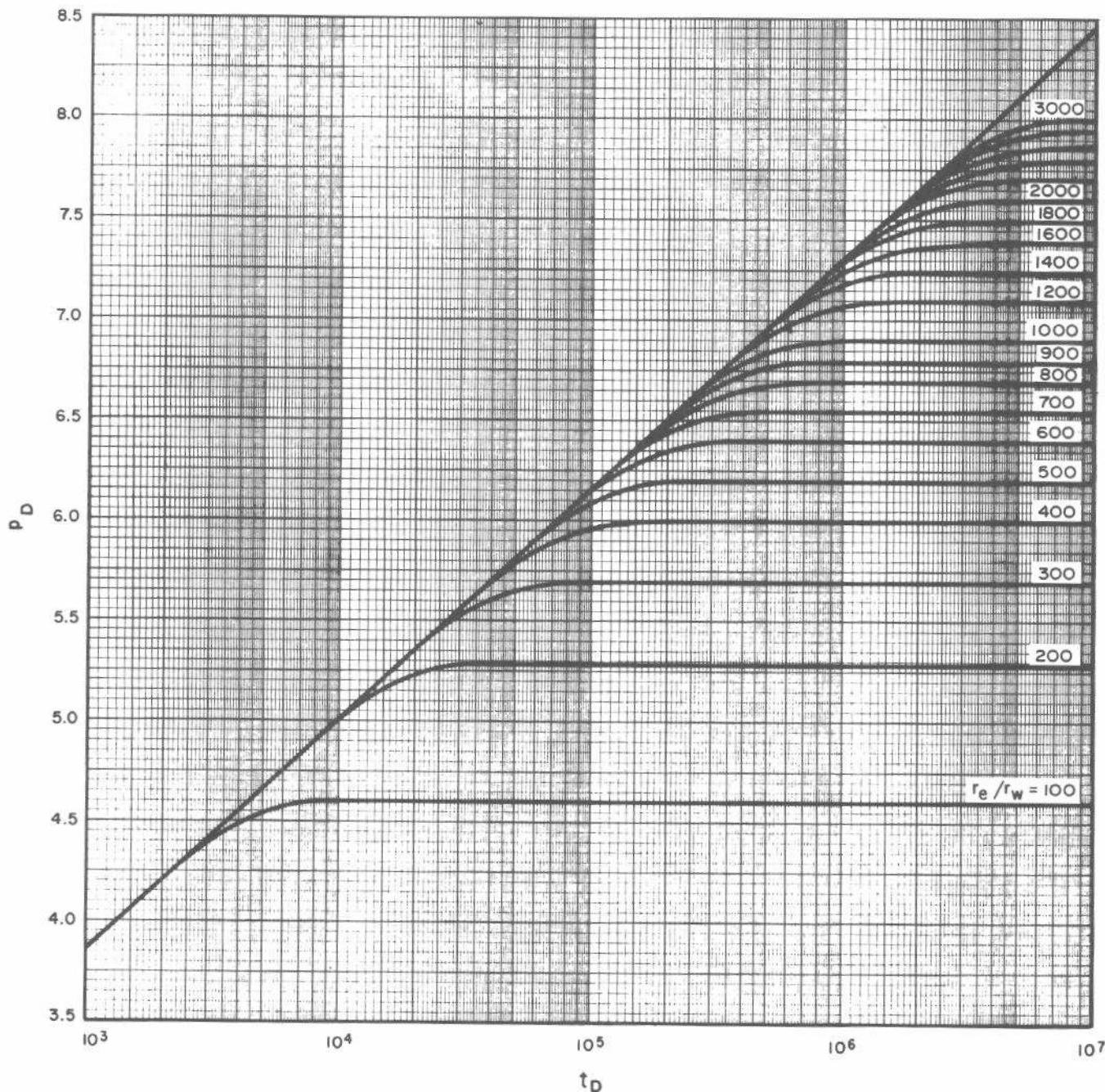


Fig. C.20 Dimensionless pressure for a well in the center of a closed circular reservoir with constant external pressure, no wellbore storage, no skin. After van Everdingen and Hurst.⁵

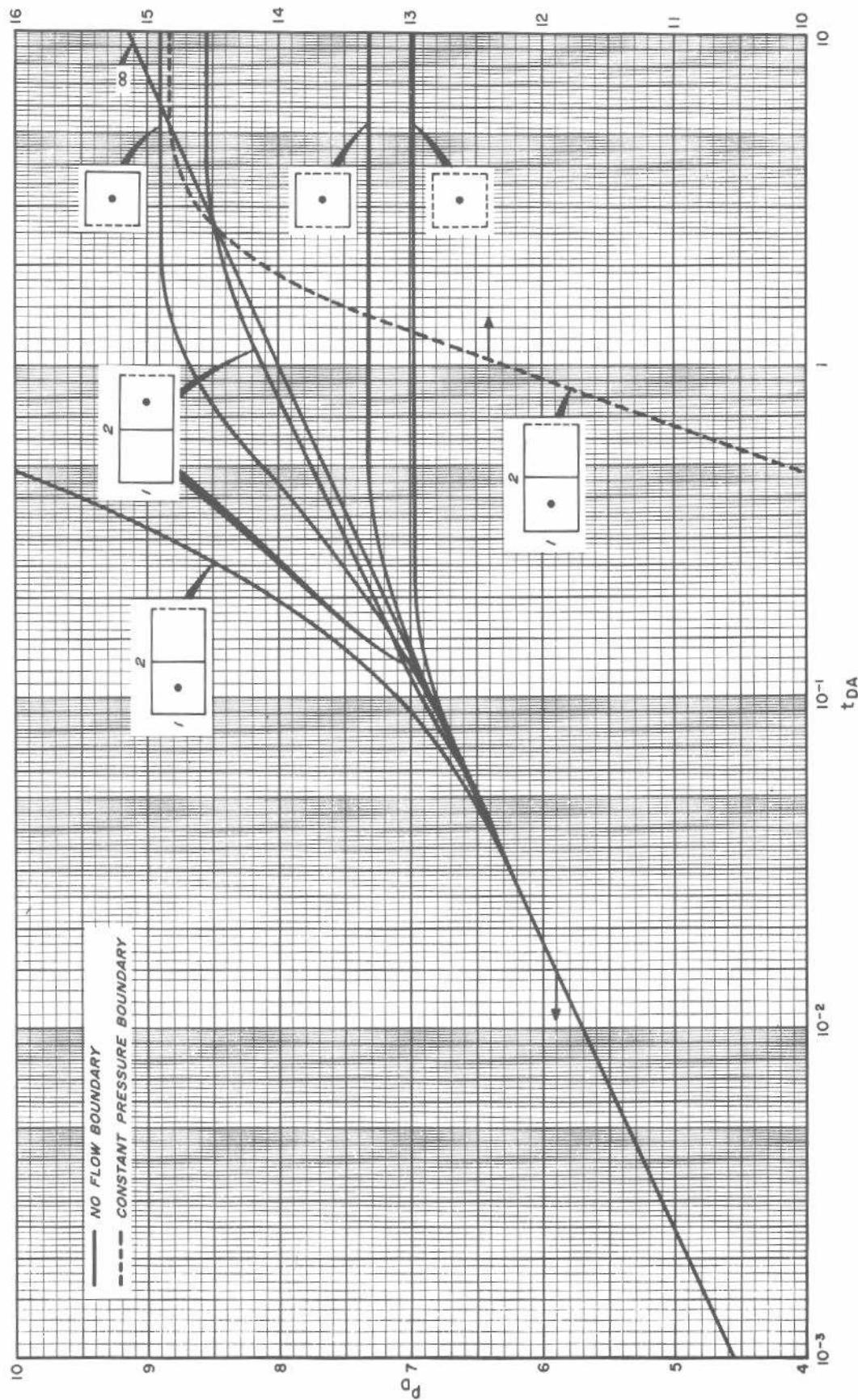


Fig. C.21 Dimensionless pressure for single wells in various rectangular shapes with one or more constant-pressure boundaries, no wellbore storage, no skin. Data of Ramey, Kumar, and Gulati.²²

the time required to reach it. They also tabulate the p_D data of Fig. C.21.

In Fig. C.21, some systems show closed-system effects (even to the point of exhibiting pseudosteady-state behavior) before the effect of the constant-pressure boundary is felt. That is particularly clear for the 2:1 rectangle with the well located three-fourths of the length from the constant-pressure boundary.

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Appendix D

Rock and Fluid Property Correlations

D.1 Introduction

This appendix provides information useful for estimating fluid and rock properties needed when analyzing transient-test pressure data. We believe that the correlations presented are among the most reliable presently available, although they are only a small sample of those available. The correlations may be used when necessary, but laboratory data measured on representative samples taken from the reservoir are always superior to general correlations and should be used whenever possible.

D.2 PVT Properties

This section presents correlations of pressure-volume-temperature (PVT) relations for reservoir fluids. The information can be used when laboratory data are not available. However, to ensure the best possible reservoir engineering and transient-test results, laboratory data should be obtained and used. It is both poor economics and poor engineering to resist obtaining good laboratory data simply because correlations are available.

Table D.1 gives physical properties of methane through decane and some other compounds commonly associated with petroleum reservoirs. More complete data are given in Ref. 1. Such information can be used to estimate some of the properties of hydrocarbon mixtures.

The pseudocritical temperature, T_{pc} , and pressure, p_{pc} , of a mixture are used in many correlations and equations in this appendix. If mixture composition is known, those quantities may be estimated from

$$T_{pc} = \sum_{i=1}^N y_i T_{ci}, \quad \dots \quad (D.1)$$

and

$$p_{pc} = \sum_{i=1}^N y_i p_{ci}, \quad \dots \quad (D.2)$$

where

N = number of components in the mixture

y_i = mole fraction of Component i

T_{ci} = critical temperature of Component i , °R

p_{ci} = critical pressure of Component i , psia.

If the system composition is not known, Figs. D.1 through D.3 may be used to estimate T_{pc} and p_{pc} . Fig. D.1 provides a way to estimate those quantities for undersaturated oil at reservoir pressure; the oil specific gravity corrected to 60 °F (the value normally reported) is used. If the API gravity is reported at other than 60 °F, it may be corrected to 60 °F

TABLE D.1—PHYSICAL PROPERTIES OF HYDROCARBONS AND ASSOCIATED COMPOUNDS.

Constituent	Molecular Weight	Normal Boiling Point		Liquid Density (lb _m /cu ft)	Gas Density at 60 °F, 1 atm (lb _m /cu ft)	Critical Temperature (°R)	Critical Pressure (psia)
		°F	°R				
Methane, CH ₄	16.04	-258.7	201	18.72*	0.04235	344	673
Ethane, C ₂ H ₆	30.07	-127.5	332	23.34*	0.07986	550	712
Propane, C ₃ H ₈	44.09	-43.8	416	31.68**	0.1180	666	617
iso-butane, C ₄ H ₁₀	58.12	10.9	471	35.14**	0.1577	735	528
n-butane, C ₄ H ₁₀	58.12	31.1	491	36.47**	0.1581	766	551
iso-pentane, C ₅ H ₁₂	72.15	82.1	542	38.99	—	830	483
n-pentane, C ₅ H ₁₂	72.15	96.9	557	39.39	—	847	485
n-hexane, C ₆ H ₁₄	86.17	155.7	615	41.43	—	914	435
n-heptane, C ₇ H ₁₆	100.20	209.2	669	42.94	—	972	397
n-octane, C ₈ H ₁₈	114.22	258.1	718	44.10	—	1,025	362
n-nonane, C ₉ H ₂₀	128.25	303.3	763	45.03	—	1,073	335
n-decane, C ₁₀ H ₂₂	142.28	345.2	805	45.81	—	1,115	313
Nitrogen, N ₂	28.02	-320.4	140	—	0.0739	227	492
Air (O ₂ + N ₂)	29	-317.7	142	—	0.0764	239	547
Carbon dioxide, CO ₂	44.01	-109.3	351	68.70	0.117	548	1,073
Hydrogen sulfide, H ₂ S	34.08	-76.5	383	87.73	0.0904	673	1,306
Water	18.02	212	672	62.40	—	1,365	3,206

*Apparent density in liquid phase.

**Density at saturation pressure.

using the technique described in Ref. 4. (In Ref. 4, Table 5 is used for hydrometer measurements at other than 60 °F; Table 7 allows correction of volume at a given temperature to volume at 60 °F.) Fig. D.2 applies to bubble-point liquids, again using the specific gravity corrected to 60 °F. The bubble-point pressure at 60 °F should be determined in the laboratory.* Fig. D.3 applies to condensate well fluids and natural gases; knowledge of the gas gravity is required to use Fig. D.3.

T_{pc} and p_{pc} are normally used to estimate the

*If only the value at reservoir temperature is known, Fig. D.5 may be used to estimate the 60 °F value by going vertically upward from the bubble-point pressure to reservoir temperature, horizontally left to 60 °F, and vertically downward to the estimated bubble-point pressure.

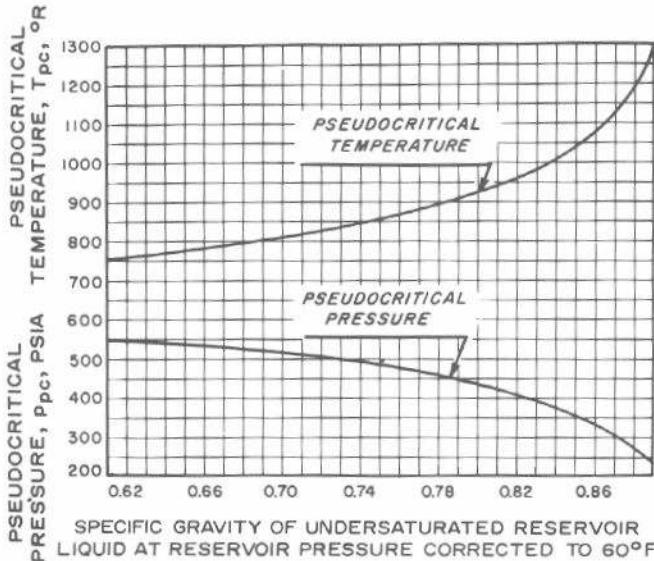


Fig. D.1 Approximate correlation of liquid pseudocritical pressure and temperature with specific gravity. After Trube.²

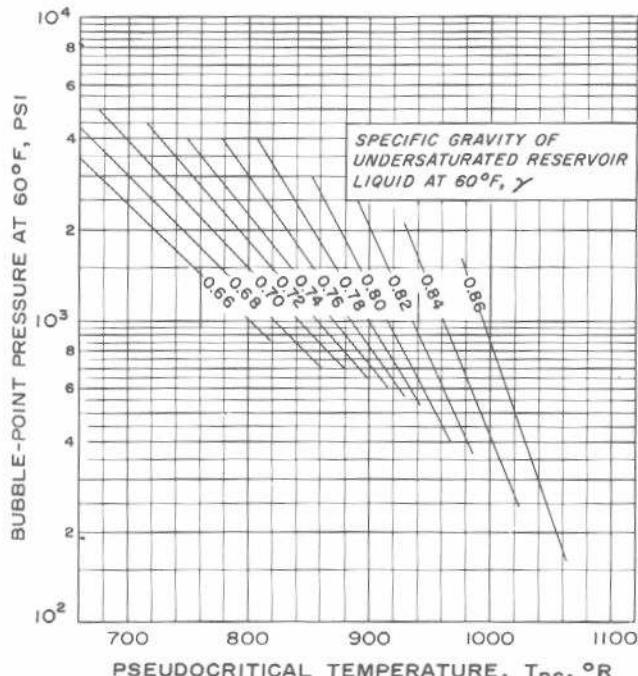


Fig. D.2 Correlation of liquid pseudocritical temperature with specific gravity and bubble point. After Trube.²

pseudoreduced temperature and pressure:

$$T_{pr} = \frac{T}{T_{pc}} , \dots \dots \dots \quad (D.3)$$

and

$$p_{pr} = \frac{p}{p_{pc}} , \dots \dots \dots \quad (D.4)$$

where

T = temperature of interest, °R

p = pressure of interest, psia.

Note in Eqs. D.1 through D.4 that temperature must be absolute temperature and pressure must be absolute pressure.

Since many correlations in this appendix use specific gravity or API gravity, it is worthwhile to restate the relationship between those two quantities:

$$^{\circ}\text{API} = \frac{141.5 - 131.5}{\gamma} . \dots \dots \dots \quad (D.5)$$

In Eq. D.5, the specific gravity, γ , must be corrected to 60 °F and atmospheric pressure.

Figs. D.4 through D.6 are Standing's⁵ correlations for properties of mixtures of hydrocarbon gases and liquids. Examples of their use are shown in the figures. Standing's correlations are based mainly on the properties of California crude oils. Cronquist⁶ gives correlations that may be useful for Gulf Coast oils.

Fig. D.7 is the well known chart of real gas deviation factor for natural gases. Pseudoreduced properties may be estimated from Eqs. D.1 through D.4.

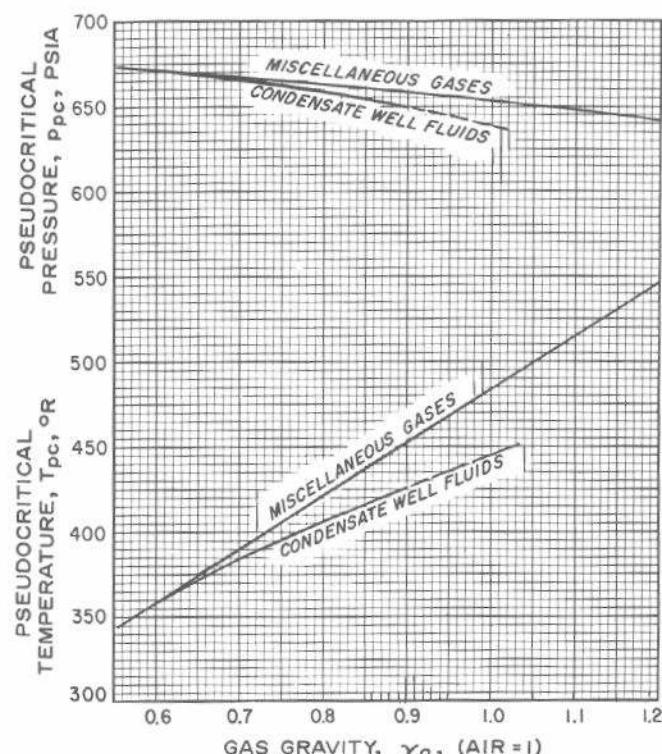


Fig. D.3 Correlation of pseudocritical properties of condensate well fluids and miscellaneous natural gas with fluid gravity.

After Brown *et al.*³

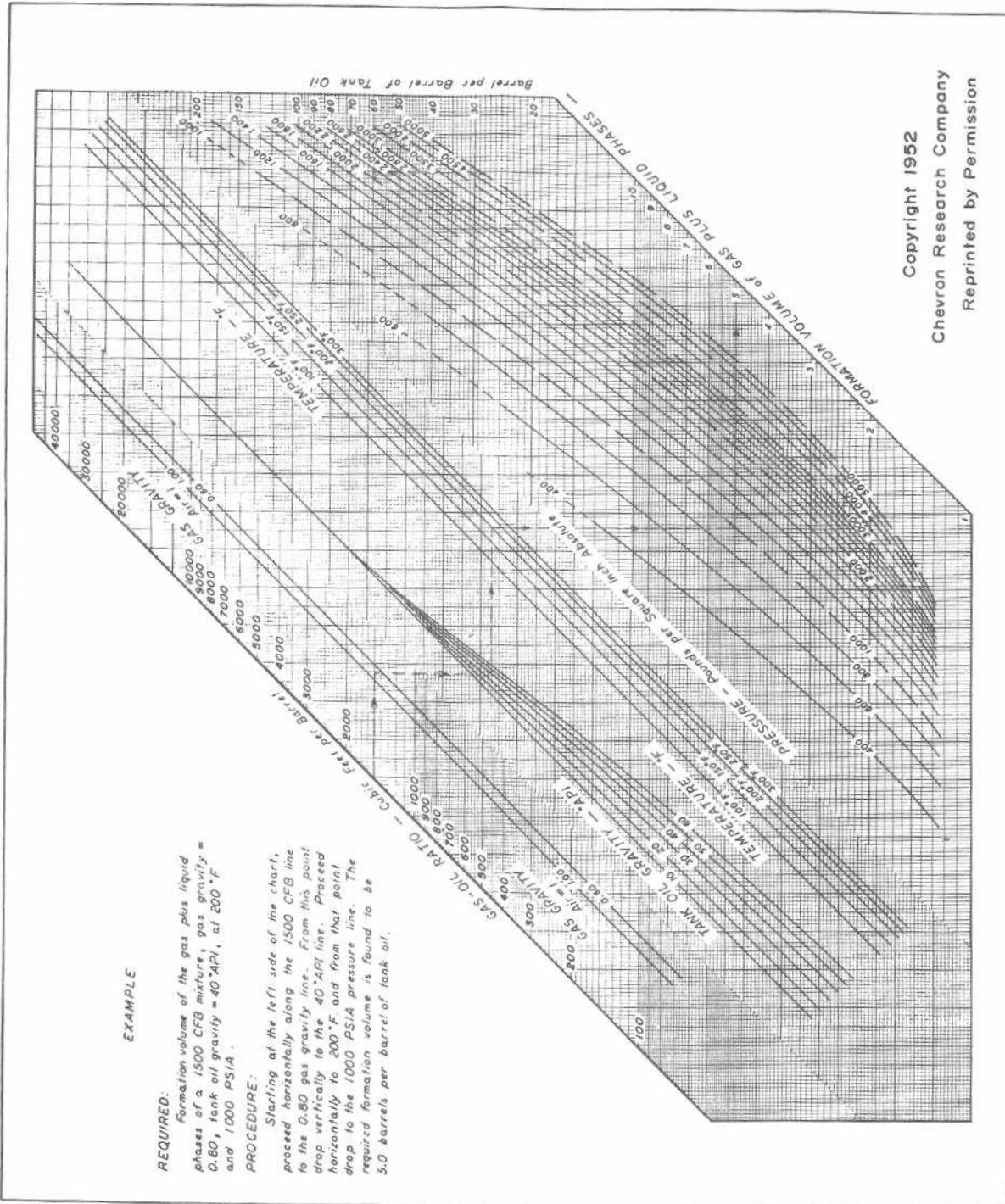
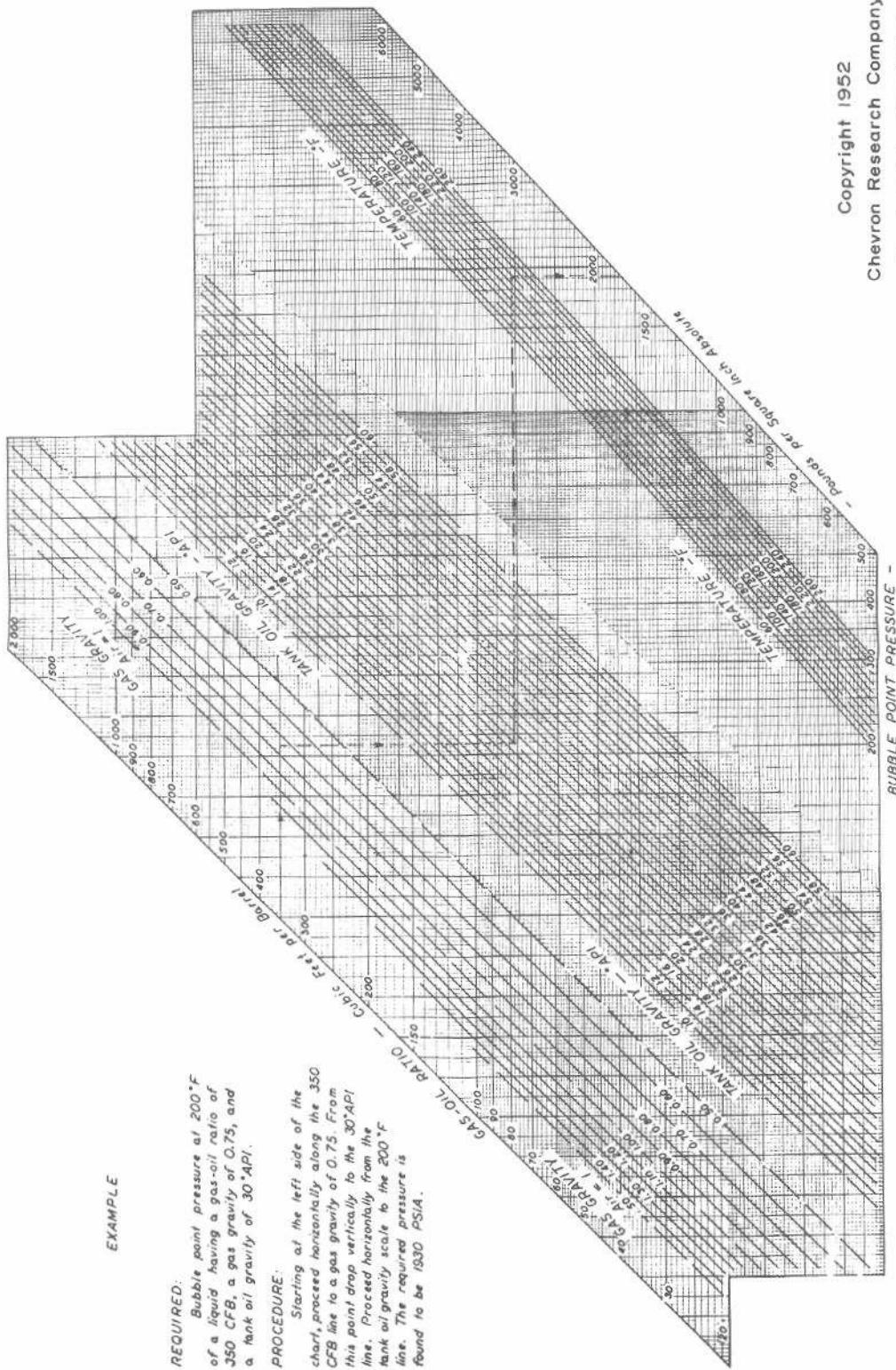


Fig. D.4 Properties of natural mixtures of hydrocarbon gas and liquids, formation volume of gas plus liquid phase. After Standing.⁵



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Fig. D.5 Properties of natural mixtures of hydrocarbon gas and liquids, bubble-point pressure. After Standing.⁵

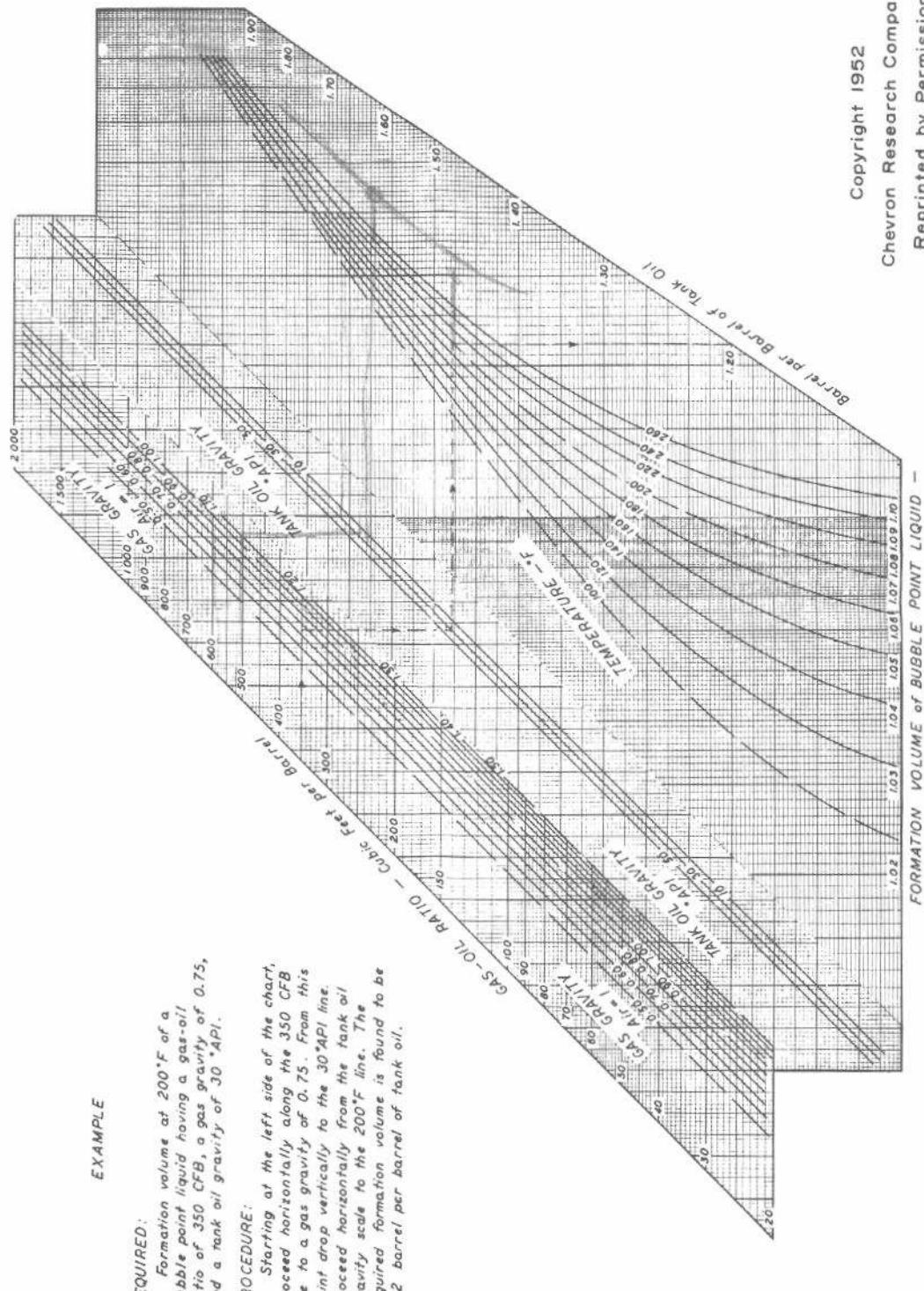


Fig. D.6 Properties of natural mixtures of hydrocarbon gas and liquids, formation volume of bubble-point liquids. After Standing.³

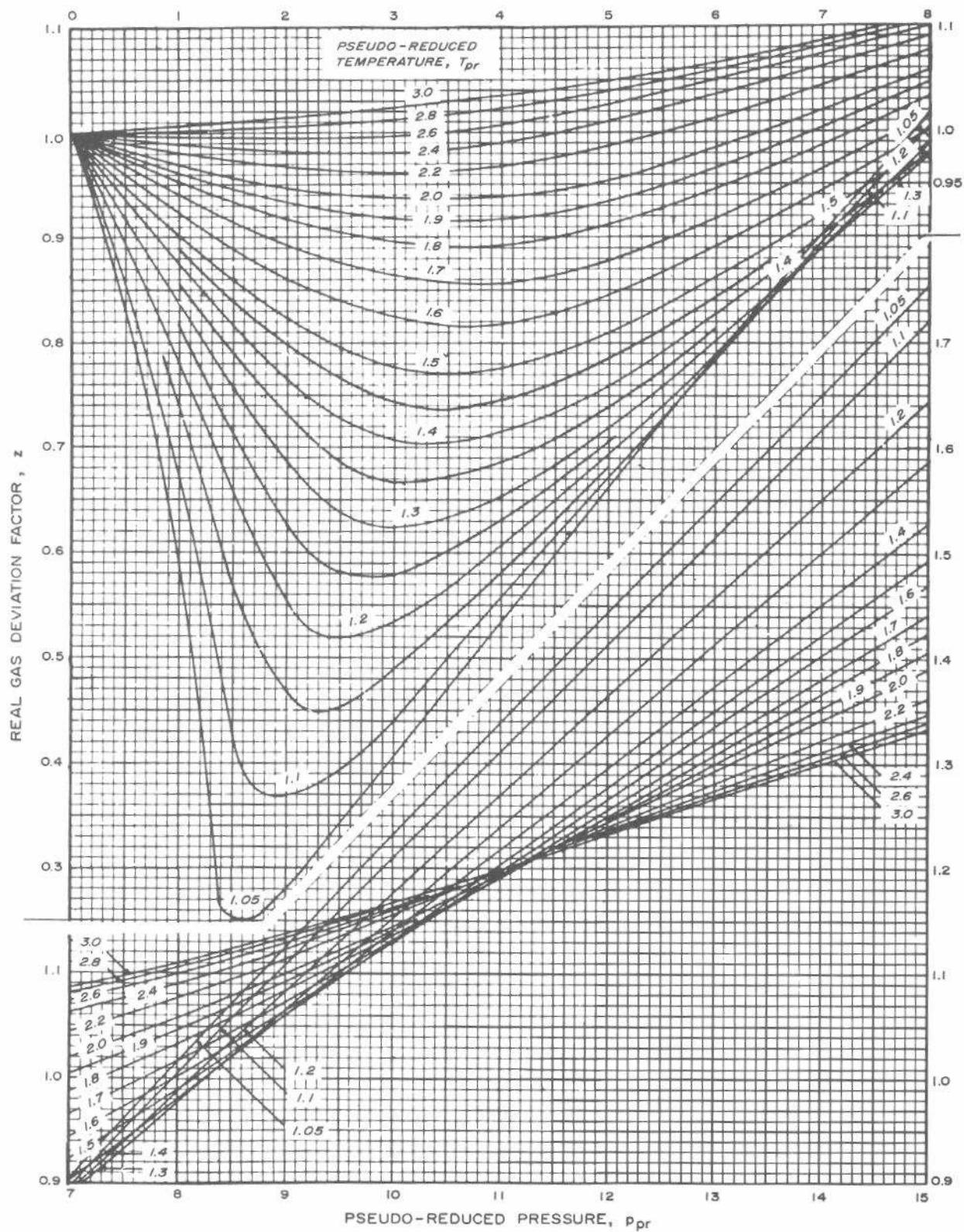


Fig. D.7 Real gas deviation factor for natural gases as a function of pseudoreduced pressure and temperature. After Standing and Katz.⁷

The gas formation volume factor may be estimated from

$$B_g = 5.039 \times 10^{-3} \frac{zT}{p}, \quad \dots \quad (\text{D.6})$$

where z is from Fig. D.7.

The water formation volume factor, B_w , may be estimated from Fig. D.8.

D.3 Rock Pore-Volume Compressibility

The isothermal formation (rock, pore volume) compressibility is generally defined as

$$c_f = \frac{1}{V_p} \left(\frac{\partial V_p}{\partial p} \right)_T \quad \dots \quad (\text{D.7})$$

The subscript T indicates that the partial derivative is taken at constant temperature. All compressibilities used in this monograph are isothermal compressibilities; the subscript is frequently omitted. Formation compressibility is defined so that it is a positive number. Thus, Eq. D.7 indicates that as fluid pressure decreases, the pore volume decreases. That occurs because the confining lithostatic pressure is essentially constant while the reservoir is depleted, thus causing compression of the rock.

Several authors⁹⁻¹¹ have attempted to correlate formation compressibility with various physical parameters. The correlations of Hall¹⁰ and van der Knaap¹¹ have been used extensively in the petroleum literature. Recently, Newman⁹ has shown that those correlations do not apply to a very wide

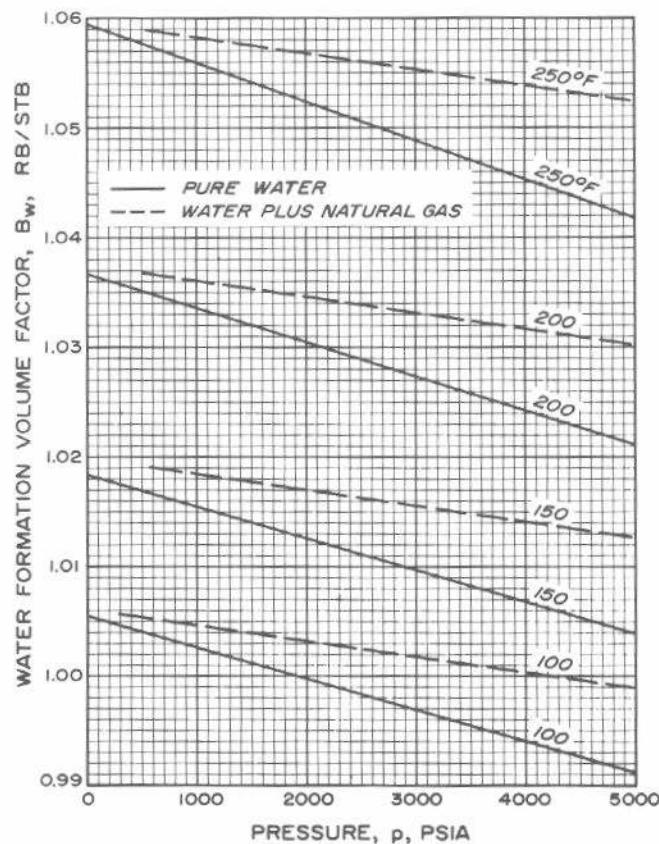


Fig. D.8 Formation volume factor of pure water and a mixture of natural gas and water. Data of Dodson and Standing.⁸

range of reservoir rocks. Fig. D.9 shows data for compressibility of limestone samples superimposed on both van der Knaap's and Hall's limestone correlations. In Fig. D.9, and in other figures in this section, the lithostatic pressure is defined as the pressure obtained by multiplying reservoir depth by 1 psi/ft.

Figs. D.10 through D.12 compare Newman's and other data with Hall's sandstone correlation. In preparing the three figures, Newman used the following definitions:⁹

1. Consolidated samples consisted of hard rocks (thin edges could not be broken off by hand).
2. Friable samples could be cut into cylinders but the edges could be broken off by hand.
3. Unconsolidated samples could fall apart under their own weight unless they had undergone special treatment, such as freezing.

As can be seen in the three figures, no correlation would provide a good description of the large suite of samples studied. It is apparent from Fig. D.11 that there is no correlation at all for the friable samples. Fig. D.12 indicates that if there is any correlation for unconsolidated samples, the trend may be opposite the trend for consolidated samples (Fig. D.10).

Unfortunately, Figs. D.9 through D.12 lead to only one conclusion: formation compressibility should be measured for the reservoir being studied. At best, correlations can be expected to give only order-of-magnitude estimates.

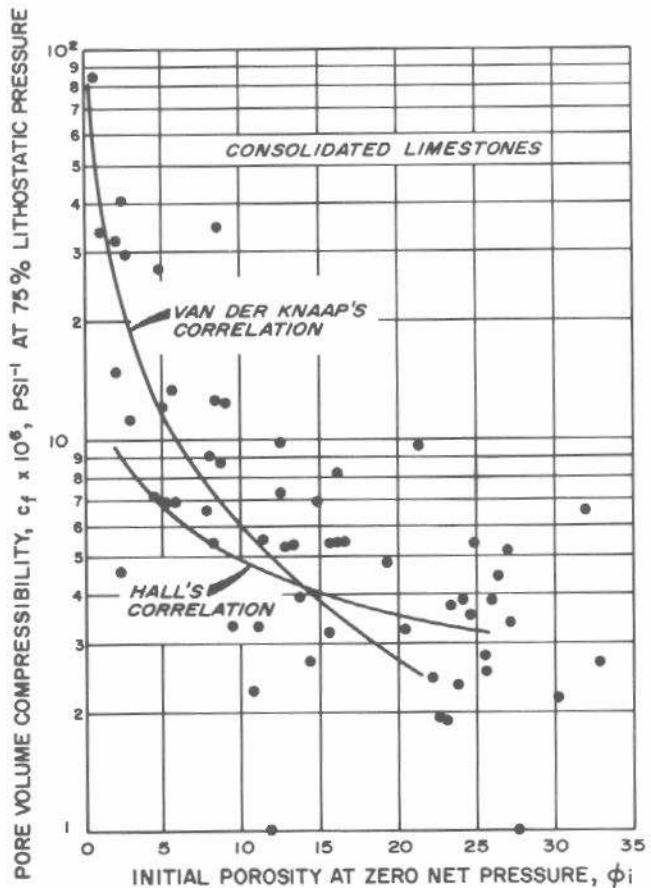


Fig. D.9 Pore-volume compressibility at 75-percent lithostatic pressure vs initial sample porosity for limestones. After Newman.⁹

D.4 Oil Compressibility

The isothermal compressibility of an undersaturated oil (oil above the bubble point) is defined as

$$c_o = -\frac{1}{V_o} \left(\frac{\partial V_o}{\partial p} \right)_T = \frac{1}{\rho_o} \left(\frac{\partial \rho_o}{\partial p} \right)_T = -\frac{1}{B_o} \left(\frac{\partial B_o}{\partial p} \right)_T. \quad (\text{D.8})$$

Since the volume of an undersaturated liquid decreases as the pressure increases, c_o is positive.

Generally, oil compressibility should be computed from laboratory PVT data for the oil existing in the reservoir being studied. The final equality in Eq. D.8 is useful for calculating c_o from such data. In some reservoirs, c_o is essentially constant above the bubble point, while in others it varies with pressure.

If laboratory data are not available, Trube's² correlation for compressibility of an undersaturated oil (Fig. D.13) may be used. It is necessary to estimate T_{pr} and p_{pr} from Fig. D.1 or Fig. D.2. The pseudoreduced compressibility, c_{pr} , is read from Fig. D.13 and the oil compressibility is estimated from

$$c_o = \frac{c_{pr}}{p_{pc}}. \quad (\text{D.9})$$

Below the bubble point, dissolved gas must be considered in computing compressibilities used in transient test and reservoir analysis. Thus, we define an *apparent* oil com-

pressibility for the region below the bubble point where oil volume increases with pressure as a result of gas going into solution:

$$c_{oa} = -\frac{1}{B_o} \frac{\partial B_o}{\partial p} + \frac{B_g}{B_o} \frac{\partial R_s}{\partial p}. \quad (\text{D.10})$$

Note that Eq. D.10 reduces to Eq. D.8 above the bubble point when R_s is constant with pressure. If available, laboratory data should be used to estimate c_{oa} ; otherwise, correlations may be used with caution. When using correlations, the $\partial R_s/\partial p$ term in Eq. D.10 may be estimated from Fig. D.14 or from

$$\frac{\partial R_s}{\partial p} \approx \frac{R_s}{(0.83p + 21.75)}. \quad (\text{D.11})$$

Eq. D.11 and Fig. D.14 are from Ramey¹² and are based on Standing's data.⁵ The gas formation volume factor may be estimated from Eq. D.6, where the z factor is estimated from Fig. D.7. The term $\partial B_o/\partial p$ in Eq. D.10 may be estimated from

$$\frac{\partial B_o}{\partial p} \approx \frac{\partial R_s}{\partial p} \cdot \frac{\partial B_o}{\partial R_s}, \quad (\text{D.12})$$

where the first term on the right-hand side is from Eq. D.11 or Fig. D.14 and the second term on the right-hand side is from Fig. D.15. Oil and gas gravities must be known to use

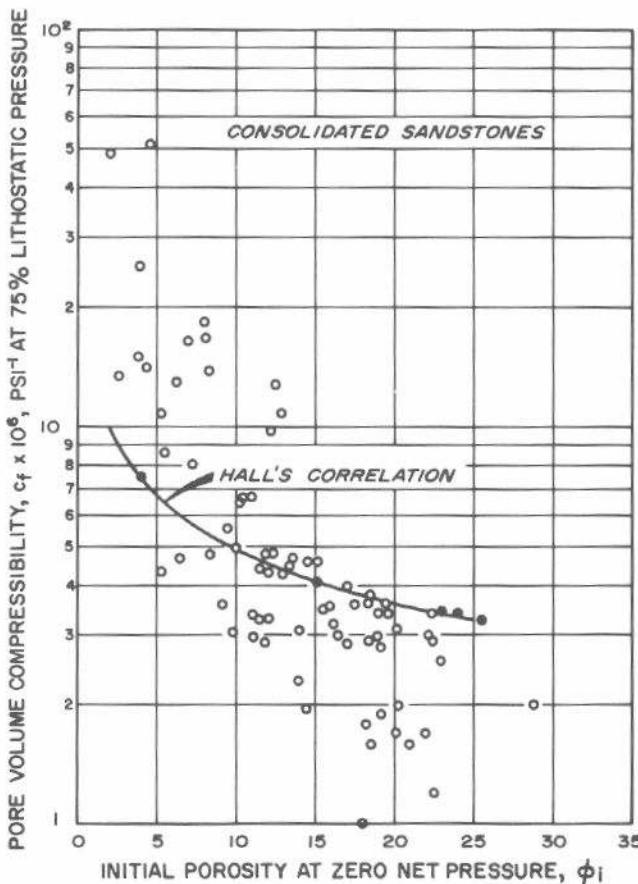


Fig. D.10 Pore-volume compressibility at 75-percent lithostatic pressure vs initial sample porosity for consolidated sandstones. After Newman.⁹

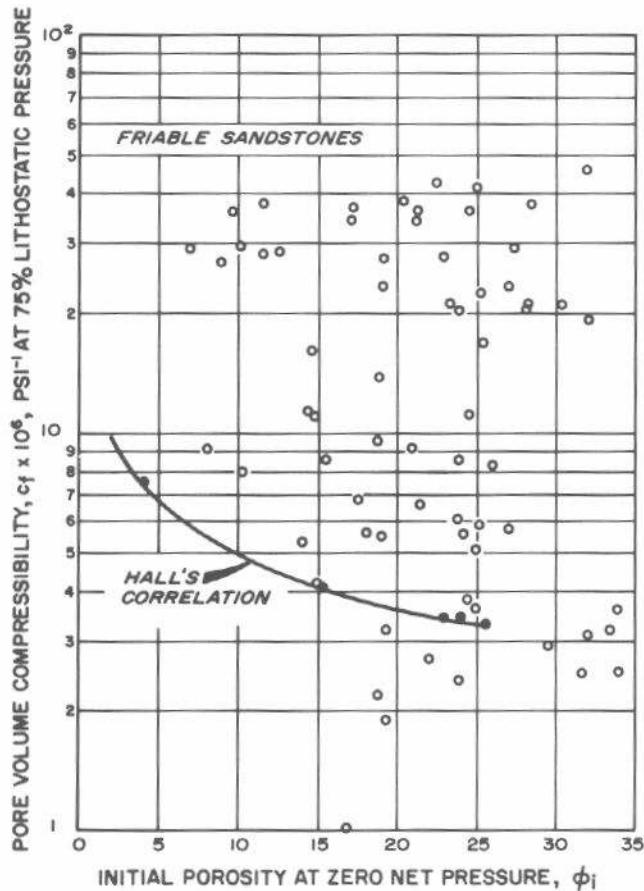


Fig. D.11 Pore-volume compressibility at 75-percent lithostatic pressure vs initial sample porosity for friable sandstones. After Newman.⁹

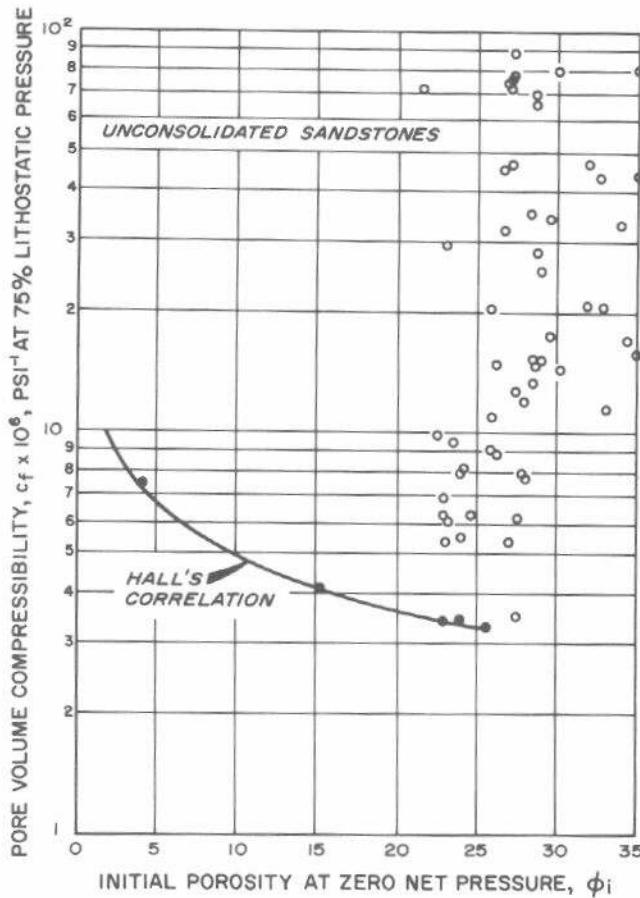


Fig. D.12 Pore-volume compressibility at 75-percent lithostatic pressure vs initial sample porosity for unconsolidated sandstones.
After Newman.⁹

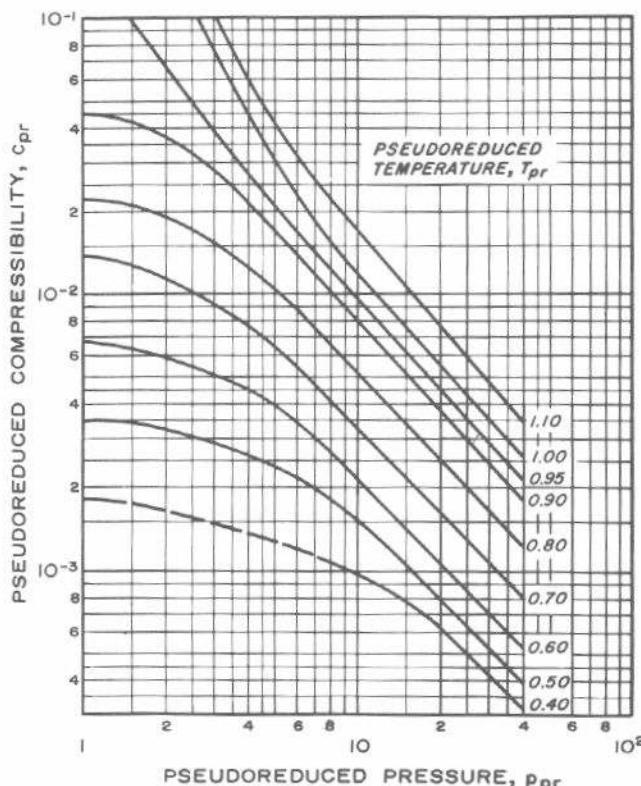


Fig. D.13 Correlation of pseudoreduced compressibility for an undersaturated oil. After Trube.²

Fig. D.15. The oil formation volume factor, B_o , may be estimated from Standing's correlation (Fig. D.6).

D.5 Water Compressibility

The water compressibility is defined analogously to the oil compressibility (Eq. D.8). The compressibility of water or brine *without* any solution gas is estimated from Figs. D.16 through D.19. Linear interpolation may be used for intermediate pressures and salinities.

To estimate the compressibility of undersaturated water or brine (that is, with solution gas), Long and Chierici¹³ recommend using

$$c_w = (c_w)_{0,n} [1 + 0.0088 \times 10^{-Kn}(R_{sw})], \dots \quad (\text{D.13})$$

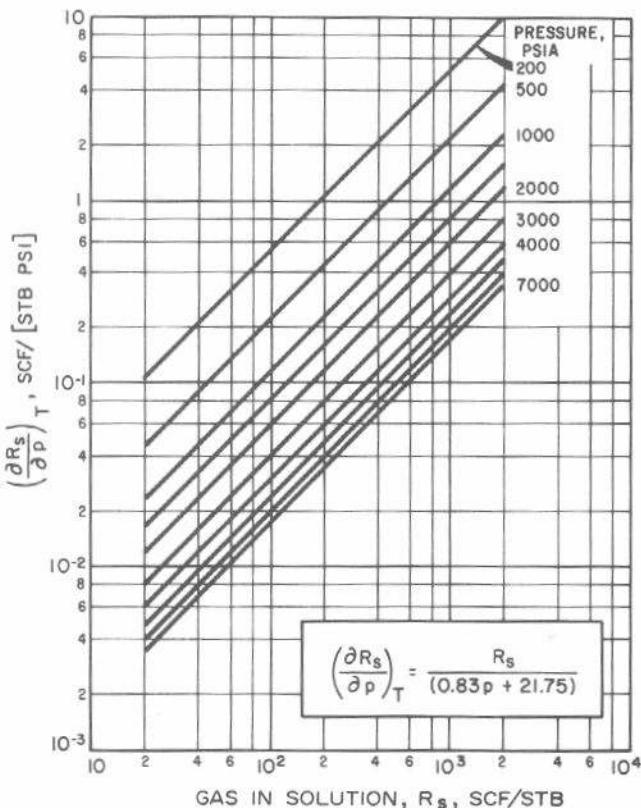


Fig. D.14 Change of gas in solution in oil with pressure vs gas in solution. After Ramey,¹² data of Standing.⁵

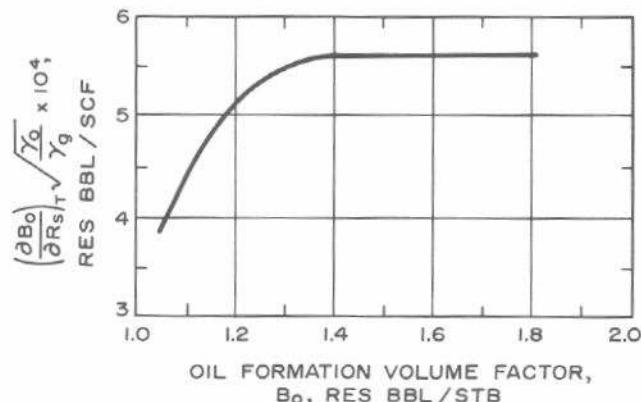


Fig. D.15 Change of oil formation volume factor with gas in solution vs oil formation volume factor. After Ramey.¹²

where

- c_w = compressibility of an undersaturated brine containing solution gas and n gram-equivalents of dissolved solids, psi^{-1}
- $(c_w)_{0,n}$ = compressibility of a *gas-free* brine containing n gram-equivalents of dissolved solids, psi^{-1} , from Figs. D.16 through D.19

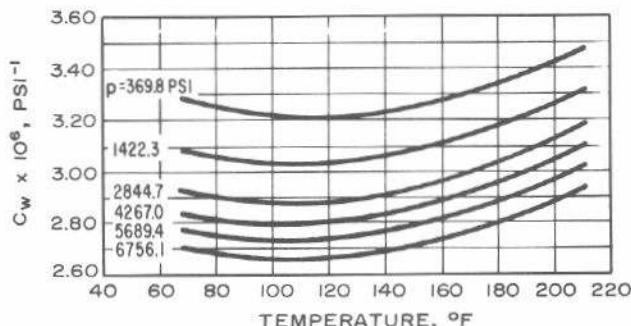


Fig. D.16 Average compressibility of distilled water. After Long and Chierici.¹³

$$n = \text{dissolved solids (ppm)} \div 58,443,$$

concentration of dissolved solids,
gram-equivalents/liter

K = Secenov's coefficient, obtained at reservoir temperature from Fig. D.20

R_{sw} = gas solubility in distilled water at the required pressure and temperature, from Fig. D.21, scf/bbl.

An alternate approach to estimating the compressibility of undersaturated water is to use Fig. D.22 to estimate water compressibility at reservoir temperature, pressure, and solution gas-oil ratio. Fig. D.23 is used to estimate the solution gas-water ratio as a function of temperature, pressure, and salinity.

The apparent compressibility of water below the bubble point is given by

$$c_{wa} = -\frac{1}{B_w} \frac{\partial B_w}{\partial p} + \frac{B_g}{B_w} \frac{\partial R_{sw}}{\partial p}. \quad \dots \dots \dots \quad (\text{D.14})$$

Again, it is best to compute c_{wa} from PVT analyses if they are available. However, since they seldom are, the use of

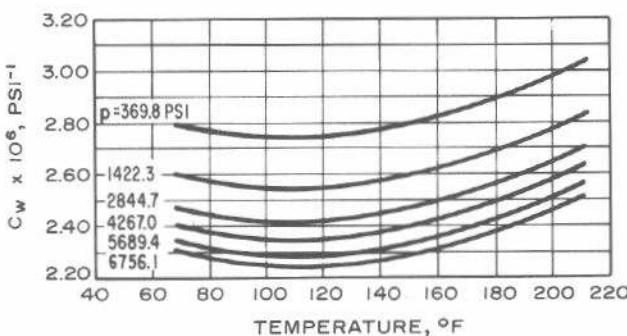


Fig. D.17 Average compressibility of 100,000-ppm NaCl in distilled water. After Long and Chierici.¹³

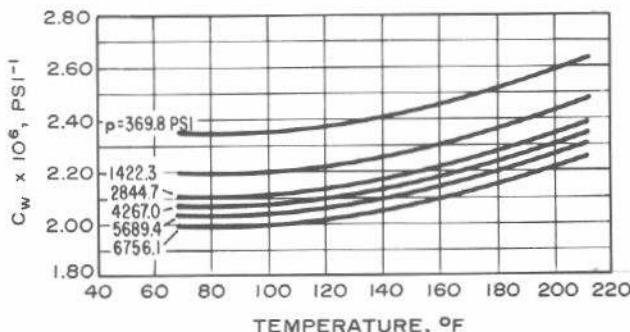


Fig. D.18 Average compressibility of 200,000-ppm NaCl in distilled water. After Long and Chierici.¹³

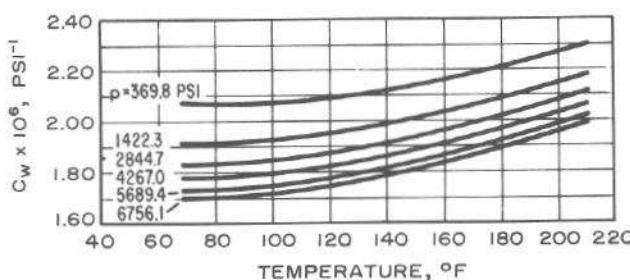


Fig. D.19 Average compressibility of 300,000-ppm NaCl in distilled water. After Long and Chierici.¹³

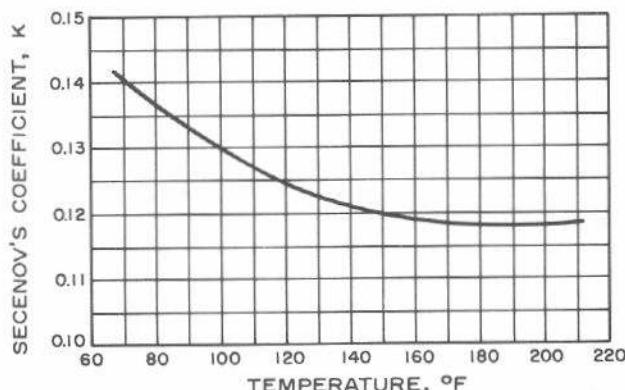


Fig. D.20 Secenov's coefficient for methane, for Eq. D.13. After Long and Chierici.¹³

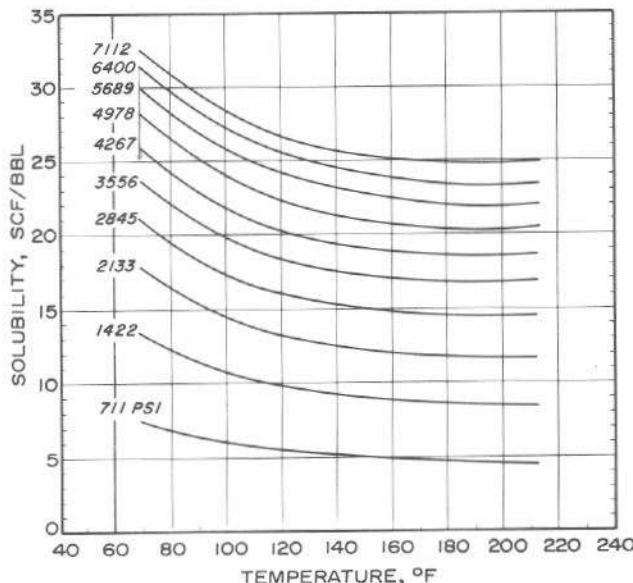


Fig. D.21 Solubility of methane in distilled water. After Long and Chierici.¹³

correlations is often required. The $\partial R_{sw}/\partial p$ term may be approximated from Fig. D.24, while B_w may be approximated from Fig. D.8. B_g is estimated with Eq. D.6. The first term on the right-hand side of Eq. D.14 must be estimated from Fig. D.22 or Eq. D.13.

D.6 Gas Compressibility

Isothermal gas compressibility is defined analogously to the oil compressibility (Eq. D.8). The gas equivalent of Eq. D.8 may be written using the real gas deviation factor, z :

$$c_g = \frac{1}{p} - \frac{1}{z} \left(\frac{\partial z}{\partial p} \right)_T \quad \dots \dots \dots \quad (\text{D.15a})$$

If pseudoreduced pressures and temperatures are introduced into Eq. D.15a, the isothermal gas compressibility may be written as

$$c_g = \frac{1}{p_{pc}} \left[\frac{1}{p_{pr}} - \frac{1}{z} \left(\frac{\partial z}{\partial p_{pr}} \right)_{T_{pr}} \right] \quad \dots \dots \dots \quad (\text{D.15b})$$

The z -factor chart, Fig. D.7, may be used directly to estimate the derivative term for Eq. D.15b.

Gas compressibility also may be estimated from the pseudoreduced-compressibility correlation shown in Figs. D.25 and D.26. The pseudoreduced compressibility is read

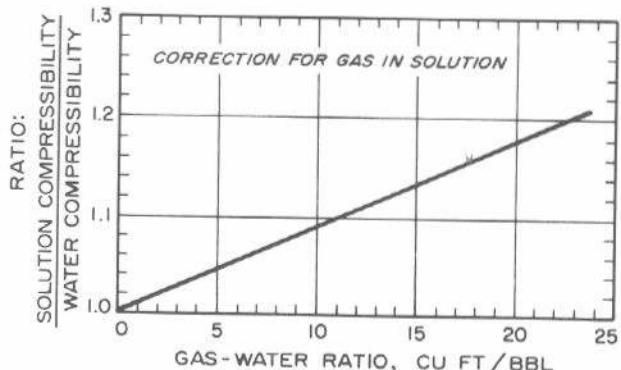
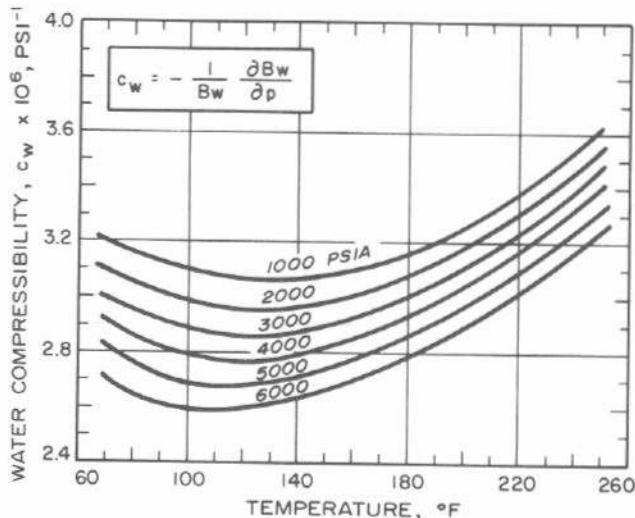


Fig. D.22 Effect of dissolved gas on water compressibility. After Dodson and Standing.⁸

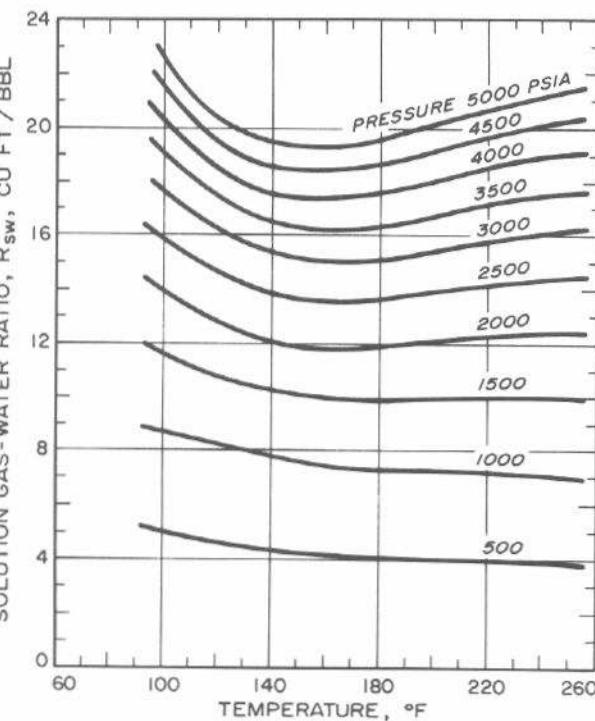


Fig. D.23 Solubility of natural gas in water. After Dodson and Standing.⁸

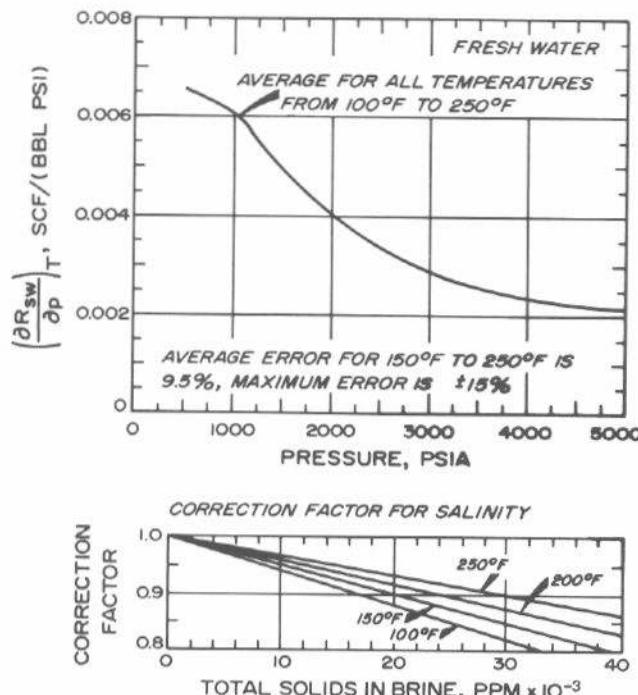


Fig. D.24 Change of natural gas in solution in formation water with pressure vs pressure. Multiply $(\partial R_{sw}/\partial p)_T$ by the correction factor to get result for brine. After Ramey,¹² data of Standing.⁵

from one of those figures and the gas compressibility is computed from

$$c_g = \frac{c_{pr}}{p_{pc}} \quad \dots \dots \dots \quad (\text{D.16})$$

D.7 Gas Viscosity

Fig. D.27 is one of the simplest hydrocarbon gas viscosity correlations available.¹ That figure gives gas viscosity as a function of gas gravity, pressure, and temperature. Its use is illustrated by the arrows. For a 0.7-gravity gas at 750 psia and 220 °F, viscosity is 0.0158 cp.

Carr, Kobayashi, and Burrows¹⁵ present a method for estimating natural gas viscosity that is widely used. That method requires knowledge of the gas composition and of the viscosity of each component at atmospheric pressure and reservoir temperature. The viscosity of the mixture at atmospheric pressure is estimated from

$$\mu_{ga} = \frac{\sum_{i=1}^N y_i \mu_i \sqrt{M_i}}{\sum_{i=1}^N y_i \sqrt{M_i}}, \quad \dots \dots \dots \quad (\text{D.17})$$

where

μ_{ga} = viscosity of the gas mixture at the desired temperature and *atmospheric* pressure, cp

y_i = mole fraction of the *i*th component

μ_i = viscosity of *i*th component at the desired temperature and *atmospheric* pressure, obtained from Fig. D.28

M_i = molecular weight of *i*th component (Table D.1)

N = number of components in the gas.

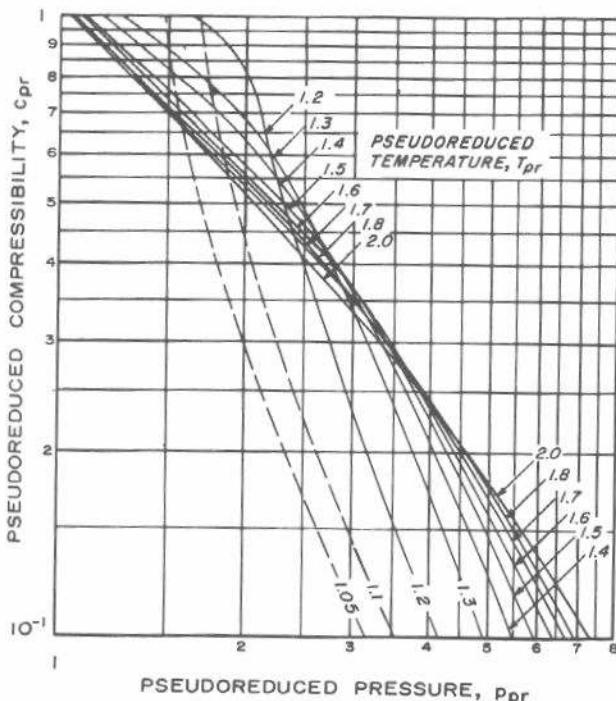


Fig. D.25 Correlation of pseudoreduced compressibility for natural gases. After Trube.¹⁴

Viscosity for many gaseous components is shown in Fig. D.28 at 1 atm and various temperatures. If the gas composition is not known, Fig. D.29 may be used with gas molecular weight to estimate the gas viscosity at reservoir temperature and atmospheric pressure. Molecular weight is related to gas gravity by

$$M \approx 29\gamma. \quad \dots \dots \dots \quad (\text{D.18})$$

The gas viscosity at reservoir pressure is estimated by determining the ratio μ_g/μ_{ga} at the appropriate temperature and pressure from Fig. D.30 or Fig. D.31. Then, that ratio is applied to μ_{ga} computed from Eq. D.17 or Fig. D.29. The pseudoreduced temperatures and pressures for use in Figs. D.30 and D.31 are estimated from Eqs. D.1 through D.4 or from Fig. D.3.

D.8 Oil Viscosity

Whenever possible, oil viscosity should be determined by laboratory measurements at reservoir temperature and pressure. Oil viscosity is usually reported in standard PVT analyses. If such laboratory data are not available, the Chew and Connally¹⁶ correlation for viscosity of gas-saturated oil, Fig. D.32, may be used. Both solution gas-oil ratio and oil viscosity at reservoir temperature and atmospheric pressure must be known to use Fig. D.32. If the dead oil viscosity is not determined from laboratory data, it may be estimated from Fig. D.33.

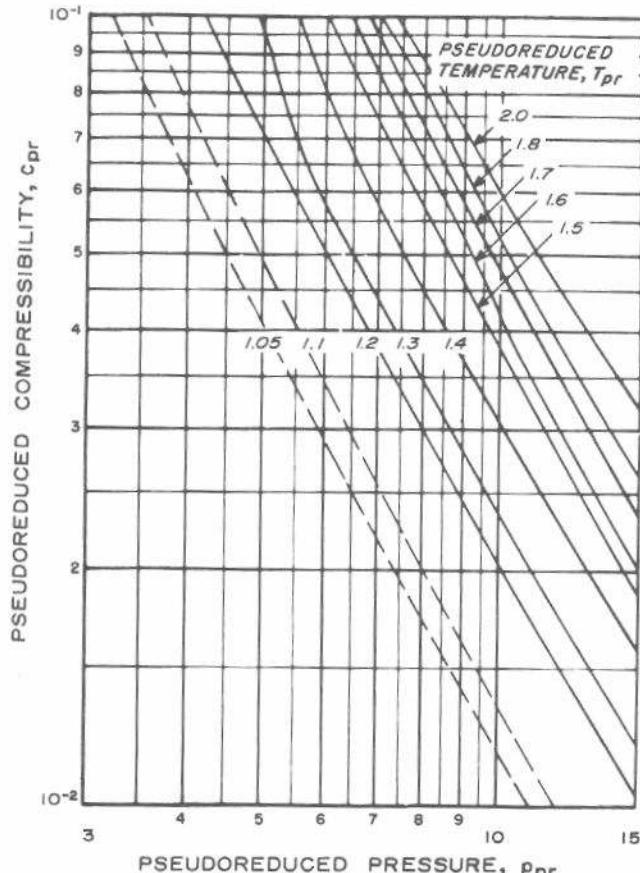


Fig. D.26 Correlation of pseudoreduced compressibility for natural gases. After Trube.¹⁴

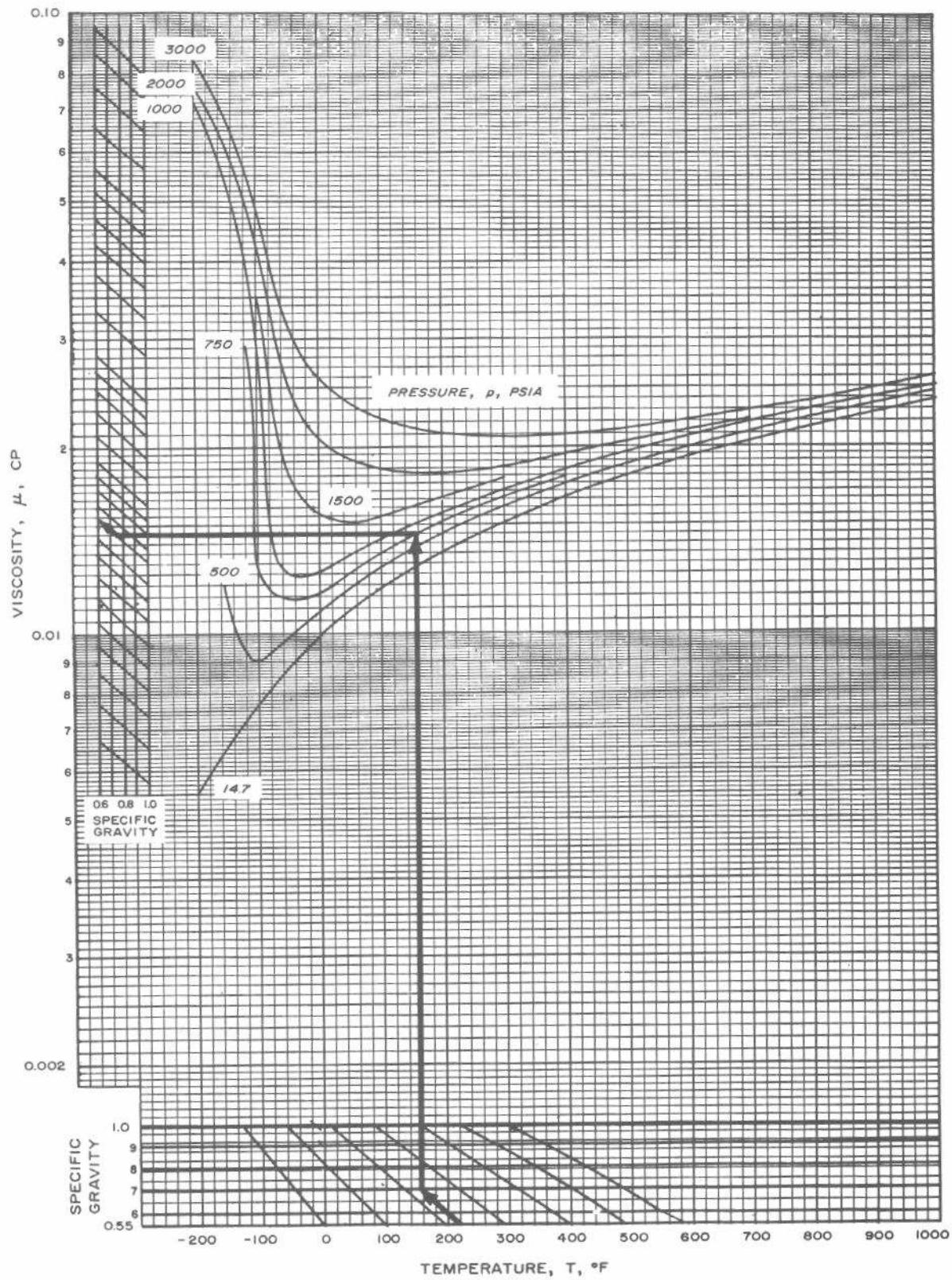


Fig. D.27 Viscosity of hydrocarbon gases.¹

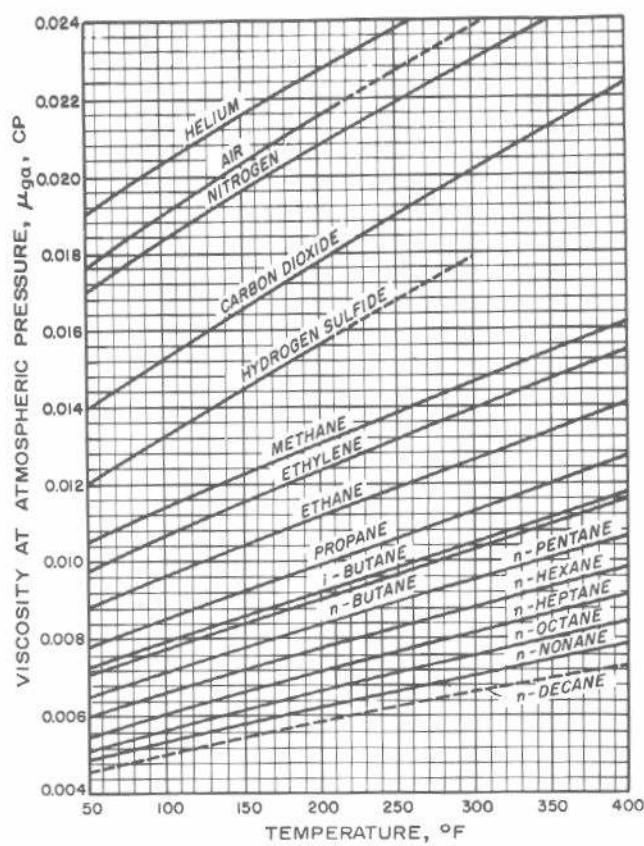


Fig. D.28 Viscosity of pure hydrocarbon gases at 1 atm. After Carr, Kobayashi, and Burrows.¹⁵

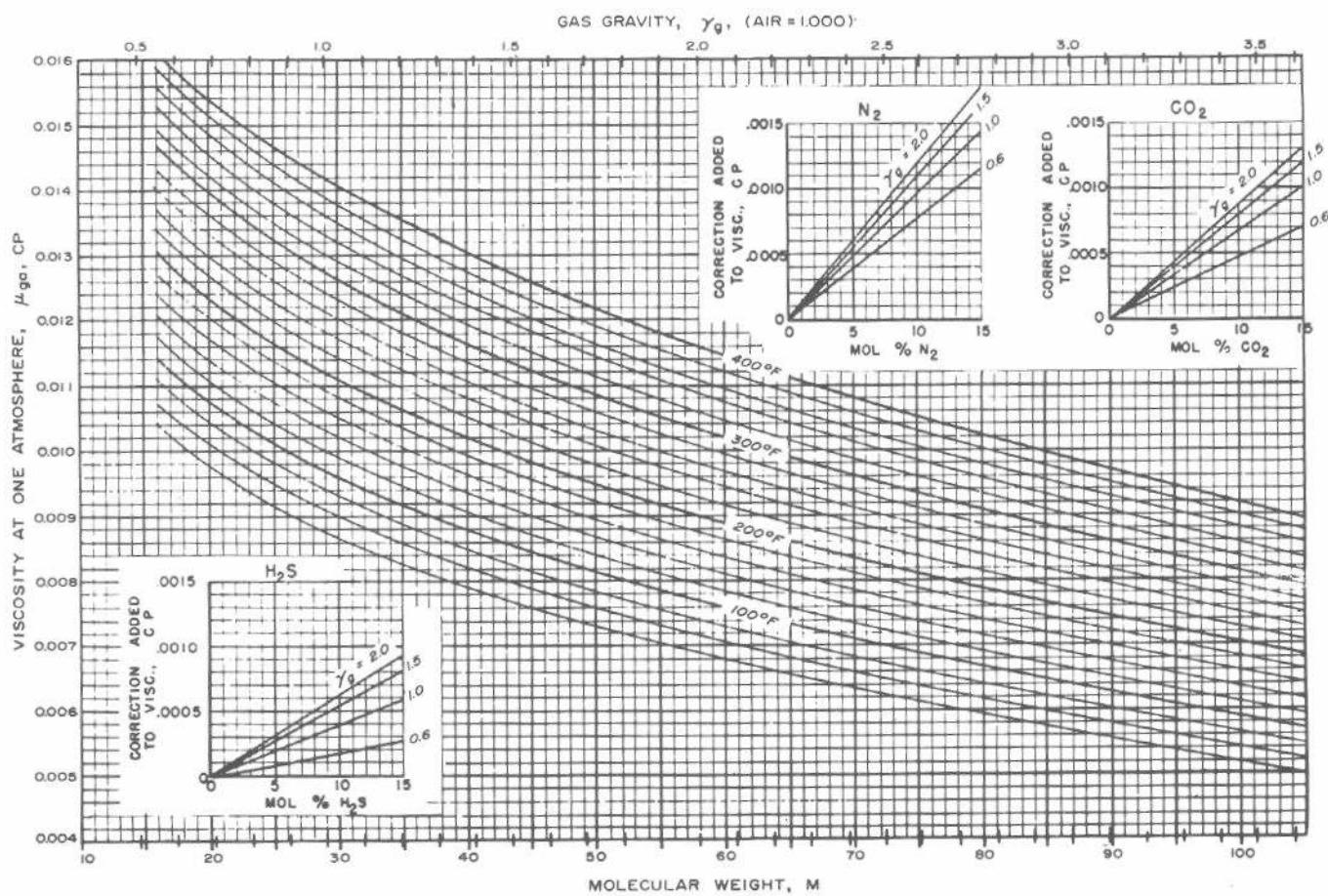


Fig. D.29 Viscosity of natural gases at 1 atm. After Carr, Kobayashi, and Burrows.¹⁵

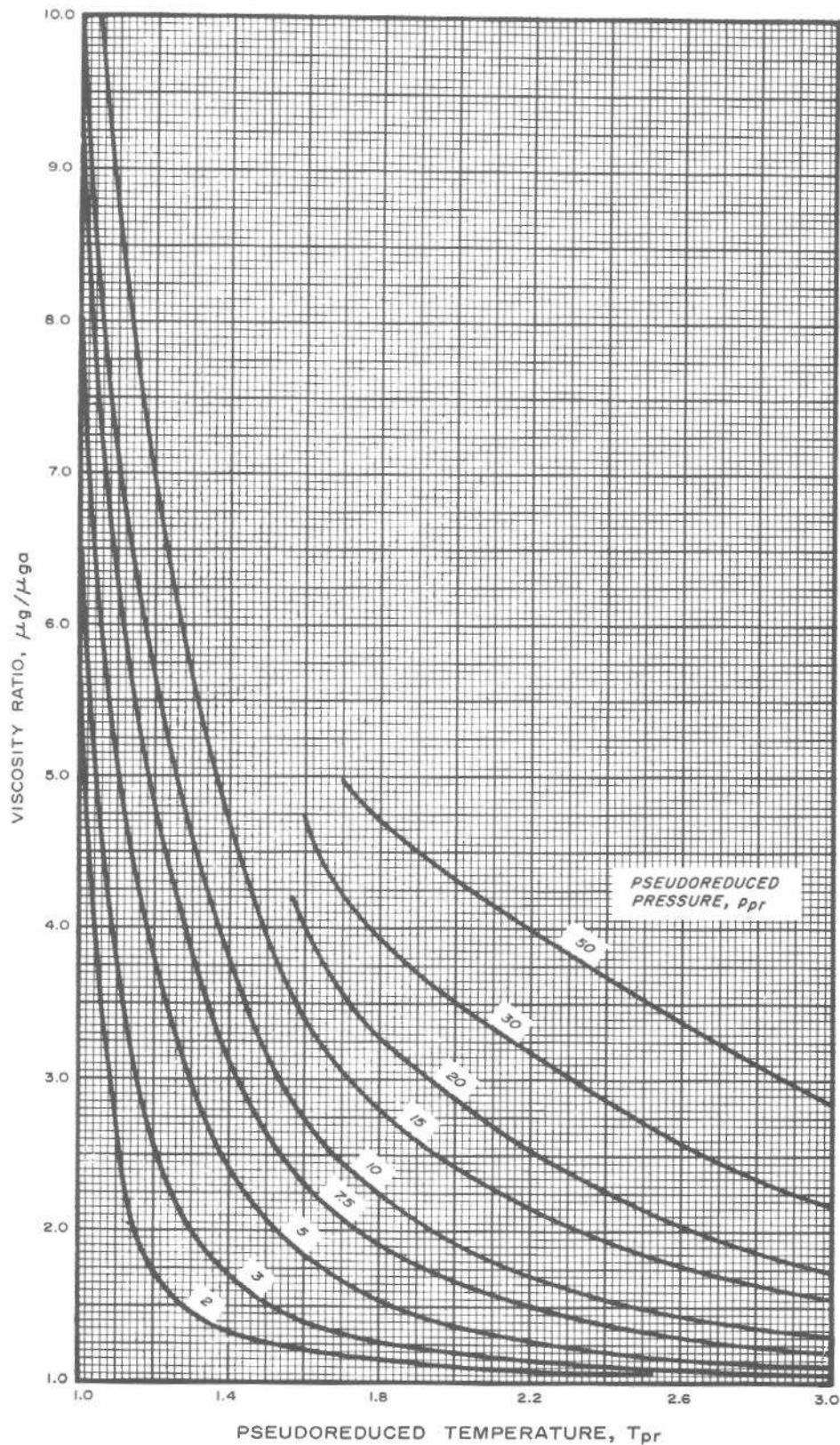


Fig. D.30 Effect of temperature and pressure on gas viscosity; μ_{ga} is estimated from Eq. D.17 or Fig. D.29. After Carr, Kobayashi, and Burrows.¹⁵

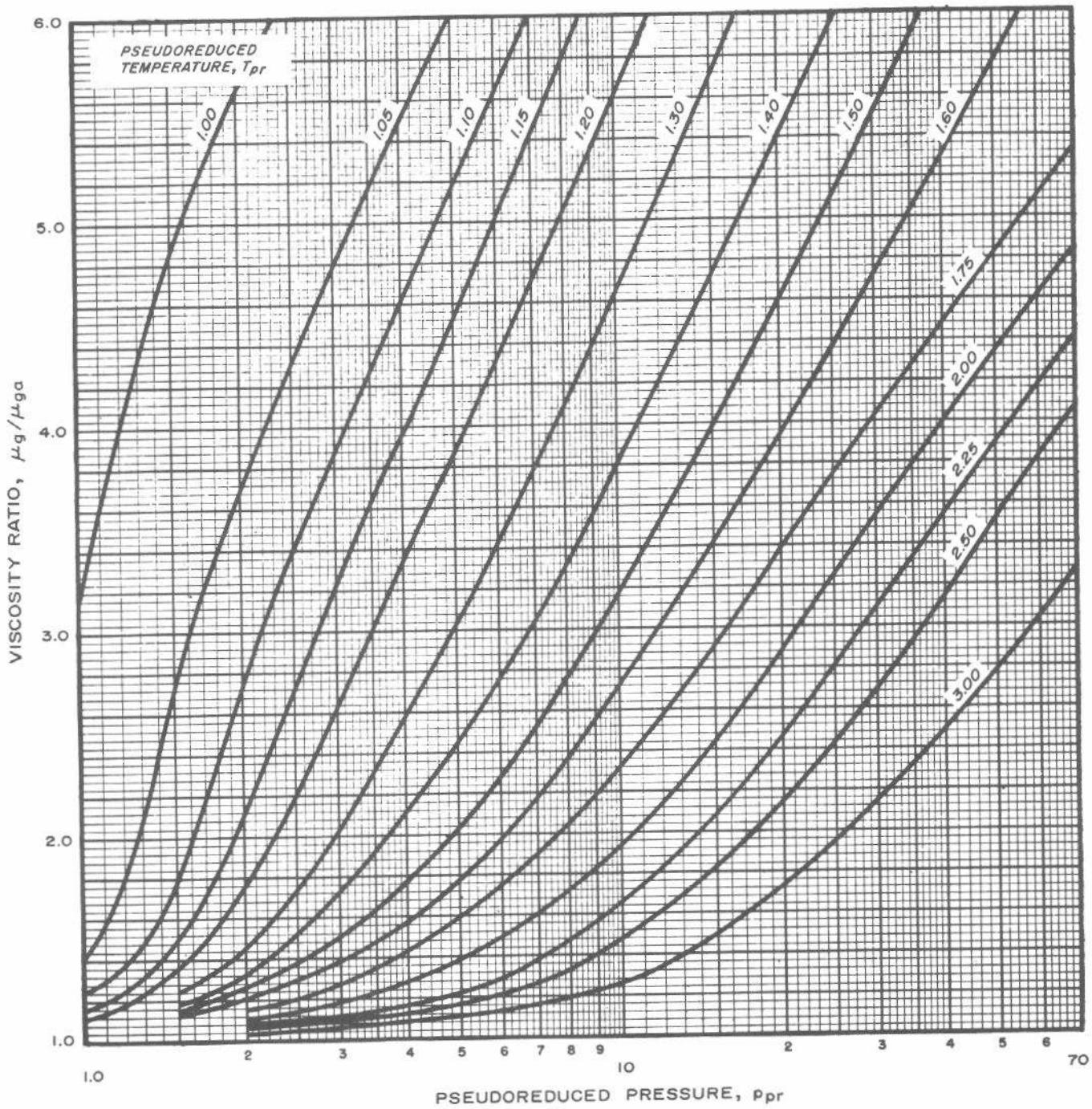


Fig. D.31 Effect of pressure and temperature on gas viscosity; μ_{ga} is estimated from Eq. D.17 or Fig. D.29. After Carr, Kobayashi, and Burrows.¹⁵

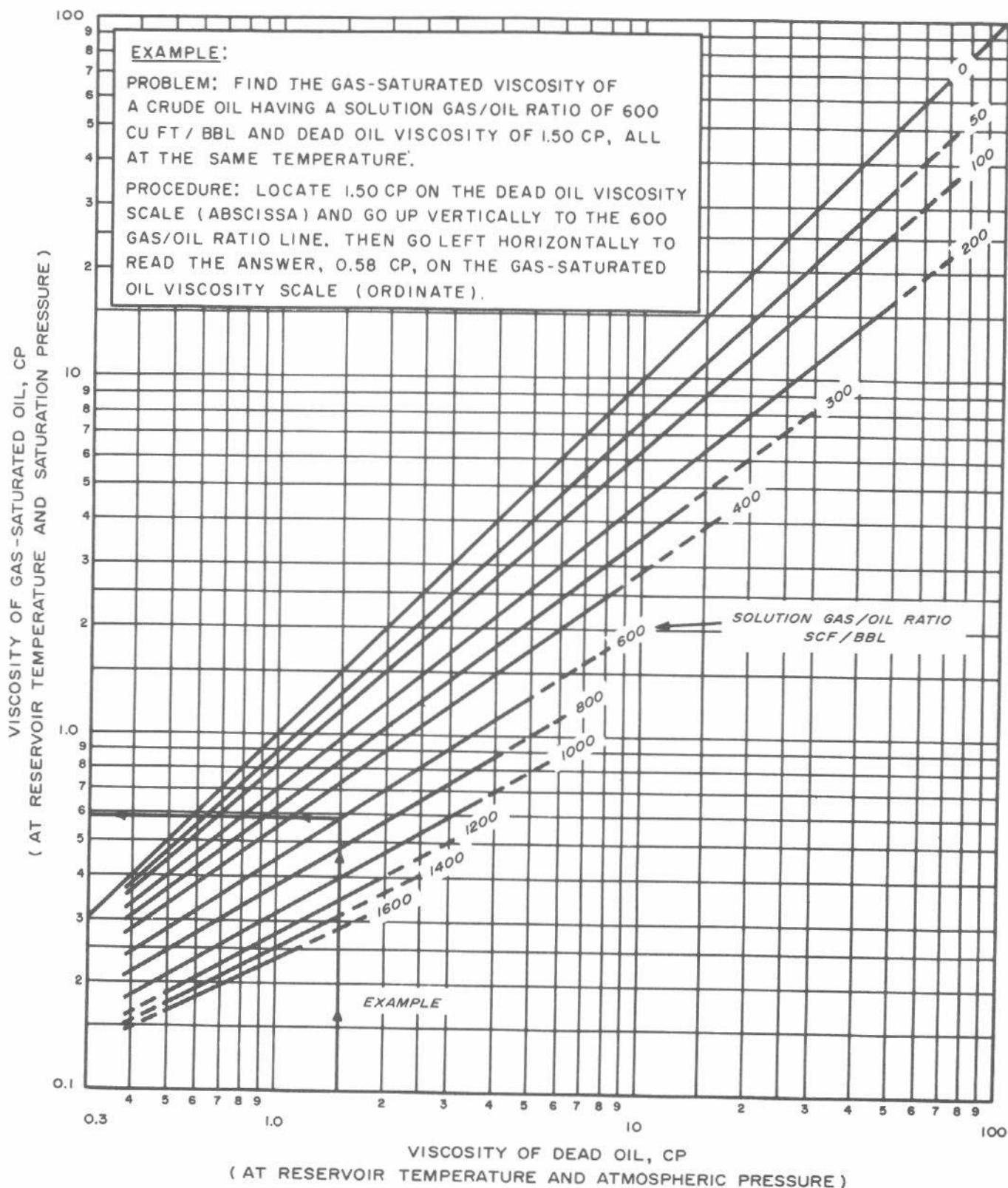


Fig. D.32 Viscosity of gas-saturated crude oil at reservoir temperature and pressure. Dead oil viscosity from laboratory data, or from Fig. D.33. After Chew and Connally.¹⁶

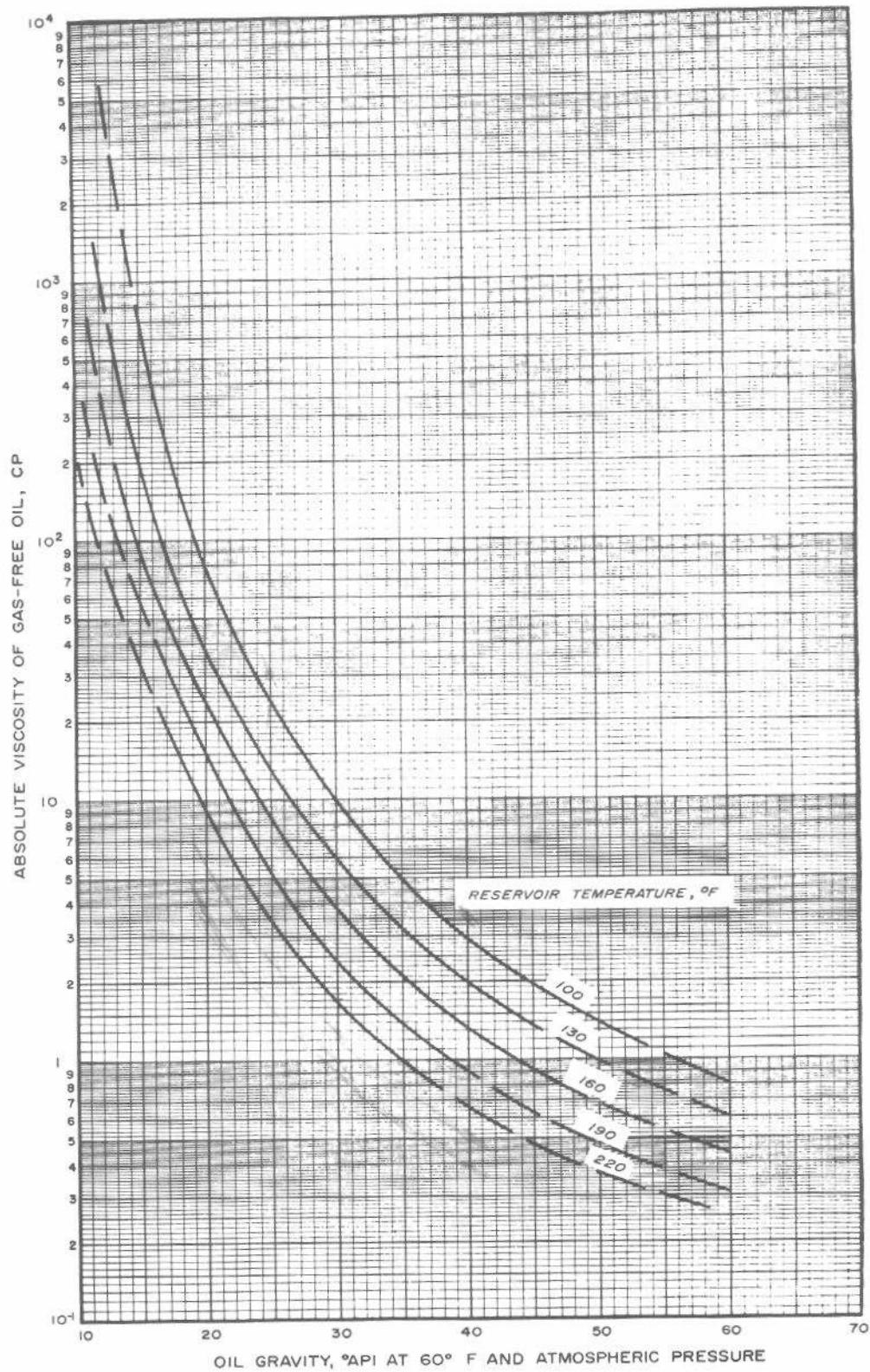


Fig. D.33 Dead oil viscosity at reservoir temperature and atmospheric pressure. After Beal.¹⁷

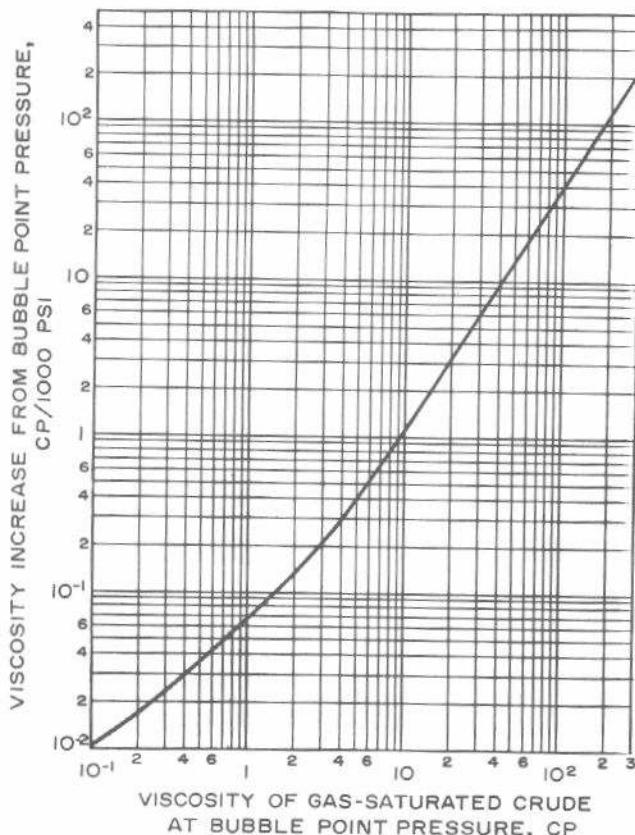


Fig. D.34 Rate of increase of oil viscosity above bubble-point pressure. After Beal.¹⁷

To estimate oil viscosity above the bubble-point pressure, use Fig. D.34. The figure shows the increase in viscosity above the bubble-point viscosity per 1,000 psi. It is based on a small amount of data, and so should be used only as a rough guide.

D.9 Water Viscosity

Fig. D.35 provides a means for estimating water viscosity as a function of salinity and temperature; a pressure correction is included. There are no provisions in Fig. D.35 for modifying the viscosity of water as a function of gas saturation. As for oil, it is best to measure water viscosity as a function of pressure at reservoir temperature. The water should have the gas saturation and salinity existing in the reservoir.

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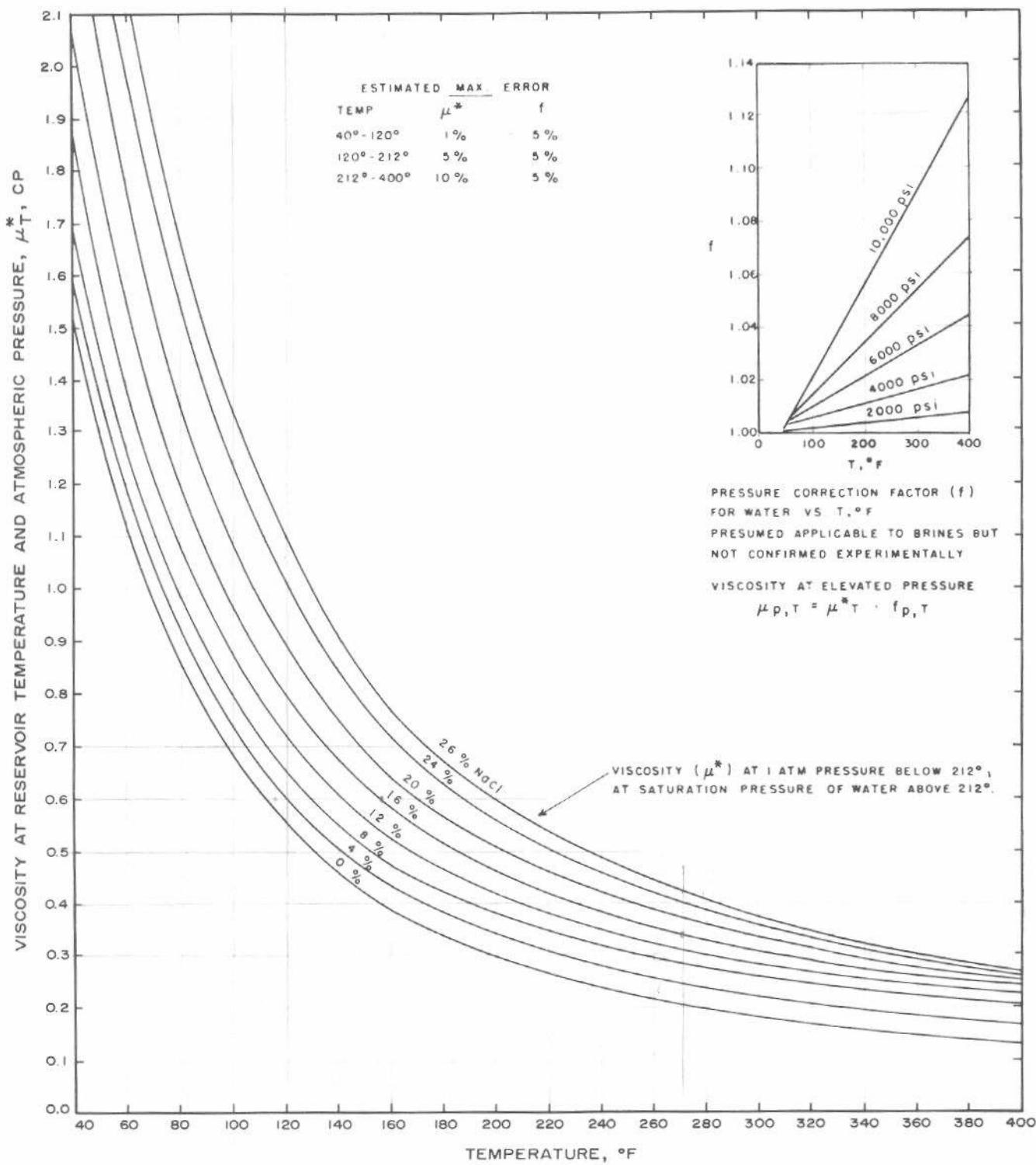


Fig. D.35 Water viscosity at various salinities and temperatures. After Matthews and Russell, data of Chesnut.¹⁸

Appendix E

Summary of Well Test Analysis Methods

E.1 Introduction

This monograph considers many types of well tests and discusses the influence of many factors on well-test response. Although some analysis techniques are unique, most have similarities. This appendix summarizes the equations and data plots for a variety of single-well-test analysis methods. Although the monograph is consistent in nomenclature and sign convention, and although many of the equations used are the same, there are some slight variations from one test type to another that are most clearly described in the tabular form presented here. This appendix also shows buildup-curve shapes resulting from various influences.

E.2 Pressure-Buildup Curve Shapes

Conceptually, graphs of pressure buildup, drawdown, injectivity, or falloff behavior in individual wells can be divided into three areas: (1) front-end effects (wellbore storage, fractures, damage); (2) the semilog straight line for which most analysis techniques apply; and (3) boundary effects. Those three portions are illustrated for a pressure buildup case in Fig. E.1. Throughout the text, criteria have been given for when each effect can be expected to be significant.

Wellbore storage effects always take priority at early time and can completely obscure early-time formation response. Thus, afterflow can be devastating to test analysis methods that depend heavily on early-time data. Fig. E.1 shows that fracture cases and large-negative-skin cases approach the semilog straight line from above when wellbore storage is small. As also shown in that figure, that behavior can be hidden by large wellbore storage effects, so the buildup curve may have the characteristic shape associated with wellbore storage only, or with a positive skin. There is no guarantee that a fractured-well pressure buildup curve will approach the semilog straight line from above.

Analysis methods that use late-time pressure data can be highly sensitive to variations in boundary conditions. Attempting to find a semilog straight line in late-time data affected by boundary or interference effects from offset wells can give highly misleading results. Generally, it is advisable to make time checks to estimate the end of the semilog straight line. Application of late-time analysis

methods, such as the Muskat method, to middle-time data can also give misleading results. In some situations, as when wellbore storage effects are extremely severe or induced fractures are deep ($x_e/x_f < 10$), a classic semilog straight line may never develop, or its slope may be incorrect so that correction factors must be applied to apparent semilog slopes. (Correction factors are given for vertically fractured wells in Section 11.3.) Sometimes type-curve matching techniques may be used for those situations as well as for situations with severe wellbore storage. Curve matching is described in detail in Section 3.3 and is illustrated in other places in the monograph.

E.3 Well-Test Analysis Equations

Table E.1 summarizes analysis equations for unfractured, single-well drawdown, buildup, injectivity, and falloff tests. The equation numbers given in that table refer to the equations listed below. Also shown are the sections containing thorough discussions of the tests, cautions, and alternate analysis techniques. Table E.1 is presented to provide a quick reference and summary only, and should not be used as a replacement for material presented elsewhere in the monograph. To do so blindly will lead to incorrect analysis results.

Time Axis

$$\sum_{j=1}^N \frac{(q_j - q_{j-1})}{q_N} \log(t - t_{j-1}), \dots \quad (E.1)$$

$$\sum_{j=1}^N \frac{q_j}{q_N} \log\left(\frac{t_N - t_{j-1} + \Delta t}{t_N - t_j + \Delta t}\right). \dots \quad (E.2)$$

Permeability

$$k = \frac{-162.6 q B \mu}{m h}, \dots \quad (E.3)$$

$$k = \frac{162.6 B \mu}{m' h}, \dots \quad (E.4)$$

$$k = \frac{162.6 B \mu}{m_q (p_i - p_{wf}) h} \quad \dots \dots \dots \quad (\text{E.5})$$

$$k = \frac{162.6 q B \mu}{m h} \quad \dots \dots \dots \quad (\text{E.6})$$

$$k = \frac{141.2 q B \mu}{h (\bar{p} - p_{ws})_{\text{int}}} p_{D \text{ Mint}}(t_{pDA}). \quad \text{Use Fig. 5.10.} \quad \dots \dots \dots \quad (\text{E.7})$$

Skin Factor

$$s = 1.1513 \left[\frac{p_{1\text{hr}} - p_i}{m} - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.2275 \right] \quad \dots \dots \dots \quad (\text{E.8})$$

$$s = 1.1513 \left[\frac{\Delta p_{1\text{hr}} - p_{ws}(\Delta t=0)}{m} - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.2275 \right] \quad \dots \dots \dots \quad (\text{E.9})$$

$$s = 1.1513 \left[\frac{b'}{m'} - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.2275 \right] \quad \dots \dots \dots \quad (\text{E.10})$$

$$s = 1.1513 \left[\frac{(1/q)_{1\text{hr}}}{m_q} - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.2275 \right] \quad \dots \dots \dots \quad (\text{E.11})$$

$$s = 1.1513 \left[\frac{p_{1\text{hr}} - p_{wf}(\Delta t=0)}{m} - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.2275 \right] \quad \dots \dots \dots \quad (\text{E.12})$$

$$s = 1.1513 \left[\frac{\Delta p_{1\text{hr}}}{m} - \log \left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.2275 \right] \quad \dots \dots \dots \quad (\text{E.13})$$

Connected Pore Volume

$$\phi h A = - \frac{0.23395 q B}{c_t m^*} \quad \dots \dots \dots \quad (\text{E.14})$$

$$\phi h A = - \frac{0.00471 k h}{\mu c_t m_M} \quad \text{Closed square.} \quad \dots \dots \dots \quad (\text{E.15a})$$

$$\phi h A = - \frac{0.00233 k h}{\mu c_t m_M} \quad \text{Square with constant-pressure boundaries.} \quad \dots \dots \dots \quad (\text{E.15b})$$

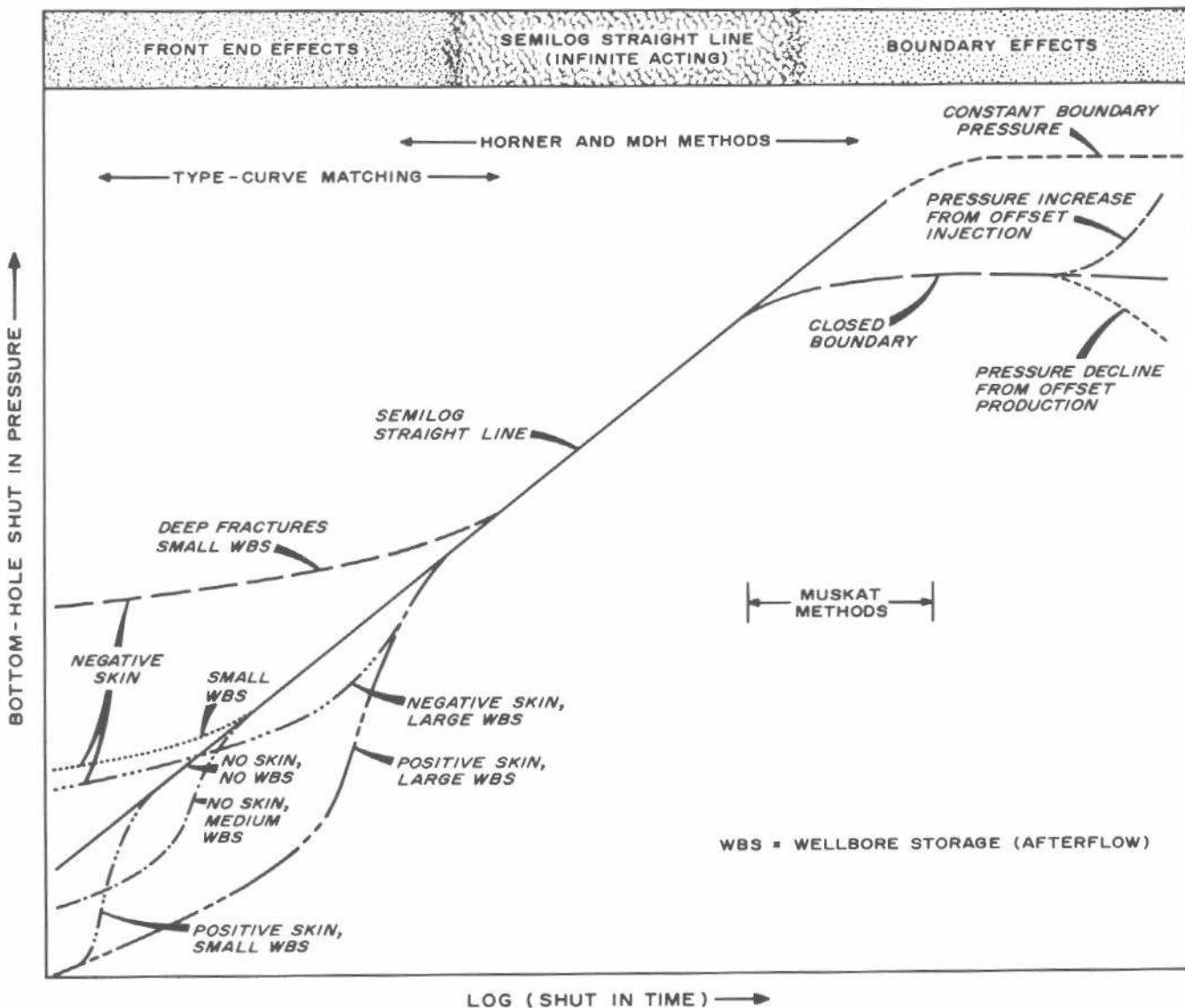


Fig. E.1 Typical bottom-hole pressure-buildup curve shapes. For production at pseudosteady state before shut-in.

TABLE E.1—SUMMARY OF COMMON WELL-TEST ANALYSIS EQUATIONS. EQUATIONS ARE IN SECTION E.3 OR REFERENCED CHAPTER.

Test Type	Analysis Method	Analysis Graph Axes		Slope Data		Connected Pore-Volume Equation		Limits of Straight-Line Equation		Section Number in Text	Comments
		Pressure	Time	Slope	Sign of $m^{(9)}$	Permeability Equation	Skin-Factor Equation	Beginning	End		
Drawdown	MDH	ρ_{wf} $(^1)\rho_{wf}-m^*\Delta t$	$\log \frac{t}{\log \Delta t}$	—	+ m	E.3	E.8	E.16	E.21	3.2	
	Developed system	ρ_{wf}	t	—	+ m	E.3	E.9	N/A	E.21	3.4	
	Reservoir limit	$(\rho_i-\rho_{wf})/q_s$		+	+ m^*	N/A	E.14	E.17	—	3.5	
	Variable rate	Eq. E.1		+	+ m^*	E.4	E.10	N/A	E.21	4.2	Many specialized forms. Line limits approximate. Also curve matching with Fig. 4.12.
Constant pressure	Constant pressure	$1/q$	$\log t$	+	+ m_q	E.5	E.11	N/A	E.18	—	4.6
	Horner	ρ_{ws}	$(^2)\log[(t_p + \Delta t)/\Delta t]$	—	- m	E.6	E.12	N/A	E.19	⁽³⁾ E.22	5.2, 5.3
	MDH	ρ_{ws}	$\log \frac{t}{\log \Delta t}$	+	+ m	⁽⁴⁾ E.7	E.12	N/A	⁽⁴⁾ E.22	5.3	Square system only.
	Muskat	$\log(\bar{p} - p_{ws})$	Δt	—	+ m_M	E.6	E.13	N/A	⁽⁶⁾ E.20	5.3	Line limits approximate.
Buildup	Developed system	$(^7)\Delta p_{\Delta t}$	$\log \frac{\Delta t}{\log \Delta t}$	+	+ m	E.6	E.9	N/A	⁽⁴⁾ E.22	5.3	Line limits approximate.
	Developed system	$(^1)\rho_{ws}-m^*\Delta t$	$\log \Delta t$	+	+ m	E.6	E.9	N/A	⁽⁴⁾ E.22	5.3	
	Variable production rate	ρ_{ws}		—	- m	E.6	E.12	N/A	—	—	Short production time only; see Section 5.4.
	Eq. E.2			+	+ m	E.3	E.8	N/A	E.16	⁽⁸⁾ E.22	
Injectivity	MDH	ρ_{wf} $(^1)\rho_{wf}-m^*\Delta t$	$\log \frac{t}{\log \Delta t}$	+	+ m	E.3	E.9	N/A	E.16	E.21	7.2
	Developed system	ρ_{wf}	t	+	+ m^*	N/A	E.14	E.17	E.21	3.4	
	Reservoir limit	$(\rho_i-\rho_{wf})/q_s$		+	+ m^*	E.4	E.10	N/A	E.16	3.5	
	Variable rate	Eq. E.1		—	+ m_q	E.5	E.11	N/A	E.18	E.21	4.2
Constant pressure	Constant pressure	$1/q$	$\log t$	—					—	—	4.6
	Horner	ρ_{ws}	$(^2)\log[(t_p + \Delta t)/\Delta t]$	+	- m	E.6	E.12	N/A	E.19	⁽³⁾ E.22	7.3
	MDH	ρ_{ws}	$\log \frac{t}{\log \Delta t}$	—	+ m	⁽⁴⁾ E.7	N/A	E.15	⁽⁴⁾ E.22	5.3	
	Muskat	$\log(\bar{p} - p_{ws})$	Δt	+	+ m_M	E.6	E.13	N/A	⁽⁶⁾ E.20	5.3	Square system only.
Falloff	Developed system	$(^7)\Delta p_{\Delta t}$	$\log \frac{\Delta t}{\log \Delta t}$	—	+ m	E.6	E.9	N/A	⁽⁴⁾ E.22	5.3	Line limits approximate.
	Developed system	$(^1)\rho_{ws}-m^*\Delta t$	$\log \Delta t$	—	+ m	E.6	E.9	N/A	⁽⁴⁾ E.22	5.3	Line limits approximate.
	Variable injection rate	ρ_{ws}		—	- m	E.6	E.12	N/A	—	—	Short injection time only; see Section 5.4.
	Eq. E.2			+	- m						

1. Requires linear pressure change before testing.
 2. t_p from Eq. 5.9
 3. Use Fig. 5.6
 4. Use Fig. 5.7
 5. Use Fig. 5.10.
 6. Use Fig. 5.11
 7. Extrapolate prior pressure trend to get $\Delta p/\Delta t$.
 8. Liquid filled, unit mobility ratio assumed.

9. Use as an equation, (slope value) = $\pm 1/n$.

10. Use Fig. 5.11.
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Beginning Time for Analyzed Straight Line

$$t \simeq \frac{(200,000 + 12,000 s) C}{(kh/\mu)} . \quad \dots \quad (\text{E.16})$$

$$t \simeq \frac{\phi \mu c_t A}{0.0002637 k} (t_{DA})_{psl} . \quad \dots \quad (\text{E.17})$$

$$t \simeq \frac{19 \times 10^6 \phi \mu c_t r_w^2}{k} . \quad \dots \quad (\text{E.18})$$

$$\Delta t \simeq \frac{170,000 C e^{0.14s}}{(kh/\mu)} . \quad \dots \quad (\text{E.19})$$

$$\Delta t = \frac{\phi \mu c_t A}{0.0002637 k} (\Delta t_{DA})_{sl} . \quad \text{Use with Fig. 5.11.} \quad \dots \quad (\text{E.20})$$

End Time for Semilog Straight Line

$$t \simeq \frac{\phi \mu c_t A}{0.0002637 k} (t_{DA})_{ela} . \quad \dots \quad (\text{E.21})$$

$$\Delta t \simeq \frac{\phi \mu c_t A}{0.0002637 k} (\Delta t_{DA})_{est} . \quad \text{Use with Figs. 5.6 and 5.7.} \quad \dots \quad (\text{E.22})$$

Nomenclature

a = distance to an image well, Appendix B, ft	h_D = dimensionless thickness for horizontal fracture cases, Appendix C and Section 11.3
a_D = dimensionless distance to an image well, Appendix B	J = productivity index, (STB/D)/psi
A = area, sq ft	J' = modified productivity index for a deliverability test
b = intercept on Cartesian plot of transient-test pressure data, psi	J^* = productivity index for a deliverability test
b' = intercept on semilog plot of transient-test pressure data normalized by rate, psi/(STB/D)	k = permeability, md
B = formation volume factor, RB/STB	k_f = fracture permeability, md
B_g = gas formation volume factor, RB/scf	k_{ma} = matrix permeability, md
B_o = oil formation volume factor, RB/STB	k_{max} = maximum directional permeability, md
B_w = water formation volume factor, RB/STB	k_{min} = minimum directional permeability, md
c = compressibility, psi ⁻¹	k_o = permeability to oil, md
c_f = formation (rock, pore volume) compressibility, psi ⁻¹	k_r = permeability in the radial (horizontal) direction, md
c_g = gas compressibility, psi ⁻¹	k_{rg} = relative permeability to gas, fraction
c_o = oil compressibility, psi ⁻¹	k_{ro} = relative permeability to oil, fraction
c_{oa} = apparent oil-phase compressibility, including effects of dissolved gas, psi ⁻¹	k_{rw} = relative permeability to water, fraction
c_t = system total compressibility, psi ⁻¹ , Eq. 2.38	k_s = permeability in the skin zone
c_w = water compressibility, psi ⁻¹	k_z = permeability in the vertical direction, md
c_{wa} = apparent water-phase compressibility, including effects of dissolved gas, psi ⁻¹	\bar{k} = average permeability for anisotropic system, md
C = wellbore storage constant (coefficient, factor) RB/psi	K = Secenov's coefficient, litre/gram-equivalent
C_A = shape constant or factor	log = logarithm, base 10
C_D = dimensionless wellbore storage constant (coefficient, factor)	ln = logarithm, base e
D = non-Darcy flow coefficient, D/Mcf	L = length or distance, ft
E_k = error in permeability estimated by simplified two-rate test analysis, fraction	m = ± slope of linear portion of semilog plot of pressure transient data, psi/cycle
E_s = error in skin factor estimated by simplified two-rate test analysis, dimensionless skin units	$m(p)$ = real gas "potential" or pseudo pressure, Eq. 2.32, psi ² /cp
Ei = exponential integral, Eq. 2.7	m_H = slope of a Hall plot, psi/(STB/D)
e = 2.7182 . . .	m_M = slope of the straight-line portion of a Muskat plot of pressure buildup data, cycle/hour
erf = error function	m_q = slope of the $1/q$ vs log t plot for a constant-pressure test, (D/STB)/cycle
exp = e	m_{HF} = slope of p_{ws} vs \sqrt{t} plot for horizontal-fracture well test data, psi/ $\sqrt{\text{hours}}$
F_{cor} = correction factor when calculating permeability for a vertically fractured well, Section 11.3	m_{VF} = slope of p_{ws} vs \sqrt{t} plot for vertical-fracture well test data, psi/ $\sqrt{\text{hours}}$
F_R = ratio of porosity-compressibility product of fracture to total porosity-compressibility product of reservoir rock	m' = slope of the data plot for a multiple-rate test, psi/(cycle STB/D)
F_{HL} = Higgins-Leighton shape factor	m'_{12} = slope (based on q_1) of the data plot for a two-rate test, psi/cycle
F' = ratio of pulse length to total cycle length, Eq. 9.13	m'_{34} = slope (based on q_3) of the data plot for a drawdown after a shut-in period, psi/cycle
g = acceleration of gravity, ft/sec ²	m'' = slope of simplified or special data plot for a multiple-rate test, psi/cycle
g_c = units conversion factor, 32.17 lb _m ft/(lb _f sec ²)	m^* = slope of the straight line on a linear plot of p_w vs t , psi/hour
G_p = primal geometric fraction for vertical pulse testing	M = mobility ratio
G_R = reciprocal geometric fraction for vertical pulse testing	M = molecular weight, lb _m /mole
G^* = geometric fraction for vertical interference testing	n = concentration of dissolved solids, gram-equivalents/litre, Appendix D
h = formation thickness, ft	

n = power in productivity-index formula	Δp_M = pressure change from transient test data at the match point for type-curve analysis, psi
p = pressure, psi	Δp_{ow} = pressure drop at a well owing to operation of other wells in the reservoir, psi
p_c = critical pressure, psia	Δp_s = pressure drop across skin, psi
p_D = dimensionless pressure	Δp_{hw} = pressure difference between wellhead and bottom hole, psi
$(p_D)_M$ = dimensionless pressure at the match point for type-curve analysis	Δp_{thr} = pressure difference on straight-line portion of semilog plot 1 hour after beginning a transient test; used in any kind of Δp vs $\log \Delta t$ plot, psi
p_{DMBH} = Matthews-Brons-Hazebroek-type dimensionless pressure	$\Delta p_{\Delta t}$ = difference between observed and extrapolated pressure at time Δt , Eq. 5.22, psi
\bar{p}_{DMBH} = Matthews-Brons-Hazebroek dimensionless pressure for a square, water-drive system based on average pressure	q = flow rate, > 0 for production, < 0 for injection, STB/D for liquid, Mcf/D for gas
p_{DMBHc} = Matthews-Brons-Hazebroek dimensionless pressure for a square, water-drive system based on boundary pressure	q_D = dimensionless flow rate
p_{DMDH} = Miller-Dyes-Hutchinson-type dimensionless pressure	$(q_D)_M$ = dimensionless flow rate at match point for type-curve matching
p_{DMin} = dimensionless pressure of extrapolated straight line at intercept of a Muskat plot	q_g = gas flow rate, Mcf/D
p_{DR} = dimensionless pressure ratio used in type-curve matching DST flow-period data	q_o = oil flow rate, STB/D
\bar{p}_D = dimensionless average reservoir pressure for water-drive reservoir	q_M = flow rate at match point for type-curve matching, STB/D
p_e = external pressure, psi	q_N = flow rate during N th rate period in a variable-rate test, STB/D or Mcf/D
p_{ext} = pressure correctly extrapolated from past behavior, psi	q_{sf} = sand-face flow rate expressed at standard conditions, STB/D
p_{ff} = final flowing pressure in a DST (subscript 1 or 2 indicates flow period), psi	q_w = water flow rate, STB/D
p_{fhm} = final hydrostatic mud pressure in a DST, psi	\bar{q} = average flow rate, STB/D
p_{fsi} = final shut-in pressure in a DST, psi	q^* = modified flow rate for pressure buildup analysis with variable rate before shut-in, STB/D
p_i = initial pressure, psi	$(1/q)_{1hr}$ = ordinate value at 1 hour on straight-line plot of $(1/q)$ vs $\log t$, D/STB
p_{if} = initial flowing pressure in a DST (subscript 1 or 2 indicates flow period), psi	r = radius, ft
p_{ihm} = initial hydrostatic mud pressure in a DST, psi	r_d = radius of drainage as defined in Section 2.12, ft
p_{int} = pressure at intercept (abscissa value = 0) of various kinds of $f(p_w)$ vs $f(t)$ plots, psi	r_D = dimensionless radial distance
p_{isi} = initial shut-in pressure in a DST, psi	r_e = external radius, ft
p_o = pressure in drillstring just before a flow period of a DST, psi	r_f = horizontal fracture radius, ft
p_{pe} = pseudocritical pressure, psia	r_{ff} = radial distance to fluid front number 1, ft
p_{pr} = pseudoreduced pressure	r_{inf} = influence radius for interference testing, ft
p_{sc} = pressure at standard conditions, psi	r_s = radius of skin zone, ft
p_{tf} = tubing or wellhead flowing pressure, psi	r_{wc} = wellbore radius, ft
p_{ts} = tubing or wellhead shut-in pressure, psi	r_{wa} = apparent or effective wellbore radius (includes effects of wellbore damage or improvement), ft
p_w = bottom-hole pressure, psi	r_{wb} = radius to a water bank, ft
$p_{wc}(\Delta t = 0)$ = bottom-hole pressure just before starting a transient well test, psi	R_s = solution gas-oil ratio, scf/STB
p_{wext} = bottom-hole pressure correctly extrapolated from past behavior, psi	R_{sw} = solution gas-water ratio, scf/STB
p_{wf} = flowing bottom-hole pressure, psi	s = van Everdingen-Hurst skin factor
p_{ws} = shut-in bottom-hole pressure, psi	s_{cp} = pseudo skin factor resulting from sand consolidation
p_{thr} = pressure on straight-line portion of semilog plot 1 hour after beginning a transient test; usually a special kind of p_{int} , psi	s_p = pseudo skin factor resulting from partial completion or restricted flow entry
\bar{p} = average reservoir pressure, psi	s_{slip} = pseudo skin factor resulting from slanted well
p^* = false pressure, pressure obtained when linear portion of the plot of p_{ws} vs $\log[(t_p + \Delta t)/\Delta t]$ is extrapolated to $(t_p + \Delta t)/\Delta t = 1$, psi	s'' = additional skin factor resulting from anisotropic effects
δp = pressure offset between two semilog straight lines in transient-test data plot for a naturally fractured system, psi	S_g = gas saturation, fraction
Δp = pressure change (or pulse response amplitude in pulse testing), psi	S_o = oil saturation, fraction
Δp_{hsf} = Δp at beginning of semilog straight line, psi	S_w = water saturation, fraction
Δp_{DV} = dimensionless response amplitude for vertical pulse testing	t = time, hours
$(\Delta p_{DV})_x$ = dimensionless response amplitude for vertical pulse testing in an infinite-acting system	t_{bst} = time to beginning of the semilog straight line, hours
Δp_{el} = Δp at end of linear flow period (half-slope log-log line, or \sqrt{t} straight line) for a vertical fracture, psi	t_D = dimensionless time
	$(t_D/r_D)^2_M$ = dimensionless time parameter from type curve at the match point for type-curve analysis
	t_{DA} = dimensionless time based on drainage area
	t_{de} = dimensionless time based on external radius, r_e
	$(t_{DA})_{pss}$ = dimensionless time at the beginning of pseudosteady-state flow
	t_{Drf} = dimensionless time based on horizontal fracture radius
	t_{Dxf} = dimensionless time based on half-fracture length of a vertical fracture

t_{ein}	= time at the end of the infinite-acting period, hours	$x_e = x$ distance from a centered well to the edge of its square drainage region (half-length of the side of a square), ft
t_{est}	= time to end of the semilog straight line, hours	
t_L	= time lag used in pulse testing, hours	$x_f = x$ distance from a well in the center of a square drainage region to the end of a vertical fracture that is parallel to the x axis (half-length of a vertical fracture), ft
$(t_L)_D$	= dimensionless time lag used in pulse testing	$x' = \text{transformed } x \text{ coordinate for an anisotropic system, ft}$
$(t_L)_\infty$	= time lag in vertical pulse testing for an infinite-acting system, hours	$\Delta x = x$ length of a grid in a reservoir simulator, ft
t_M	= time value from transient-test data at the match point for type-curve analysis, hours	$y = y$ coordinate, ft
t_p	= equivalent time well was on production or injection before shut-in, hours	$y_D = \text{dimensionless } y \text{ coordinate, Fig. 2.15 and Appendix B}$
t_p^*	= modified production time for pressure buildup analysis with variable rate before shut-in, hours	$y_i = \text{mole fraction of Component } i \text{ in the gas phase}$
t_{pDA}	= dimensionless production time based on drainage area	$y' = \text{transformed } y \text{ coordinate in an anisotropic system, ft}$
t_{pss}	= time at the beginning of pseudosteady-state flow, hours	$\Delta y = y$ length of a grid in a reservoir simulator, ft
t_R	= readjustment time, hours	$z = \text{real gas deviation factor}$
t_s	= stabilization time, hours	$z_i = \text{real gas deviation factor at initial conditions}$
t_x	= intersection time of two semilog straight-line segments on transient-test data plot, hours	$\Delta Z_p = \text{vertical distance from upper formation boundary to center of upper perforations; for vertical well testing; Fig. 10.25; ft}$
t_1	= any time in a transient test, hours	$\Delta Z_R = \text{vertical (response) distance between upper and lower perforations; for vertical pulse testing; Fig. 10.25; ft}$
t_2	= any time in a transient test, hours	$\Delta Z_{lf} = \text{vertical distance from lower formation boundary to flow perforations; for vertical well testing; Fig. 10.25; ft}$
t^*	= time that transient-test data start deviating from semilog straight line, hours	$\Delta Z_{ws} = \text{vertical distance from lower formation boundary to observation (static) perforations; for vertical well testing; Fig. 10.25; ft}$
Δt	= running testing time, hours	$\Delta = \text{difference}$
Δt_C	= total cycle length in pulse testing, hours	$\gamma = \text{specific gravity; referenced to water for liquids, to air for gases}$
$(\Delta t_{DA})_{est}$	= dimensionless time at end of Horner or Miller-Dyes-Hutchinson straight line for pressure buildup test analysis	$\epsilon = \text{interporosity flow parameter}$
$(\Delta t_{DA})_{sl}$	= dimensionless time at beginning or end of Muskat straight line for pressure buildup analysis	$\theta = \text{angle between positive } x \text{ axis and direction of } k_{\max} \text{ in an anisotropic reservoir, degrees}$
Δt_{dfx}	= dimensionless intersection time of two semilog straight lines for falloff test in a composite system	$\lambda = \text{mobility, md/cp}$
Δt_{df1}^*	= dimensionless time for deviation of data from first semilog straight line for falloff test in a composite system	$\lambda_g = \text{mobility of gas phase, md/cp}$
Δt_{dyn}	= time for reading dynamic pressure (used in reservoir simulation) from straight line of a buildup plot, hours	$\lambda_o = \text{mobility of oil phase, md/cp}$
Δt_{fx}	= time of intersection of two semilog straight lines for falloff test in a composite system, hours	$\lambda_t = \text{total flowing mobility, md/cp}$
Δt_{f1}^*	= time of deviation of data from first semilog straight line for falloff test in a composite system, hours	$\lambda_w = \text{mobility of water phase, md/cp}$
$(\Delta t)_M$	= time at match point for type-curve matching, hours	$\mu = \text{viscosity, cp}$
$\Delta t_{\bar{p}}$	= shut-in time corresponding to Dietz's average reservoir pressure, hours	$\mu_g = \text{gas viscosity, cp}$
Δt_p	= pulse length used in pulse testing, hours	$\mu_{ga} = \text{gas viscosity at atmospheric pressure and reservoir temperature, cp}$
Δt_{pdv}	= dimensionless pulse length used in vertical pulse testing	$\mu_{gi} = \text{gas viscosity at initial conditions, cp}$
Δt_{si}	= shut-in time before drawdown test, hours	$\mu_o = \text{oil viscosity, cp}$
T	= temperature, °R	$\mu_w = \text{water viscosity, cp}$
T_c	= critical temperature, °R	$\rho = \text{density, lb}_m/\text{cu ft}$
T_{pc}	= pseudocritical temperature, °R	$\rho_w = \text{water density, lb}_m/\text{cu ft}$
T_{pr}	= pseudoreduced temperature	$\phi = \text{porosity, fraction}$
T_{sc}	= temperature at standard conditions, °R	
V	= volume, bbl	
V_p	= pore volume, bbl	
V_{pi}	= drainage pore volume of Well i , bbl	
V_{pt}	= total system pore volume, bbl	
V_p	= volume produced, bbl	
V_u	= wellbore volume per unit length, bbl/ft	
V_w	= wellbore volume, bbl	
ΔV	= change in volume, bbl	
W_i	= cumulative water injection, bbl	
x	= x coordinate, ft	
x_D	= dimensionless x coordinate, Fig. 2.15 and Appendix B	

Subscripts

a	= apparent
b	= base
bsl	= beginning of semilog straight line
C	= calculated
dyn	= dynamic pressure value for use in reservoir simulation
D	= dimensionless
e	= external
$eria$	= end of infinite-acting period
el	= end of linear flow period
esl	= end of straight-line portion
ext	= on extrapolated pressure trend
E	= estimated
f	= flowing
f	= in fracture

F = future
 g = gas
 i = initial, index, component number
int = intercept value, value of ordinate at zero abscissa
 value
 j = index
 ma = in formation matrix
 M = match point in type-curve matching
 n = total in summation
 N = last rate interval in a multiple-rate flow test
 N = total in summation
 N = number of components in a mixture
 o = oil
 OB = observed value

s or si = shut-in or static
 s = skin zone
 sl = beginning or end of straight-line portion
 t = total
 tr = true
 w = water
 w = well
 wb = wellbore
 x = intersection point of two semilog straight-line
 segments on transient-test data plot
 $1hr$ = data from straight-line portion of semilog plot at
 1 hour of test time, extrapolated if necessary
 $1, 2$ = layer, zone numbers, or time numbers
 ∞ = infinite-acting system

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